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Computational Study of Concrete Corbel

Liew Bi Hong[1], **Koh Heng Boon**[1]*

[1]Faculty of Civil Engineering and Built Environment,
Universiti Tun Hussein Onn Malaysia, Parit Raja, Johor, 86400, MALAYSIA

*Corresponding Author Designation

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Abstract: Corbel is a precast connection that protrudes perpendicularly from columns to connect beams and columns together. This paper presents the finite element analysis (FEA) of shallow and deep concrete corbels. The shallow reinforced concrete corbels were analysed using strut-and-tie modelling (STM) and design using Eurocode 2. In the FEA, the concrete corbels were modelled as a surface element subjected to an incremental concentrated load. Nonlinear concrete material was assigned to predict the structural behavior of shallow and deep corbels with various shear span to an effective depth ratio. From the FEA, it was found that the most tensile stress concentration is at the top re-entrant corner for both shallow and deep corbels. The crack was started from the re-entrant corner and extended diagonally to the column.

Keywords: Corbel, Finite Element Analysis, Strut-And-Tie Model

1. Introduction

In conjunction to the Industrialised Building System (IBS) Roadmap 2003-2010 and IBS Roadmap 2011-2015, the construction industry in Malaysia has begun to shift the construction methods from conventional to prefabrication. The growing of precast concrete construction increases the necessity to support members such as reinforced concrete corbel. Corbel is a short cantilever that protrudes perpendicularly from the face of a column to support other structural elements such as beams and slabs. It is one of the most widely used precast elements in the construction industry to connect beams and columns together. Figures 1 and 2 display a single-sided and double-sided reinforced concrete corbels respectively.

The design and construction of a connection like corbel are a vital consideration in precast concrete structures due to their roles to transmit forces between structural members to provide stability and robustness (Elliott, 2016) [1]. Typically, the design of corbels can be simplified by using the strut-and-tie modelling technique. Kassem [2] clarified that the strut-and-tie modelling method is possible to be used for corbel designation as the tension and compression zones arise in corbels can be modelled like a truss. In the tension zones, steel ties are mounted to combat the tensile forces, while compressive struts are represented by concrete between cracks. There are also some tension zones along the corbel

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*Corresponding author: koh@uthm.edu.my

height, but not as concentrated as in the tie region. In this case, some Standard Codes recommend adding additional horizontal stirrups along the corbels to withstand these tensile stresses.

In this paper, a design table for a typical shallow reinforced concrete corbel based upon the strut-and-tie modelling (STM) method in accordance to BS EN 1992-1-1 (Eurocode 2) [3], PD 6687-1:2010 [4] and Elliott & Jolly [5] was produced. The design table can act as a design aid in future prefabrication construction. Moreover, combined with engineering examples from numerous journal articles, books, theses and standard code of practice, the structural behaviours of both shallow and deep corbel are being explored through the finite element analysis software LUSAS Modeller.


(a) (b)

Figure 1: Single-sided reinforced concrete corbel at the basement carpark of Boulevard Shopping Mall, Kuching Sarawak.


(a) (b)

Figure 2: Double-sided reinforced concrete corbel at the basement carpark of The Spring Shopping Mall, Kuching Sarawak.

2. Methodology

2.1 Analysis and design of Shallow Corbels

For the design table, certain parameters are kept constant throughout the analysis to identify the main reinforcements and shear links required by certain shallow reinforced concrete corbel dimensions to withstand the proposed design loads. The characteristic yield strength of steel reinforcement f_{yk} and concrete cover are fixed at 500N/mm^2 and 50mm respectively. Besides, the distance to the effective position of load action a_c is kept constant at 200mm and the ratio of h_y/h_c is 0.5. The reinforcements in the design table are based on main steel bar of 16mm diameter and link size of 12mm diameter.

In the design table, there will be three concrete grades being used. The characteristic compressive cylinder strength of concrete f_{ck} chosen are 40N/mm^2, 45N/mm^2 and 50N/mm^2. This allows engineers to choose suitable and economic designs for reinforced concrete corbel based on a variety of choices of concrete grades.

The manipulated variables in this study are the design load V_{Ed} and corbel front face depth h_y. The variables of the design table are summarised in Table 1. Figure 3 illustrates a simple model of shallow reinforced concrete corbel.

Table 1: Summary of variables for design table

Corbel Depth, h_y (mm)	Design Load, V_{Ed} (kN)
500	300
	400
550	500
	600
600	700
	800

Notes: f_{yk} **= 500 N/mm²** , a_c **= 200 mm** , $\frac{h_y}{h_c}$ **= 0.5** , **Ø**$_{\text{bar}}$ **= 16 mm** , **Ø**$_{\text{link}}$ **= 12 mm**

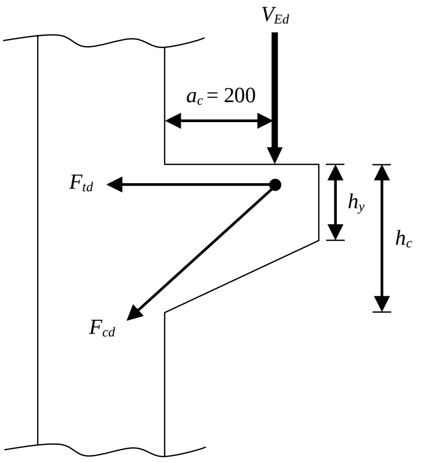


Figure 3: A simple shallow reinforced concrete corbel

2.2 Finite Element Analysis of Shallow and Deep Corbels

Finite element software LUSAS Modeller version 14.1 has been used in the parametric study. The numerical simulation involves both linear elastic analysis and nonlinear analysis. For the linear elastic analysis, concentrated load of 1kN is applied to the node at the upper edge of the corbel, while for the nonlinear plastic analysis, the load is applied incrementally. Since Canha [6] stated that the mesh size does not affect significantly the behaviour of the corbels, medium mesh size which is 50mm is used for the finite element analysis to avoid an excessive computational effort and loss of precision in characterisation of the cracking pattern. An example of finite element model of shallow corbel is illustrated in Figure 4.

In the numerical study, two types of corbels were investigated which are the shallow corbel and deep corbel. The corbels are modelled in 2-dimensional (2D). Besides, the finite element simulation is employed out solely on the corbels connecting to their supporting columns without considering the steel reinforcement. Steel bars are not modelled in the simulation to give an overall idea of the stress distributions and cracking patterns in the corbels when loading is applied.

Few numbers of corbel models are simulated in this study. Corbels with different a_c/h_c ratios ranging from 0.30 – 0.50 are subjected to a concentrated load of 1kN in the linear study and an increment of loadings in the nonlinear control. The f_{ck} is fixed at 40 N/mm^2 in the simulation. Through simulation by using FEA, researcher can identify the stress distribution and cracking patterns over the shallow and deep corbels. The range of a_c/h_c ratios for both shallow and deep corbels set for simulation purpose are shown in Table 2. This can help to discover the critical spots and stress changing in the corbels so that safety measures can be conducted to prevent failures of corbels.

Also, the concept of strut-and-tie modelling is being applied in the finite element analysis for determining the stress distribution, cracking patterns and load-carrying capacity of reinforced concrete corbels. The strut-and-tie developed in the corbel body is portrayed through the tension and compression zones happen in the corbel structure when subjected to a load. A strut-and-tie analogy is shown in Figure 5. Kassem [2] stated that the strut-and-tie modelling (STM) is a general truss analogy in which a structural continuum is transformed into a discrete truss with compressive forces being resisted by concrete and tensile forces by reinforcement. STM is a simple but powerful method which effectively expresses complex stress patterns as triangulated models. It is based on the lower bound theorem of plasticity in which equilibrium is satisfied, the structure has adequate ductility for the assumed struts and ties to develop, and the struts and ties are proportioned to resist their design forces.

Table 2: Range of $\frac{a_c}{h_c}$ for shallow and deep corbels for simulation purpose

Corbels	$\frac{a_c}{h_c}$
Shallow	0.30
	0.40
	0.50
Deep	0.30
	0.40
	0.50

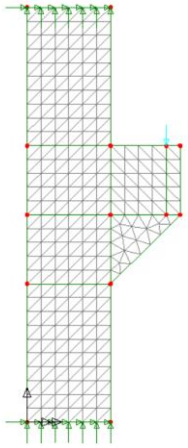


Figure 4: Meshed shallow corbel model by using LUSAS

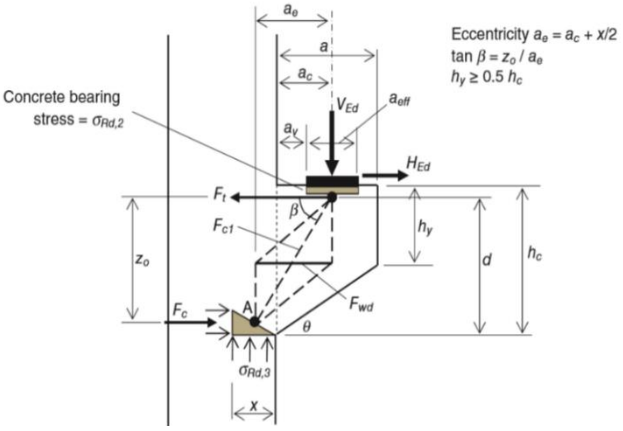


Figure 5: Strut-and-tie analogy [5]

2.3 Finite Element Analysis Validation

In order to validate the numerical analysis, three corbels of different authors are selected to illustrate the stress distribution and cracking pattern of corbel. The numerical simulation provides a good response related to the cracking for the analysed corbel.

Yin et al. [7] analysed the ultimate bearing capacity of deep bracket under normal working condition. Due to the load position near the edge of the corbel, Yin et al. [7] stated that the load creates a certain bending moment on the corbel, as a result the leftmost region of the corbel bears a great tensile tie force. On the other hand, the maximum compression is located along the inclined plane of the corbel, indicating that diagonal compression struts develop. Figure 6 displays the principal stress contour of the deep bracket.

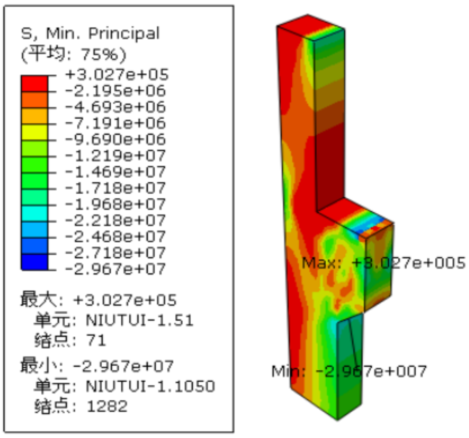


Figure 6: Principal stress contour of deep bracket [7]

Canha et al. [6] compared the experimental results from various literatures with the numerical results generated using the nonlinear finite element program ATENA. Figure 7 presents the cracking, minimum principal stress and strain patterns of shallow corbel YO-E3. Shallow corbel YO-E3 has a lower shear span-to-effective depth ratio a/d. It can be observed that a steep compression stresses and strains fields from the plate to the lower corner of the corbel-column interface. Besides, a smeared cracking in the upper region of the corbel-column junction is observed as the experimental cracking pattern. Therefore, it can be concluded that the cracks started to appear at the contact surface between the column and corbel. The outcome indicates that the intersection is under greater stress than the other region of the corbel model.

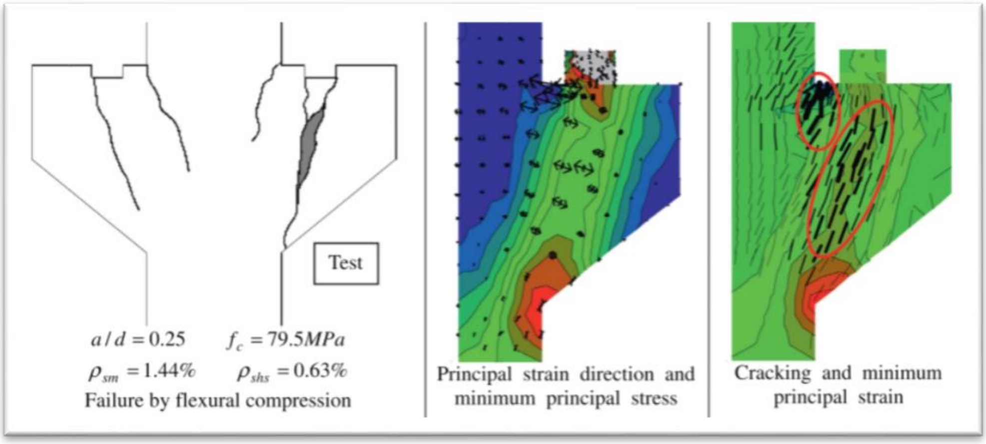


Figure 7: Cracking, minimum principal stress and strain at failure load of corbel YO-E3 [6]

3. Results and Discussion

3.1 Standard Design Table

The section presents the outcome of design and analysis of shallow reinforced concrete corbel based on the strut-and-tie modelling (STM) method in accordance to Eurocode 2 [3], PD 6687-1:2010 [4] and Elliott & Jolly [5]. A Microsoft Excel spreadsheet is created to ease the calculations for different parameters. The design procedures and equations follow those listed in the methodology which include determination of horizontal tie force, sloping angle, main reinforcement, shear link, tension lap and corbel detailing. The standard design table and detailing of shallow reinforced concrete corbel is inserted in Appendix A.

According to the standard design table, when the same concrete grade is being used, the corbel with larger corbel depth can withstand greater design load. The area of main reinforcement required $A_{s,req}$ increases gradually with the design load. Hence, the main reinforcement provided $A_{s,prov}$ also getting bigger to resist the tensile forces and to give sufficient support to the corbel. Also, the shear link provided increases when the design load increases to carry the shear force induced by the applied load onto the corbel.

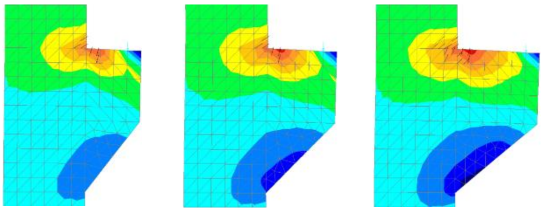
By comparing among the three concrete grades, it can be observed that the main reinforcement required $A_{s,req}$ are almost the same for all the corbel depths and design loads. The differences between them are small, thus the main reinforcement provided $A_{s,prov}$ for the corbels of three different f_{ck} are the same for all. Same goes to the shear reinforcement, they are all the same except the corbel of h_y = 500mm subjected to 800kN load and under f_{ck} of 50N/mm^2. Shear link of 8H12 (905mm^2) is provided as it is adequate to prevent shear failure.

3.2 Parametric Study

The finite element analysis is executed on the shallow and deep corbels of a_c/h_c ratios of 0.3, 0.4 and 0.5. The simulation is implemented exclusively on the corbel and the column supporting it without considering the steel reinforcement and linear elastic steel plates. Material nonlinearity analysis are employed to determine the stress distribution and cracking patterns of the corbel under a number of incremental load factor up to 300. The concrete of the corbels including the columns are meshed using the element size of 50.0mm (medium mesh) to avoid an excessive computational effort and loss of precision in characterization of the cracking patterns. Only one mesh size is used because Canha et al. [6] concluded that the mesh size did not affect the behaviour of the corbel significantly. The element represents a homogeneous region with the same properties. As the third out-of-plane dimension little affects the corbel behavior, only the two-dimensional (2D) analysis is considered.

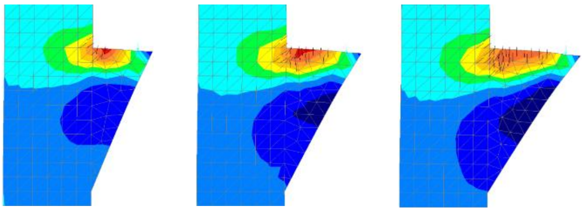
3.2.1 Stress Distribution

The stress distribution of the corbels is portrayed through the colours of contour shown on the corbel models in the software. Through the analysis of the stress distribution on both shallow and deep corbels, it is found that the stress patterns of the corbels of different a_c/h_c ratios is similar when subjected to the same load. Tensile force concentrates at the column-corbel interface and the tension distribution is almost constant between the point of loading and the column face. As stated by Yin et al. [7], due to the load position near the edge of the corbel, the load creates a certain bending moment on the corbel, as a result the leftmost region of the corbel bears a great tensile tie force. On the other hand, the maximum compression is located along the sloping edge of the corbel, indicating that diagonal compression struts develop. Thus, the “tensile stress zone” and “compression stress zone” are formed in the corbel structure [7]. Examination of the stress distribution on corbels of three different a_c/h_c ratios reveals that the greater the a_c/h_c ratio, the greater the area of tension at the column-corbel interface until the ultimate loading at rupture. The tensile stress region is critical and will lead to flexural cracking and deformation.



(a) $\frac{a_c}{h_c} = 0.3$ (b) $\frac{a_c}{h_c} = 0.4$ (c) $\frac{a_c}{h_c} = 0.5$

Figure 8: Stress distribution in shallow corbel of load factor approximately 79.0000



(a) $\frac{a_c}{h_c} = 0.3$ (b) $\frac{a_c}{h_c} = 0.4$ (c) $\frac{a_c}{h_c} = 0.5$

Figure 9: Stress distribution in deep corbel of load factor approximately 73.0000

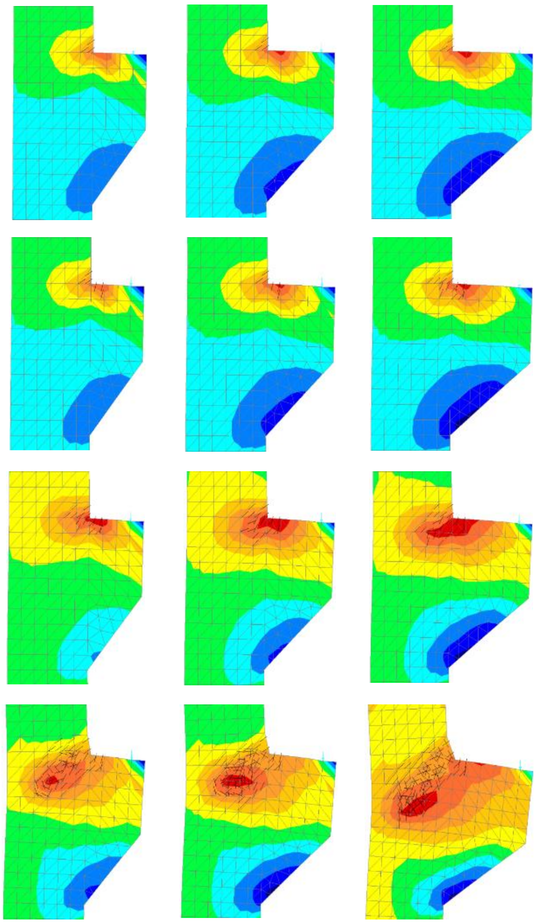
3.2.2 Cracking Patterns

The numerical simulation provides a good response related to the cracking in the shallow and deep shallows being analysed. The first cracking, in addition to the failure loads are recorded. Figure 10 and 11 present the distinctive cracks development between the shallow and deep corbels of various a_c/h_c ratios when the applied axial load is increased.

The overall cracking patterns of all the three shallow models of a_c/h_c ratios of 0.3, 0.4 and 0.5 are the same. The first flexural cracks on the shallow corbels initiate diagonally from the column-corbel junctions when low level of loading is applied. While the first crack sustained spreading along the column face, more and more cracks progress towards the inner part of the column. With more loading, the inclined cracks widened and continued propagating with the appearance of new cracks until failure. Thus, through the material nonlinearity simulation, the hazardous zones in shallow corbels are identified at the column-corbel interfaces. This problem can be solved by installing sufficient normal steel reinforcement to prevent the flexure and shear failures or using prestressed steel strand to strongly constrain the central region of the corbel [7].

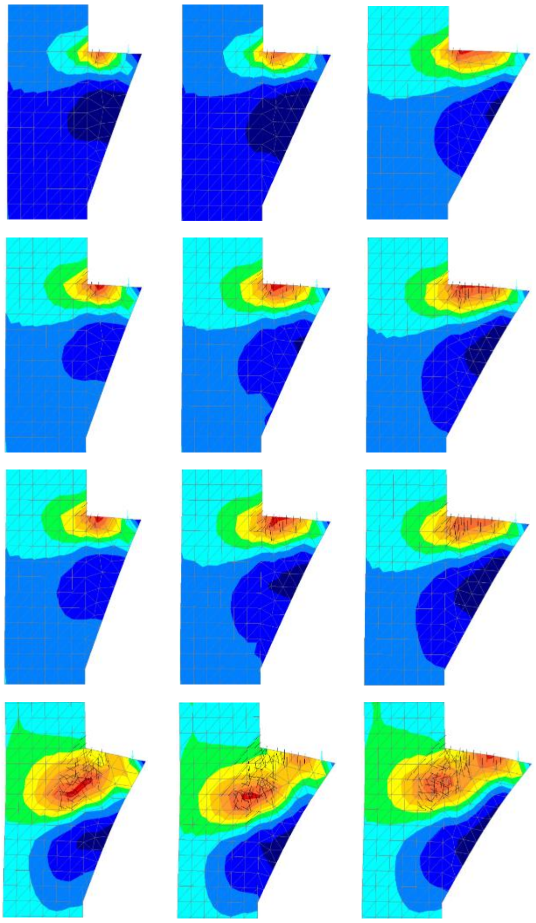
For deep corbels, the diagonal cracks are initially form from the top edge of the corbels near to the columns. The cracks then propagate downwards to the middle region of the corbel bodies and dispersed to the column-corbel interfaces. It is observed that the fractures emerge along the distances between the top edge of the corbels and the points of load action. Hence, through the material nonlinearity analysis, the weak zones in deep corbels are defined at the corbel top edges and column-corbel junctions.

Generally, the examination of cracking patterns for both corbel models indicates that the flexural tension cracks with wide openings propagated and clustered at the column-corbel interfaces as more loading is applied [6], [9]. Besides, diagonal shear cracks that are characterised by small openings are noticed in this failure too. It is also known as flexure shear failure by Fattuhi and Hughes [10]. Canha [6] commented that this is the ideal situation for design, since that this failure is gradual and rather ductile, allowing the repair before the collapse of the structure.



(a) $\frac{a_c}{h_c} = 0.3$ (b) $\frac{a_c}{h_c} = 0.4$ (c) $\frac{a_c}{h_c} = 0.5$

Figure 10: Evolution of cracking in shallow corbel under increment of flexural loading



(a) $\frac{a_c}{h_c} = 0.3$ (b) $\frac{a_c}{h_c} = 0.4$ (c) $\frac{a_c}{h_c} = 0.5$

Figure 11: Evolution of cracking in deep corbel under increment of flexural loading

4. Conclusion

This paper targeted to develop a design table for shallow reinforced concrete corbel. The design table depicts the main reinforcement and shear link of certain corbel depths subjected to the designated loadings. Basically, the deeper the corbel depth, the bigger the reinforcement required to resist flexure and shear deformations.

Besides, LUSAS Modeller is being adopted to implement the finite element analysis (FEA) on both shallow and deep corbels. Material nonlinearity analysis is involved with an incremental loading. The reinforcement of the corbels is ignored in the simulation. The FEA aims to provide an overall idea of the stress distribution and cracking patterns in the shallow and deep corbels of different a_c/h_c ratios. The findings obtained show that as loading is applied to both corbels, the tensions concentrate at the column-corbel interfaces, while the compressions distribute along the sloping edge of the corbels. This indicates the development of the strut-and-tie models in the corbel structures.

The critical regions in both shallow and deep corbels are highlighted at the tension zone where permanent flexural cracks are identified. The cracks in shallow corbel initiates diagonally from the column-corbel interface, while the first cracking in deep corbel forms at the top edge of the corbels near to the column. Basically, under the same load, when the a_c/h_c ratio increases, the deformation of corbel is larger, and more cracks are developed. This will generate a bigger tensile tie force at the column-corbel junction and bigger compression area at the corbel incline edge. Moreover, this paper deduces that corbel with lower a_c/h_c ratio has higher strength as cracking happens at a higher loading value.

5. Recommendation

The future research of reinforced concrete corbel could be directed into two parts which are experimental and analytical works. Firstly, conduct experimental works for both shallow and deep reinforced concrete corbels of various dimensions to simulate actual practices in order to obtain true results and more accurate strength and stiffness of the corbel structures. Furthermore, measure the crack width opening in the corbel critical zones to interpret the variation of flexural and shear failures. Next, implement finite element analysis on both shallow and deep reinforced corbel models with reinforcement detailing to visualise the stress distribution and fractures on the connections more accurately.

Acknowledgement

The authors would also like to thank the Faculty of Civil Engineering and Built Environment, Universiti Tun Hussein Onn Malaysia for its support.

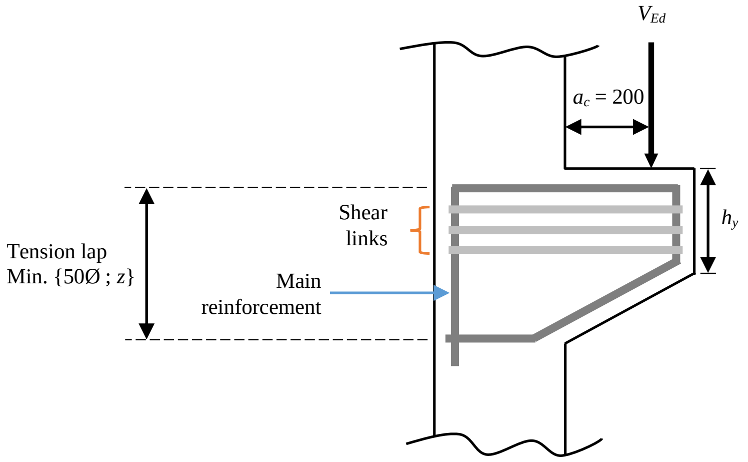
Appendix A

Standard Design Table for Shallow Reinforced Concrete Corbel

f_{ck} (N/mm^2)		40			45			50		
Corbel Depth, h_y (mm)	Design Load, V_{Ed} (kN)	Main Reinforcement		Shear Link	Main Reinforcement		Shear Link	Main Reinforcement		Shear Link
		$A_{s,req}$ (mm^2)	$A_{s,prov}$ (mm^2)	$A_{s,link}$ (mm^2)	$A_{s,req}$ (mm^2)	$A_{s,prov}$ (mm^2)	$A_{s,link}$ (mm^2)	$A_{s,req}$ (mm^2)	$A_{s,prov}$ (mm^2)	$A_{s,link}$ (mm^2)
300	300	445	3H16 (603)	4H12 (453)	441	3H16 (603)	4H12 (453)	438	3H16 (603)	4H12 (453)
	400	613	4H16 (805)	6H12 (679)	606	4H16 (805)	6H12 (679)	600	3H16 (603)	6H12 (679)
400	500	633	4H16 (805)	6H12 (679)	625	4H16 (805)	6H12 (679)	620	4H16 (805)	6H12 (679)
	600	780	4H16 (805)	7H12 (792)	768	4H16 (805)	7H12 (792)	761	4H16 (805)	7H12 (792)
500	700	789	4H16 (805)	7H12 (792)	777	4H16 (805)	7H12 (792)	768	4H16 (805)	7H12 (792)
	800	922	5H16 (1006)	9H12 (1018)	907	5H16 (1006)	9H12 (1018)	896	5H16 (1006)	8H12 (905)

Notes: $f_{yk} = 500$ N/mm^2 , $a_c = 200$ mm , $\frac{h_y}{h_c} = 0.5$, $Ø_{bar} = 16$ mm , $Ø_{link} = 12$ mm

Reinforcement in Shallow Reinforced Concrete Corbel



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