

Empirical Mathematical Model of Multiple Interaction Cracks

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Abstract: As Stress Intensity Factors (SIF) may be used to measure the fracture force of the faulty structure quantitatively, the engineering application of fracture mechanics relies largely on knowledge of stress intensity factors. Many researcher methods have so far been proposed to measure stress intensity factors. Over the past few years, because of their reducing structural integrity, the issue of extended operations of aging aircraft structures is becoming pressing. Therefore, this study aims to investigate the SIF for multiple cracks growth by using software such Microsoft excel. It detailed about the existing study of multiple cracks and how to formulate the empirical mathematical model to predict the SIF. The findings of relationship between independent variable and dependent variable of multiple cracks with different load types. The parameters of the model are a/D and a/b which are crack depth ratio and cracks aspect ratio respectively. By using different cracks shapes of 0.1,0.2,0.3 and 0.4. For a/b , the range are from 0.2 and 1.2 with the increment of 0.2. The variable of c/l is 0.005, 0.01, 0.02,0.04, 0.08,0.16, 0.32. In conclusion, the equation has been made for SIF. The objective was achieved with seven equations are shown in this thesis from different C/L , a/D , a/b values

Keywords: Stress Intensity Factor, Empirical Mathematical Model, Multiple Cracks, Independent Variable, Dependent Variable

1. Introduction

Researchers have done a lot of work to boost the fracture strength of ceramic materials over the years. One of the results is the microcracking technique, which has increasingly been used as a mechanism to improve the fracture strength of ceramic materials. This toughening effect typically occurs as a result of the contact between microcracks. Microcracks may either strengthen or shield a macrocrack, depending on the location and orientation of the microcracks. As a result, the issue of interaction effects between multiple cracks in fracture mechanics has drawn the attention of many researchers [1]. In most situations, due to corrosion and fatigue several cracks are possibly initiated.

Conventional tests are focused on the concept of fracture mechanism based on a single crack configuration for multiple crack problems. Multiple cracks are usually re-evaluated as one bigger crack, following strict rules & requirements, even though they use current design code like BS7910 [2]. Over the past few years, because of their reducing structural integrity, the issue of extended operations of aging aircraft structures is becoming pressing. This reduction is primarily due to many damages at the site, this also occurs in riveting riveted longitudinal and circumferential joints, beginning with the initiation and propagation of crack at multiple locations of fuselage pressure cycling fatigue loads [3]. These numerous cracks may interact, and rapid crack bonding may occur, decreasing the overall structural integrity of the system, which may even lead to catastrophic failure. Prediction of structural residual strength and of the crack rate of growth demand correct measurement of the stress intensity factor (SIF), as it is one of the key parameters in the mechanical analysis of fractures. It determines the stress field close to the crack tip adequately and provides basic details about how the crack propagates.

2. Multiple cracks and Methods

From previous study, A tensile fatigue test was carried out in a servo fatigue electric [4] SDS-100 with maximum tensile strength of 72 KN and frequency of 10 Hz in the air at room temperature. A tensile fatigue test under load control was performed [5]. Two tensile loading block sequences were applied on the specimen in order to create consistent beach marks on the fatigue fracture surface. In the 10th base line block the specimen ultimately failed, suffering a total of 157.8×10^4 cycle [6], with the last beach mark matching the 9th beach loading block. Figure 1 indicates that the fatigue fracture can easily be found in the fatigue zone and the immediate fracture zone.

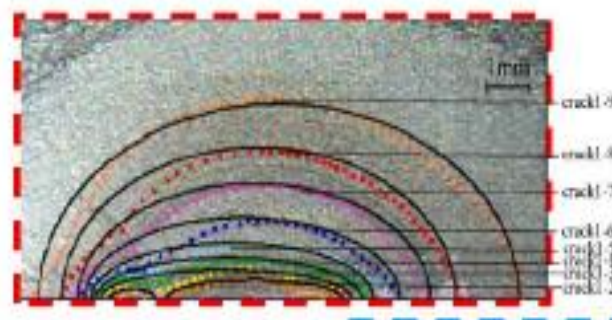


Figure 1: Multiple cracks at specimen

2.1 Multiple regression

The method of statistical analysis used to forecast a consequence of the variable, based on two or more variables, is a multilinear regression. It is also known as multiple regression, and an extension of linear regression. The predictable variable is the dependent variable and is known as the independent or explanatory variables to predict the value of the dependent variable. The multiple linear regression helps analysts to evaluate the model variance and the relative contribution of each independent variable. There are two distinct types of multiple regression, linear and nonlinear.

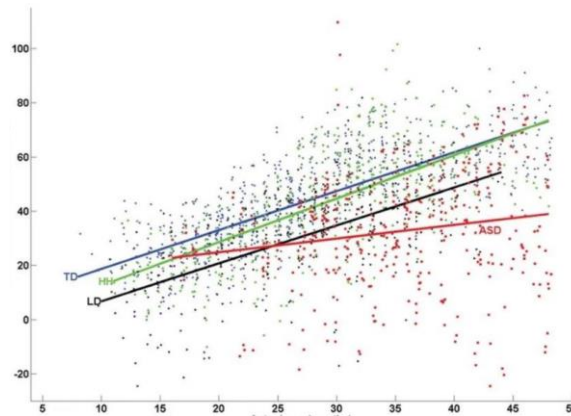


Figure 2: Scatter plot of multiple regression

2.2 Regression analysis

Using Microsoft Excel software, go to data tab data analysis and click regression. Choose the independent variable and dependent variable from the data. Then select the output in the output range box. From the data analysis, the regression table will come out to predict the interaction between the independent variable and dependent variable.

3. Results and Discussion

This chapter presents the findings of relationship between independent variable and dependent variable of multiple cracks with different load types. The parameters of the model are a/D and a/b which are crack depth ratio and cracks aspect ratio respectively. By using different cracks shapes of 0.1, 0.2, 0.3 and 0.4. For a/b , the range are from 0.2 and 1.2 with the increment of 0.2. The variable of c/l is 0.005, 0.01, 0.02, 0.04, 0.08, 0.16, 0.32. In the beginning, the data was taken from Ansys and import into excel to find the relationship between both variables

3.1 Significant regression analysis for multiple cracks

As we can see, Figure 3 is the summary output from $a/D = 0.1$ and $a/b=0.2$ with $C/L=0.005$. From the figure 3, The Multiple R is 95 %, the R Square is 92% and the Adjusted R Square is 90% from 14 observation from this model The Significance F in Anova table shown the regression is equal to $9.01E-07$. The P-value is 0.5147(intercept), 0.1319(variable 1), 0.0128 (variable 2). Figure 4 is the summary output from $a/D=0.1$ and $a/b=0.2$ with $C/L=0.04$. From the figure 4, The Multiple R is 98 %, the R Square is 97% and the Adjusted R Square is 97% from 14 observation from this model. The Significance F in Anova table shown the regression is equal to $4.69E-10$. The P-value is 0.0044(intercept), 0.001(variable 1), 0.0002(variable 2) Figure 5, R is 98 %, the R Square is 97% and the Adjusted R Square is 96% from 14 observation from this model. The Significance F in Anova table shown the regression is equal to $3.63E-09$. The P-value is $4.75E-06$ (intercept), 0.004 (variable 1), 0.0001 (variable 2).

SUMMARY OUTPUT								
	C/L=0.005	A/D=0.1	a/b=0.2					
<i>Regression Statistics</i>								
Multiple R	0.959378							
R Square	0.920407							
Adjusted R Square	0.905936							
Standard Error	0.007287							
Observations	14							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	2	0.006755	0.003378	63.60159	9.01E-07			
Residual	11	0.000584	5.31E-05					
Total	13	0.007339						
	<i>Coefficient</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	0.16082	0.238903	0.673158	0.51474	-0.365	0.686642	-0.365	0.686642
X Variable	0.091342	0.056135	1.627188	0.131976	-0.03221	0.214894	-0.03221	0.214894
X Variable	0.376803	0.126939	2.968385	0.012782	0.097413	0.656194	0.097413	0.656194

Figure 3: Summary output a/D = 0.1 and a/b = 0.2 for C/L = 0.005

SUMMARY OUTPUT								
	C/L=0.04	a/D=0.1	a/b=0.2					
<i>Regression Statistics</i>								
Multiple R	0.989884							
R Square	0.97987							
Adjusted R Square	0.97621							
Standard Error	0.006161							
Observations	14							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	2	0.020323	0.010161	267.7286	4.69E-10			
Residual	11	0.000417	3.8E-05					
Total	13	0.02074						
	<i>Coefficient</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	-2.55903	0.717827	-3.56497	0.004434	-4.13896	-0.97911	-4.138961	-0.97911
X Variable	0.665929	0.150626	4.42109	0.001027	0.334405	0.997454	0.3344046	0.997454
X Variable	1.456475	0.278931	5.221623	0.000285	0.842551	2.070399	0.8425511	2.070399

Figure 4: Summary output a/D = 0.1 and a/b = 0.2 for C/L = 0.04

SUMMARY OUTPUT								
	C/L=0.01	a/D=0.2	a/b=0.2					
<i>Regression Statistics</i>								
Multiple R	0.985291							
R Square	0.970799							
Adjusted R Square	0.965489							
Standard Error	0.001533							
Observations	14							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	2	0.000859	0.00043	182.8473	3.63E-09			
Residual	11	2.58E-05	2.35E-06					
Total	13	0.000885						
	<i>Coefficient</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	0.469291	0.056743	8.270507	4.75E-06	0.344401	0.59418	0.344401	0.59418
X Variable 1	0.032409	0.008979	3.609246	0.004103	0.012645	0.052173	0.012645	0.052173
X Variable 2	0.119412	0.02039	5.856439	0.00011	0.074534	0.164289	0.074534	0.164289

Figure 5: Summary output a/D = 0.2 and a/b = 0.2 for C/L = 0.01

From the regression statistic, we can see Multiple R, R square and Adjusted R square are below 98% to show the model is significance in the 14 observations of variable. From the Anova table, the significance F is below than $\alpha = 0.05$ so that we would rejecting our null hypothesis and we should have a good regression. The value of P-value is lower than 0.05 so the variable is acceptable to put in the equation

3.2 Non-Significant regression analysis for multiple cracks

Based on the Figure 6, it shown the summary output from $a/D = 0.2$ and $a/b=0.6$ with $C/L=0.01$. From the figure 6, The Multiple R is 52%, the R Square is 27% and the Adjusted R Square is 13% from 14 observation from this model The Significance F in Anova table shown the regression is equal to 0.1756. The P-value is $4.8E-08$ (intercept), 0.1639(variable 1), 0.0834 (variable 2). Figure 7 is the summary output from $a/D=0.3$ and $a/b=0.2$ with $C/L=0.32$. From the figure 3.5, The Multiple R is 65 %, the R Square is 43% and the Adjusted R Square is 32% from 14 observation from this model. The Significance F in Anova table shown the regression is equal to 0.0452. The P-value is $7.81E-05$ (intercept), 0.5199(variable 1), 0.3985(variable 2). Figure 8 is the summary output from $a/D=0.4$ and $a/b=0.2$ with $C/L=0.005$. From the figure 8, The Multiple R is 86 %, the R Square is 75% and the Adjusted R Square is 70% from 14 observation from this model. The Significance F in Anova table shown the regression is equal to 0.000461. The P-value is 0.1202(intercept), 0.5684(variable 1), 0.4448(variable 2).

SUMMARY OUTPUT		C/L=0.01	a/D=0.2	a/b=0.6				
Regression Statistics								
Multiple R	0.520711							
R Square	0.27114							
Adjusted R Square	0.13862							
Standard Error	0.001625							
Observations	14							
ANOVA								
	df	SS	MS	F	Significance F			
Regression	2	1.08E-05	5.41E-06	2.046027	0.175608			
Residual	11	2.91E-05	2.64E-06					
Total	13	3.99E-05						
Coefficients								
	Coefficient	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	0.638219	0.04882	13.07282	4.8E-08	0.530766	0.745671	0.530766	0.745671
X Variable	0.018185	0.012193	1.491465	0.16395	-0.00865	0.045021	-0.00865	0.045021
X Variable	0.009568	0.005027	1.903415	0.083466	-0.0015	0.020633	-0.0015	0.020633

Figure 6: Summary output $a/D = 0.2$ and $a/b = 0.6$ for $C/L = 0.01$

SUMMARY OUTPUT		C/L=0.32	a/D=0.3	a/b=0.2				
Regression Statistics								
Multiple R	0.656103							
R Square	0.430471							
Adjusted R Square	0.32692							
Standard Error	0.002103							
Observations	14							
ANOVA								
	df	SS	MS	F	Significance F			
Regression	2	3.68E-05	1.84E-05	4.157103	0.045221			
Residual	11	4.86E-05	4.42E-06					
Total	13	8.54E-05						
Coefficients								
	Coefficient	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	0.891169	0.14626	6.093044	7.81E-05	0.569253	1.213086	0.569253	1.213086
X Variable	0.007781	0.011706	0.664722	0.51992	-0.01798	0.033546	-0.01798	0.033546
X Variable	0.023756	0.027048	0.878296	0.398562	-0.03578	0.083287	-0.03578	0.083287

Figure 7: Summary output $a/D = 0.3$ and $a/b = 0.2$ for $C/L = 0.32$

SUMMARY OUTPUT	C/L=0.005	a/D=0.4	a/b=0.2				
Regression Statistics							
Multiple R	0.867518						
R Square	0.752587						
Adjusted R Square	0.707603						
Standard Error	0.001446						
Observations	14						
ANOVA							
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>		
Regression	2	6.992E-05	3.5E-05	16.73003	0.000461		
Residual	11	2.299E-05	2.09E-06				
Total	13	9.29E-05					
	<i>Coefficient</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>
Intercept	0.50472	0.299625	1.684504	0.120212	-0.15475	1.16419	-0.15475
X Variable	0.014434	0.0245493	0.587978	0.568422	-0.0396	0.068467	-0.0396
X Variable	0.038194	0.0481931	0.792513	0.444818	-0.06788	0.144266	-0.06788

Figure 8: Summary output a/D = 0.4 and a/b = 0.2 for C/L = 0.005

From the regression statistic, we can see Multiple R, R square and Adjusted R square are below 90% to show the model is not significance in the 14 observations of variable. From the Anova table, the significance F is over than $\alpha = 0.05$ so that we would cannot rejecting our null hypothesis and we should not have a good regression. The value of P-value is above that 0.05 so the variable is not acceptable to put in the equation.

4. Conclusion

The results of SIFs were obtained before regression analysis been done, then were normalized to generalize the results to put in the analysis. Before the analysis been done, dependent and independent variables have to choose to predict Stress Intensity Factor (SIF). The objective was achieved but the results were inconsistent. Half of the results were found significant and the other half or non-significant because of the P-value is above 0.05. From the significant regression analysis, the equation has been made for Stress Intensity Factor (SIF). The objective was achieved with seven equations are shown in this thesis from different C/L, a/D, a/b values.

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