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Numerical Analysis in MHD of Casson Fluid Flow Over Stretching Sheet

Muhammad Salman Ahmadi[1], Noorzehan Fazahiyah Md Shab[1*]

[1] *Faculty of Mechanical and Manufacturing Engineering, Universiti Tun Hussein Onn Malaysia (UTHM), 86400 Batu Pahat, Johor, MALAYSIA*

[2]

*Corresponding Author: fazahiya@uthm.edu.my
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Abstract

This paper about study the numerical analysis in MHD of Casson fluid induced by a stretching sheet. Casson fluid plays important roles for application in industry and technology. The aim of the present study is to investigate the physical behaviour of velocity, thermal fields, skin friction and Nusselt number that affected by the parameters (magnetic fields parameter (M), Casson fluid parameter (β) and Prandtl number (Pr)) on Casson fluid that flow over a stretching sheet. First, the governing partial differential equations are transformed into a system of ordinary differential equations by similarity transformation. The system is solved numerically using a shooting method with RK45 with the help of Maple software. Then, velocity and thermal fields are graphically discussed. Furthermore, the numerical results of skin friction and Nusselt number are also shown in the tabular form. As a result, it is revealed that velocity decreased and thermal increased for the larger magnetic field parameter. Moreover, velocity and thermal fields will be decreasing but the skin friction coefficient is increasing when the Casson fluid parameter and Prandtl number are increasing. Nusselt number increases when the larger value Prandtl number is used. The opposite situation is noticed for higher magnetic fields parameter.

1. Introduction

The research background will focus on the fundamental principles and characteristics of fluid flow, Casson fluid, magnetohydrodynamics (MHD), and stretching sheets. This section aims to provide an in-depth understanding of these concepts, their physical behaviors, and their significance in various industries, engineering disciplines, and technological applications. Fluid flow is a motion or movement of a fluid such as liquids and gases [1]. These motions will always move as long as the forces are applied to it. There are different types of fluid flow such as viscous or non-viscous, steady or unsteady flow and others. Application of fluid flow in our daily life including the flowing of the river and blood circulation in the human body. Numerous industries and scientific fields find practical applications for magnetohydrodynamics (MHD). The behaviour, characteristics, and motion of fluids under varied circumstances are covered in this study, which is crucial to many engineering and scientific fields [2]. Recent advances in MHD research demonstrating the versatility and ongoing relevance of MHD in modern science and technology [3]. While laminar flow is one of the several types of fluid flow. Non-Newtonian fluids have a viscosity that changes with the applied stress, whereas Newtonian

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fluids have a constant viscosity regardless of the tension. This different behavior means that non-Newtonian fluids do not obey Newton's viscosity law, which states that the rate of strain is directly proportional to the applied stress [4]. However, non-Newtonian fluids are not subject to this law. Casson fluid is one of the non-Newtonian fluids. Casson fluid is a non-Newtonian fluid differentiated by its rheological behaviour, particularly its yield stress features. Stretching sheet in fluid dynamics is a surface or plane over which fluid flows while simultaneously experiencing stretching or extension in one or more directions [5]. This idea can be illustrated as tension being created along the surface of a thin, elastic material, such as a sheet of rubber or plastic, by pushing or extending it.

2. Mathematical Modelling

Consider the boundary layer of a two-dimensional, steady-state magnetohydrodynamic (MHD) Casson fluid flow across a stretched sheet. According to the model, u and v are the velocity components in the x and y directions, while U_w represents the stretching velocity, which correspond to the Cartesian coordinates measured along the sheet and normal to it, respectively. Additionally, B_0 denotes the applied magnetic field. The flow is generated by the stretching sheet. The effects of Joule heating and viscous dissipation are considered, and a low Magnetic Reynolds number is assumed. Figure 1 shows model of the problem by refer to Rehman *et. al* [6].

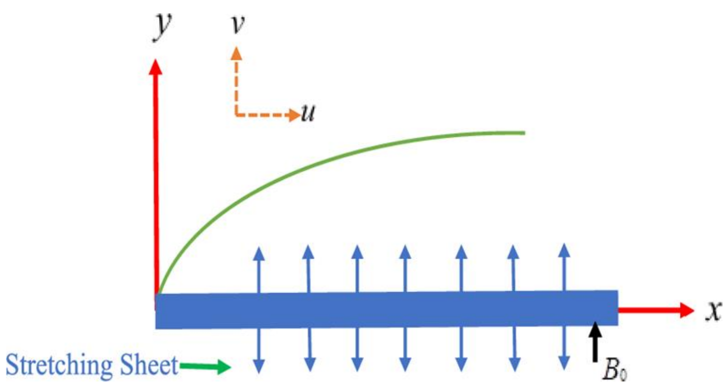

Figure 1. physical model of the problem

2.1 Governing Equation

According to the problem hypothesis, the two-dimensional flow expressions of Tamoor et al. [7] and Mukhopadhyay et al. [8] are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 u}{\partial y^2}\right)\frac{-\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=k\frac{\partial^2 T}{\partial y^2} \tag{3}$$

The boundary conditions are:

$$u=U(x), v=V_w(x)=0, T=T_w(x) \text{ at } y=0,$$
$$U\rightarrow 0, T\rightarrow T_\infty \text{ as } y\rightarrow \infty. \tag{4}$$

v_w represents mass transfer velocity in the equation above. $T_w(x) = T_\infty + \frac{cx^2 T_0}{2v}$, where T_0 represents the reference temperature (slit temperature at x = 0), T_∞ is the continuous free stream temperature, and c is the initial stretching rate. The Casson fluid parameter is $\beta = \frac{\mu_B \sqrt{2\pi_c}}{p_y}$, where μ_B is the plastic dynamic viscosity of the non-Newtonian fluid, π_c is the critical value of this product based on the non-Newtonian model, and p_y is the yield stress of the fluid. In addition, v represents the fluid's kinematic viscosity, ρ its density, σ its constant electrical conductivity, T its temperature, and κ its thermal diffusivity. The sheet emerges from a slit at the origin (x = 0, y = 0) and moves at a non-uniform velocity $U(x) = cx$, where $c > 0$ is a constant.The governing equations (1) - (3), subject to the boundary condition in equation (4), can be written in a simplified manner by introducing the following stream function of ψ in terms of similarity variable and similarity function:

$$u = \frac{\partial x}{\partial y}, v = \frac{-\partial \Psi}{\partial x} \text{ and } \theta(\eta) \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

And with the help of

$$\eta = \sqrt{\frac{c}{v}} y, \tag{6}$$

$$\Psi = \sqrt{vc} x f(\eta), \tag{7}$$

$$T = \left(T_\infty + \frac{T_0 c x^2}{2v}\right) \theta(\eta). \tag{8}$$

2.2 Transformation of partial differential equation to ordinary differential equation

Using similarity transformation to reduce the partial differential equations of the continuity equation, energy equation, and momentum equation, or equations (1) – (3), to their most basic form (ODE).

2.2.1 Method of Solutions

The partial differential equation in equation (2) and (3) were transformed into ordinary differential equations which are

$$f'^2 - f f'' = \left(1 + \frac{1}{\beta}\right) f''' - M f', \tag{9}$$

$$-f \theta' = \left(\frac{1}{Pr}\right) \theta''. \tag{10}$$

Subject to the boundary conditions

$$f' = 1, f = 0, \theta = 1 \text{ at } \eta = 0 \text{ and}$$
$$f' \to 0, \theta \to 0 \text{ as } \eta \to \infty. \tag{11}$$

2.3 Methodology

The study begins by formulating the governing equations. Next, the partial differential equations are transformed into ordinary differential equations through similarity transformations. Then, numerical solutions are obtained using the Runge-Kutta-Fehlberg method with a shooting technique. Shooting method is known as a technique of reducing a boundary value problem to an initial value problem. Considering that it is a trial and error way to get the solution. The boundary value problem will be converted to an initial value problem which can be easily solved by Runge-Kutta-Fehlberg method.

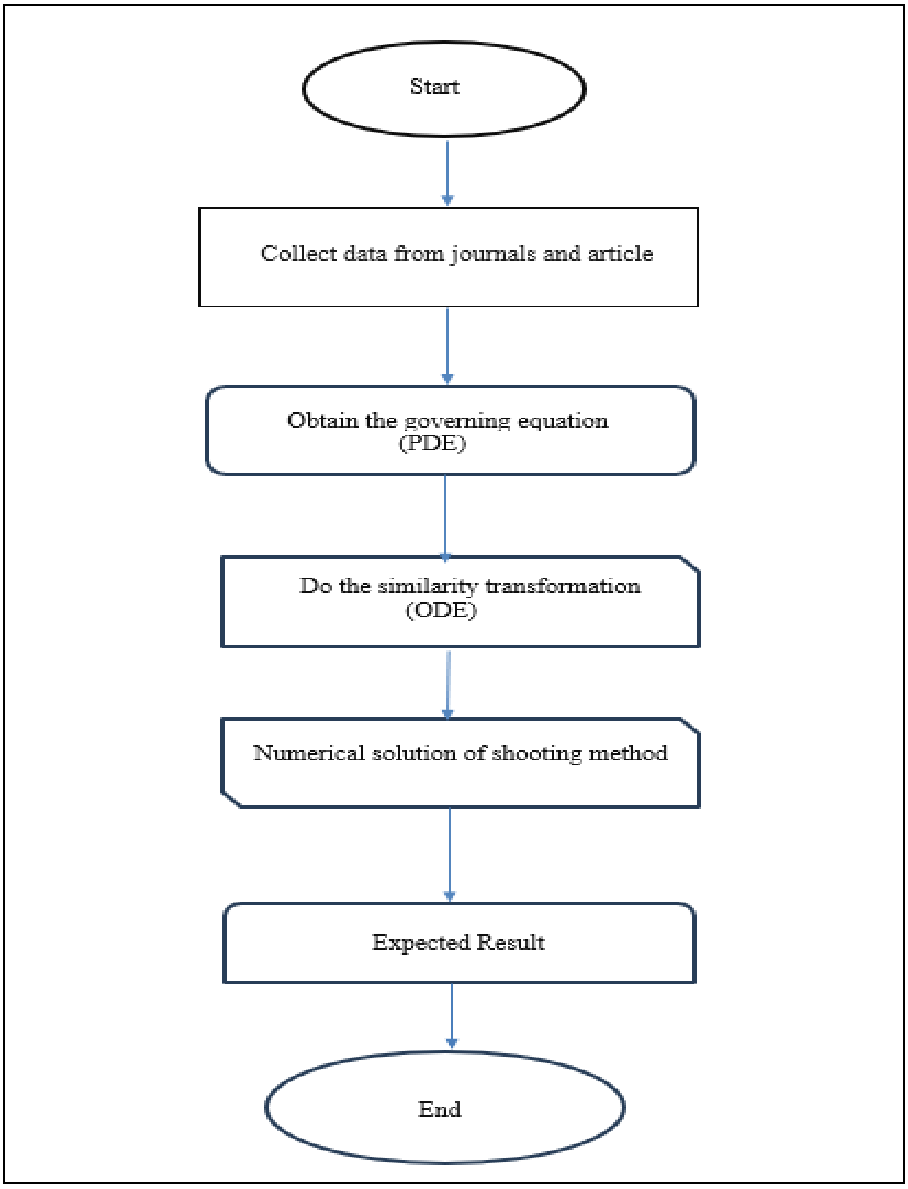

Figure 2: Methodology chart

3. Result and Discussion

Numerical solutions to the governing ordinary differential equations (9) and (10) with the boundary conditions (11) were obtained by Maple2024 using a shooting method with RKF45. The behaviour of physical variables on velocity, thermal fields, skin friction and Nusselt number will be affected by the magnetic field parameter (M), Casson fluid parameter (β) and Prandtl number (Pr), referring [4] of the values to obtain the numerical solution of all the parameters. The velocity and thermal fields graphs for different values of governing parameters were carried out. Figure 3 – Figure 7 verify the roles of all the parameters. The results obtained are displayed through Figure 3 – Figure 4 and Figure 5 – Figure 6 for velocity and thermal fields respectively. Besides, the computed numerical results are recorded in Table 1 and 2 for the behaviour of skin friction and Nusselt number. Table 1 is about the values of skin friction $f''(0)$ for the different values of governing parameters (Casson fluid parameter (β) and magnetic field parameter (M). Moreover, Table 2 is computed about the numerical values of Nusselt number $\theta'(0)$ for the different values of the parameters (magnetic field parameter, M and Prandtl number, Pr).

3.1 The Effects of Parameters on Velocity, $f'(\eta)$

Figure 3 elucidates the behaviour of Casson fluid parameter, β on the velocity, $f'(\eta)$. The different values β are set as β = 0.7, β =1.4, β = 2.0 and β = 3.0. The figure shows that the increasing Casson fluid parameter caused the velocity and thickness of the boundary layer to decrease. As the increment of Casson fluid parameter, the yield stress becomes decreases and as a result, the velocity of the fluid is suppressed. Larger values of the non-Newtonian parameter can produce resistance in the motion of the fluid. Meanwhile, Figure 4 shows the features of the magnetic field parameter, M on the velocity, $f'(\eta)$. The magnetic field parameter, M is set up by referring the values from [4] as M = 0.0, M = 0.4, M = 0.8 and M =1.2. It is of interest to note that both the magnitude of the velocity and boundary layer thickness will be decreased if larger magnetic field parameter is used. This phenomenon occurs when the conductive fluid's magnetic field produces an opposite force on the fluid at the boundary layer called the Lorentz force. This force can slow down the fluid's movement.

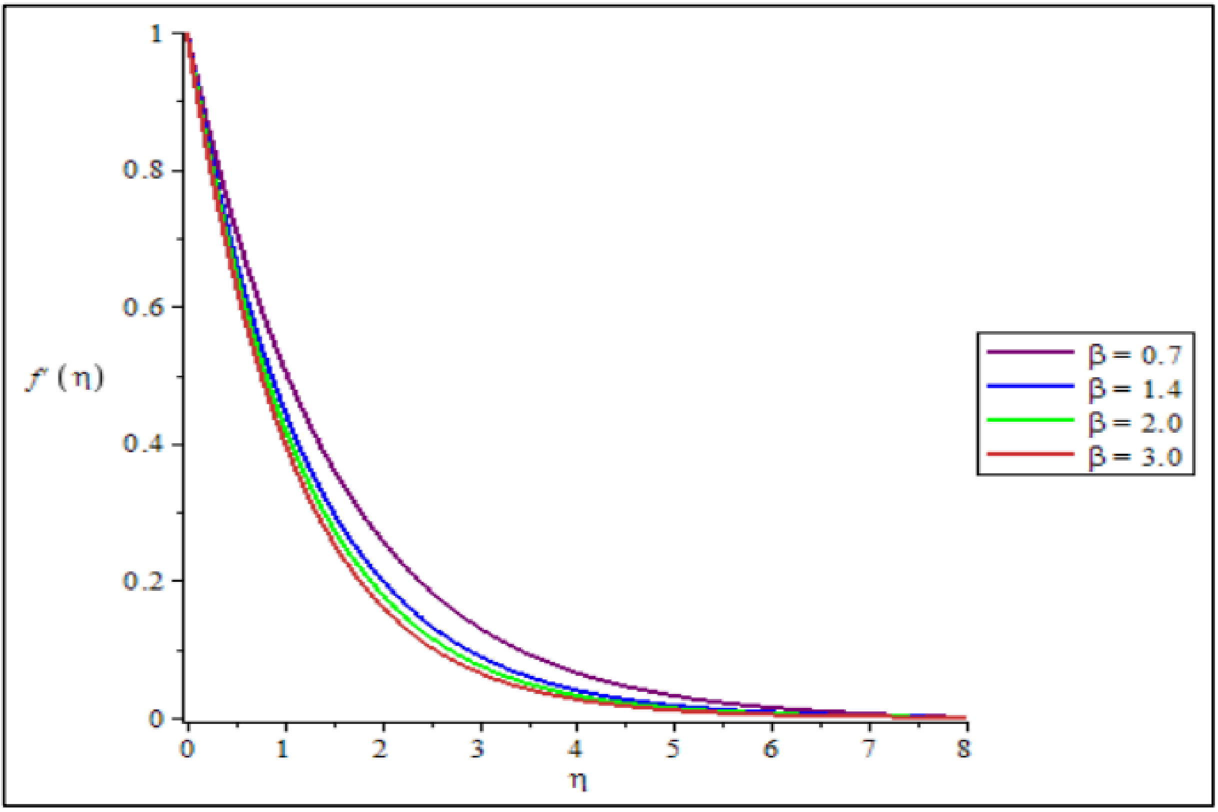

Figure 3 : Impact of Casson fluid parameter, β on the velocity, $f'(\eta)$ when magnetic field parameter, M = 0.1.

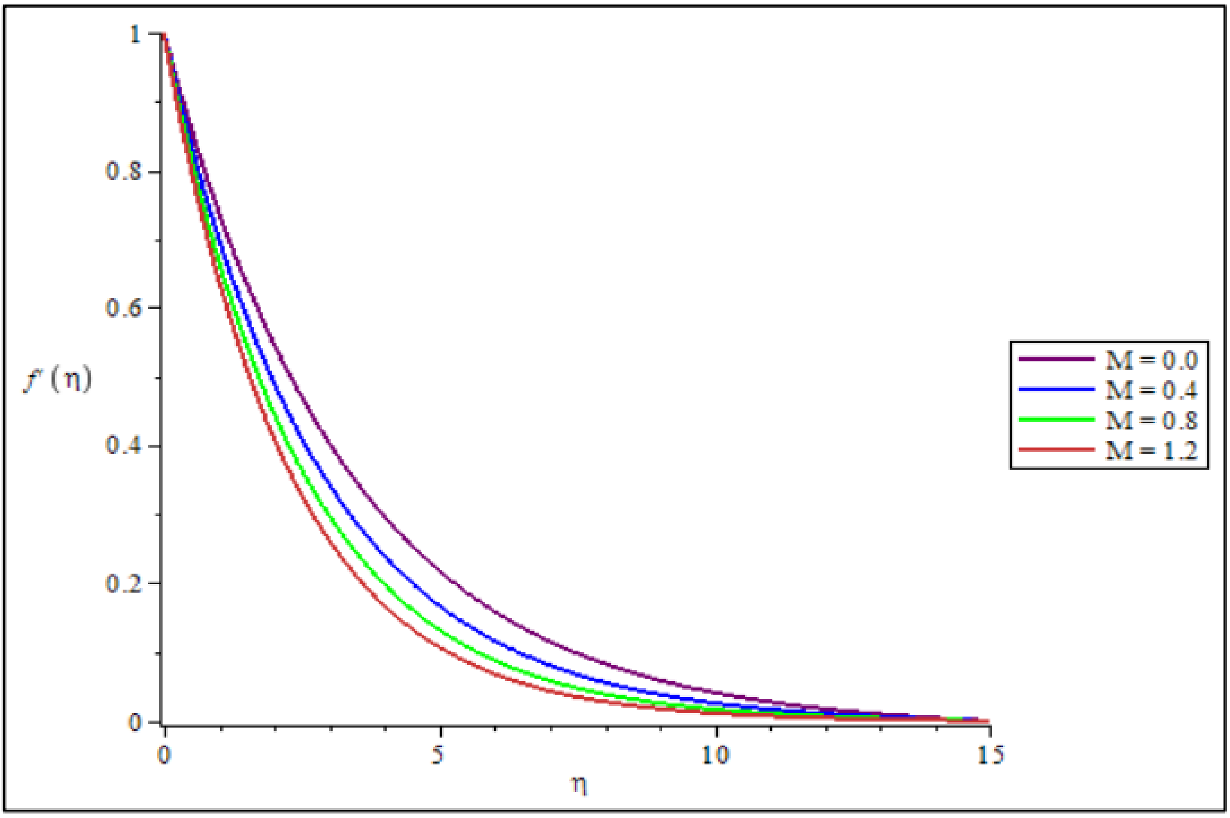

Figure 4: Impact of magnetic field parameter, M on the velocity, $f'(\eta)$ when Casson fluid parameter, $\beta = 0.1$.

3.2 The Effects of Parameters on Thermal Fields, $\theta(\eta)$

Figure 5 is set up that M = 0.0, M = 0.4, M = 0.8 and M =1.2 when the Casson fluid parameter, β = 2.0 and Pr= 1.2. This figure demonstrates that larger magnetic field parameter, M corresponds to higher thermal fields, $\theta(\eta)$ and thickness of boundary layer. In addition, due to the larger magnetic field parameter, the Lorentz force strengthens because it provides resistance to fluid motion and converts some valuable energy into heat. This process is responsible for the thermal field increases. On the other hand, a variation of Prandtl number, Pr on the thermal fields, $\theta(\eta)$ when magnetic field parameter, M =1.0 and the Casson fluid parameter, β = 2.0 is disclosed in Figure 6. The values of the Prandtl number are set up as Pr= 0.9, Pr =1.3, Pr =1.7 and Pr =2.1. It is explored that the increasing of Prandtl number, Pr caused the thermal fields, $\theta(\eta)$ and boundary layer thicknesses to decrease. Prandtl number is a thermal diffusivity ratio of kinematic viscosity. Therefore, it reduces the thermal diffusivity for higher Pr values.

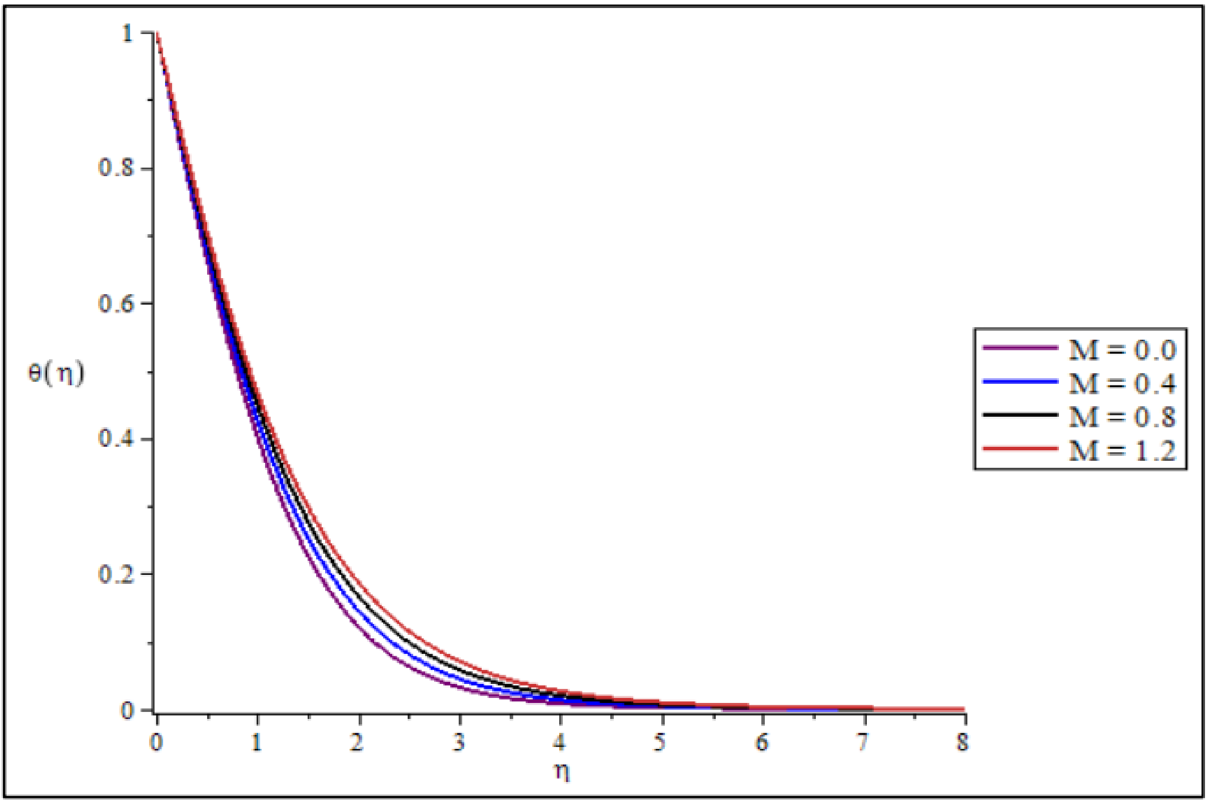

Figure 5: Impact of magnetic field parameter, M on the thermal fields, $\theta(\eta)$ when Casson fluid parameter, β = 2.0 and Prandtl number, Pr 1.2.

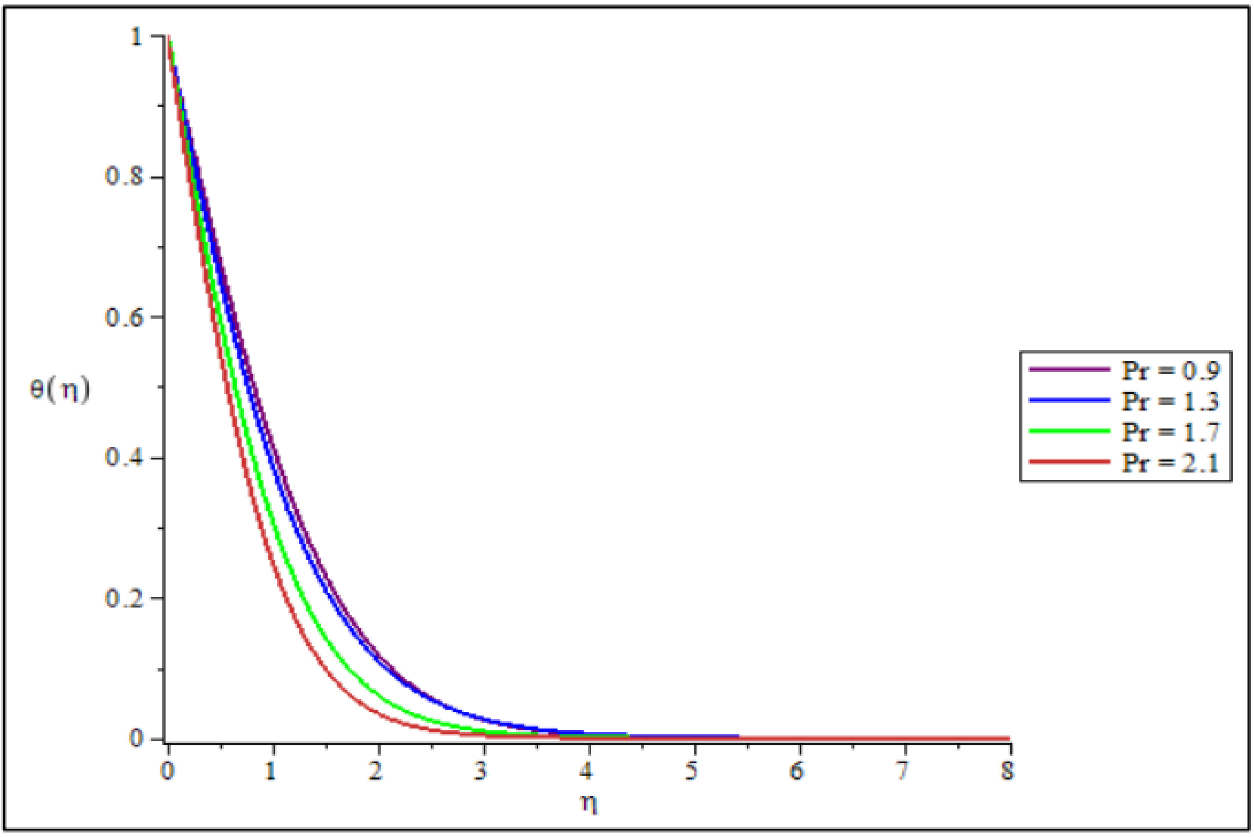

Figure 6: Impact of Prandtl number, Pr on the thermal fields, $\theta(\eta)$ when magnetic field parameter, M = 0.1 and Casson fluid parameter, β = 2.0

3.3 Skin Friction and Nusselt Number

Table 1 and 2 discuss the effects of governing parameters on skin friction coefficient $f''(0)$ and Nusselt number $\theta'(0)$. The behaviour of skin friction coefficient and Nusselt number are determined by Maple18 using shooting method in a tabular form for different values of physical variables.

β	M	$-f''(0)$
1.0	0.2	0.7748
1.2	0.2	0.8092
1.4	0.2	0.8368
2.0	0.2	0.8945
2.0	0.0	0.8167
2.0	0.3	0.9310
2.0	0.6	1.0328

Table 1: Values of skin friction $f''(0)$ at the wall for the different values of governing parameters (Casson fluid parameter, β and magnetic field parameter, M)

M	Pr	$-\theta'(0)$
0.1	1.2	0.6882
0.2	1.2	0.6800
0.3	1.2	0.6722
0.0	1.2	0.6967
0.4	1.2	0.6648
0.5	1.2	0.6576
0.5	1.3	0.6931
0.5	1.4	0.7273
0.5	1.5	0.7603
0.6	1.6	0.7852
0.7	1.6	0.7784
0.8	1.6	0.7718
0.9	1.6	0.7654
1.0	1.6	0.7593
2.0	1.6	0.7058

Table 2: Numerical values of Nusselt number $\theta'(0)$ for the different values of the variables (magnetic field parameter, M and Prandtl number, Pr)

Finally, the behaviour of the skin friction coefficient is determined for the Casson fluid parameter, β and magnetic field parameter, M that constructed in Table 1. The coefficient of skin friction $f''(0)$ is calculated for the different values of β and M when the Prandtl number, Pr = 1.2. Based on Table 1, increasing the values of Casson fluid parameter, β and magnetic field parameter, M lead to skin friction to increase. The larger skin friction will produce more drag force caused by the friction of fluid along the surface of the stretching sheet. It found that non-Newtonian fluid has more friction because of the effects of viscosity.

Meanwhile Table 2 shows the numerical values of Nusselt number $\theta'(0)$ for the different values of the variables M and Pr when Casson fluid parameter, β = 2.0. It is clearly illustrated that the Nusselt number will be increased if the larger values of the Prandtl number is used. However, the opposite situation for the higher M is noticed. If the magnetic field parameter, M is increased, the Nusselt number will decrease. In conclusion, all the parameters β, M and Pr have their own role that affects the velocity and temperature.

4. Conclusion

In this study, magnetohydrodynamic (MHD) effects in stretching sheet flow of Casson fluid is investigated to analyse the behaviour of velocity and thermal fields. The problem has been solved using a shooting method with RK45 numerically in Maple2024. Transformation of the governing partial differential equations into ordinary differential equations (2.2) and (2.3) with the boundary conditions (2.9) by a similarity transformation was obtained through Maple2024. The main research objective is to obtain the numerical solutions that represent velocity, thermal fields, skin friction and Nusselt number and to investigate the effects of the magnetic field parameter, Casson fluid parameter and Prandtl number. The effects of all the parameters on velocity, thermal fields, skin friction and Nusselt number were investigated and discussed. From the result it can be concluded that the increment of Casson fluid parameter, β the yield stress become decreases and as the result the velocity of the fluid and thickness of the boundary layer decreases. Besides that, both the magnitude of the velocity and boundary layer thickness will be decreased if larger magnetic field parameter, M is used. For thermal fields, larger the magnetic field parameter, M corresponds to higher thermal fields and thickness of the boundary layer. However, the increasing of Prandtl number ,Pr may cause the thermal field to decrease. Last but not least, the increasing values of Casson fluid parameter, β and magnetic field parameter, M lead to skin friction to increase. Then, the behaviour of the Nusselt number will be increased if the larger values of the Prandtl number, PR is used but the opposite situation occurs for the higher magnetic field parameter, M.

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Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Muhammad Salman Ahmadi, Noorzehan Fazahiyah Md Shab;* ***data collection:*** *Muhammad Salman Ahmadi;* ***analysis and interpretation of results:*** *Muhammad Salman Ahmadi, Noorzehan Fazahiyah Md Shab;* ***draft manuscript preparation:*** *Muhammad Salman Ahmadi, Noorzehan Fazahiyah Md Shab. All authors reviewed the results and approved the final version of the manuscript.*

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