

Investigation of Explicit Analytical Solution for Colebrook - White Equation of Airflow in Circular Pipe with or without Air Filter

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Abstract

One popular model is the Colebrook-White equation, which is used to determine the friction factor in turbulent flow through rough pipes. Yet, because it is implicit, many explicit approximations have been developed to make its use simpler. As the bulk of known explicit formulations for the Colebrook-White equation, thirty "30" explicit analytical solutions are the focus of this work, which tries to examine and assess their accuracy. The research utilises a methodical assessment technique that is predicated on several factors, such as formula uncomplicatedness, maximum deviations, and comprehensive coverage of the Moody diagram spectrum. As a result of the study, which ranks the examined models from highest to lowest accuracy, the most realistic and correct explicit solutions are found. The results provide guidance on the selection of the appropriate explicit approximation for various engineering applications and flow conditions, which benefits researchers and engineers working in fluid flow-related fields. They also advance the understanding and application of explicit analytical solutions for the Colebrook-White equation.

1. Introduction

Colebrook - White equation is a method that is used to calculate the friction factor of airflow in circular pipe. The Colebrook-White equation is commonly utilised in various engineering domains where fluid flow might occur, including mechanical engineering for pipe flow analysis. Since 1947, several formulations have been put forth to make the Colebrook-White equation more practical, eliminate the need for iterative techniques, and simplify the determination of the friction factor. Thirty (30) equations are listed, representing most of the explicit formulae currently used for calculating the friction factor for turbulent flow in rough pipes. This work aims to evaluate the correctness of each model and provide a ranking from highest to lowest accuracy based on a suggested technique that combines three criteria: formula simplicity, maximum deviations, and coverage of the whole Moody diagram range [1].

2. Methodology

This chapter includes methodology, an analytical framework consisting of techniques for acquiring data, and research validity. The study reviewed and evaluated a number of explicit formulae for calculating the friction

factor in flow pipes. These ten explicit equations will be compared with the Colebrook-White equation to see which equation is the most suitable for calculating the friction factor in a flow pipe. Three criteria were used to evaluate the formulas: coverage of the whole Moody's diagram range, correctness, and simplicity. After establishing the greatest relative error for each approximation, two tests were conducted, one on the suggested range of the approximation and the other on the full range of Moody's diagram. Ten's equations emerged from the comparison study as the best approximations. Table 1 shows the ten formula simplified from thirty sample equations.

Table 1 Ten (10) of explicit equation that has Error ($\Delta f/f$)% from 0.146% to 2.13%

No	Author	Error ($\frac{\Delta f}{f}$)%
1	Shacham correlation (1980)	0.88%
2	Zigrang – Sylvester (1982)	0.17%
3	Haaland (1983)	1.41%
4	Romeo et al. (2002)	0.16%
5	Achour (2002)	2%
6	Vatankhah and kouchakzadeh (2009)	0.16%
7	Papaevangelou correlation (2010)	0.76%
8	Fang correlation (2011)	0.54%
9	Brkic correlation (2011)	2.13%
10	Ali R. Vatankhah formula (2014)	0.146%

Local Exhaust Ventilation is designed to determine the airflow velocity that satisfied all the correlations above. Several locations must be found to get velocity data in order to determine the velocity distribution inside local exhaust ventilation. Because the velocity distribution inside local exhaust ventilation is not uniform for each part, many locations must be established in order to get correct velocity data. Every point in this research study has to be situated in the middle of each pipe. A Hot Wire Anemometer is being utilised to measure the velocity distribution. Every position will be marked, and a hole will be drilled to insert the anemometer probe. The locations of the LEV's points are displayed in Fig. 1.

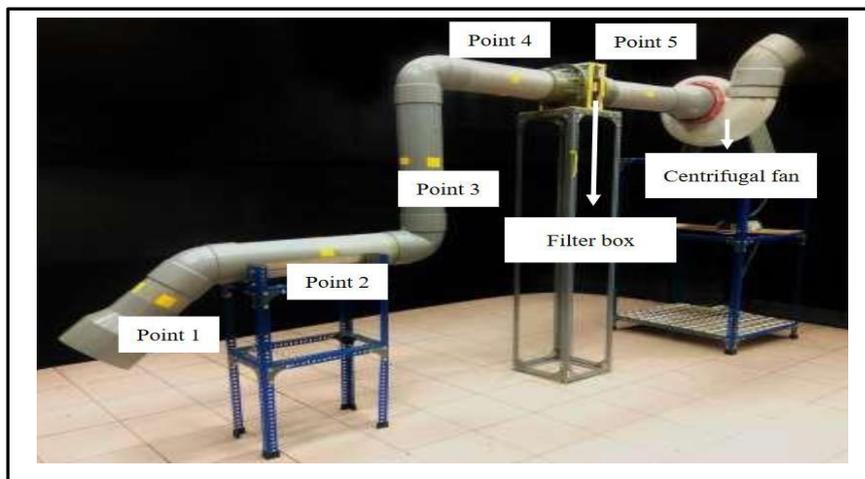


Fig. 1 Exact locations of Local Exhaust Ventilation

Point 1, as indicated in the above diagram, is located at the side of the local exhaust ventilation, near the middle of a 210mm PVC pipe between the 45° elbow pipe and the catch hood. Points 2 are situated in the middle of a 640mm PVC pipe, which is situated between 45° and 90° elbow pipes. Points 3 are, therefore, situated in the middle of a 670mm PVC pipe, perpendicular to the 640mm PVC pipe adjustment. Two 90° elbow pipes are linked to a 670mm PVC pipe for the up and down. Point 4 is at the 1145mm PVC pipes, which are situated between the blower and the 90° elbow pipe. Point 5 is located after the filter box and near the centrifugal fan.

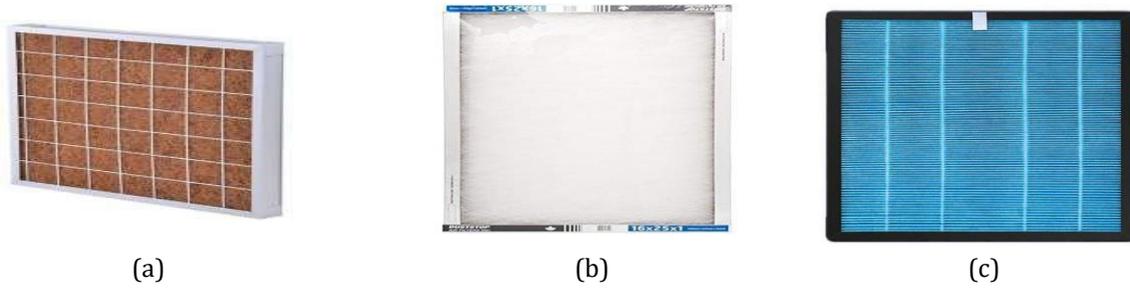


Fig. 2 Types of air filters (a) Coconut Husk filter, (b) Fibreglass filter, (c) HEPA filter

The types of air filters (Fig. 2), such as No filter, Coconut Husk filter, Fibreglass filter and HEPA filter, are used to determine the air filter's suction that affects the airflow velocity. The fan speed (RPM) is also being manipulated from 500 to 1488 to see how the fan speed affects the type of airflow, either laminar or turbulent flow.

For calculating Reynold number, friction factor, and relative error for each correlation, formulas in a spreadsheet such as Microsoft Excel were used to analyse and evaluate the explicit formulae for estimating the friction factor in flow pipes. "A formula in Microsoft Excel is a mathematical phrase that is used to operate on data or carry out computations inside of a spreadsheet," the sentence writes. Excel formulas for equations start with the equal sign (=) and can have a range of values, operators, cell references, and functions. Many functions that are already included in Excel can be used in formulas to do specific tasks or calculations. These functions can perform operations such as summation, average calculation, cell counting, and more [2].

3. Result and discussion

The study examines the investigated relationship between the Reynold number and fan speed (Fig. 3), whether the airflow is laminar or turbulent, and the relationship between the correlations (friction factor) without or with the air filter. The experiment was conducted from the lowest fan speed (RPM), 500 to the highest fan speed, 1488.

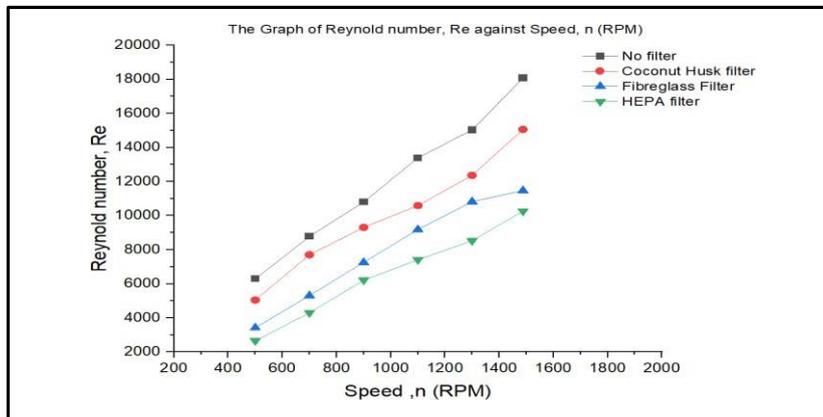


Fig. 3 Reynold number, Re against Speed, n (RPM)

The graph shows the link between Reynolds number and speed (RPM) for different types of air filters. The Reynolds number grows in proportion to the speed for each displayed filter choice. The No filter has the highest Reynolds numbers, and this makes sense because there isn't much airflow resistance when there isn't a filter in place. Higher Reynolds numbers are the outcome of more air flowing freely in the absence of a filter. The filter made of Coconut husk has the second-highest Reynolds number. This is because the material used to make coconut husk filters is usually porous, which means that airflow is not significantly obstructed. Higher airflow through the filter is made possible by this media's reduced resistance to airflow [5]. Fibreglass filter has the third highest Reynold number due to its packing density of fibres in fibreglass filter media is typically lower than that of other filter types [6]. This decreased packing density decreases overall airflow resistance through the filter. Because it has the most dense and restrictive filter media of the three, the HEPA filter has the highest Reynolds numbers throughout the speed range. Airflow is severely impeded by the densely packed fibres [7].

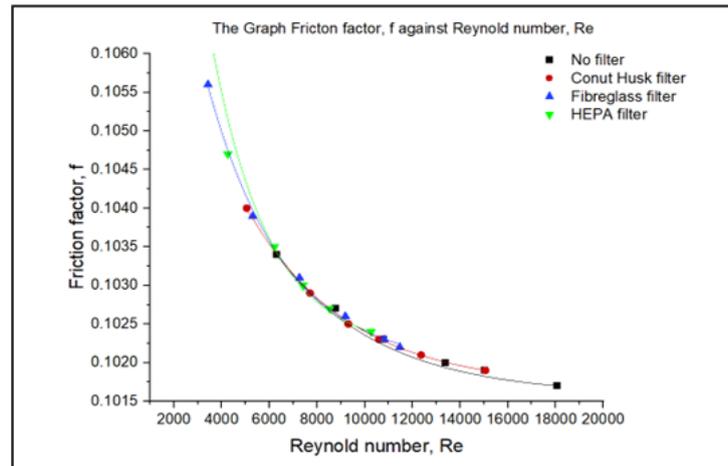


Fig. 4 Friction factor, f against Reynold number

Fig. 4 displays the relationship for different types of air filters between the Reynolds number, Re and the friction factor, f . The friction factor drops for each displayed filter option as the Reynolds number rises. The HEPA filter shows the highest friction factor over the whole range of Reynolds numbers due to its densely packed fibres [7], which provide an extremely high airflow resistance; the HEPA filter has the most restrictive and dense filter media of the three. A HEPA filter will have the lowest air velocity of the three to achieve the same volumetric flow rate as the other filter types because of the dense filter media's strong resistance. No filter shows the lowest friction factor due to a lack of porous structures, fibrous materials, or winding flow paths obstructing the airflow. The Fibreglass filter and the Coconut Husk filter casing are in the middle of these two extremes. A higher friction factor means that the Fibreglass filter offers more resistance to airflow. Moreover, the friction factor in the Coconut Husk filter is slightly lower than in the Fibreglass filter. This is because of the porous and open structure of the coconut husk, which provides little resistance to airflow; the coconut husk filter has the least restrictive filter media of the three, allowing for the maximum air velocity to pass through the filter. [5].

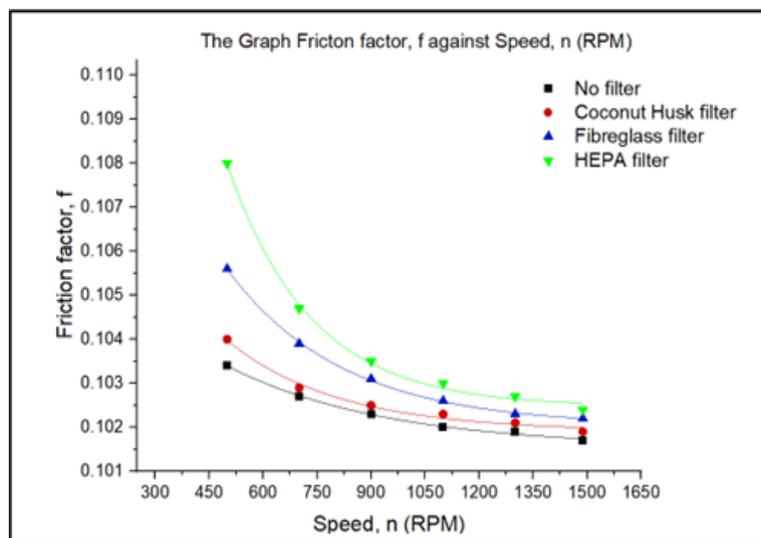


Fig. 5 Friction factor, f against Speed, n (RPM)

The relationship between the speed (RPM) and the friction factor, f for different types of air filters is depicted in the graph. For every filter option displayed, the friction factor decreases with increasing speed. The friction factors of the various filter types range, with the choice to not filter having the lowest friction factor and the filters made of Coconut husk, Fibreglass, and HEPA in increasing order of friction factor. The graph offers helpful information to help choose the right filter type for a given system running speed by balancing the intended needs for energy/power (which are influenced by the friction factor) and filtration efficiency.

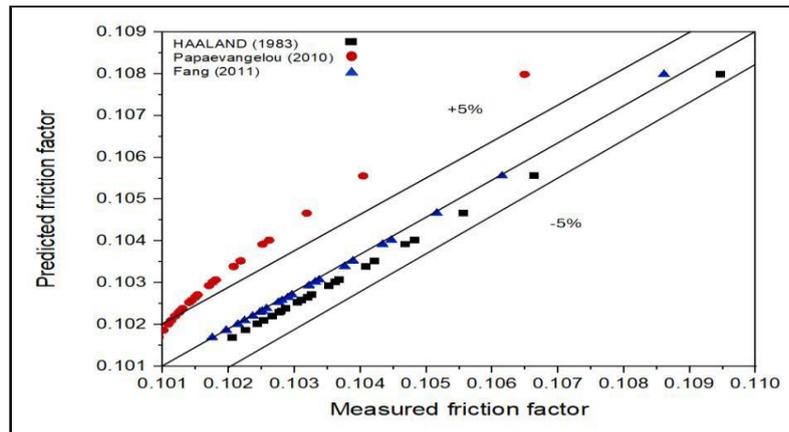


Fig. 6 Predicted friction factor against Measured friction factor

The graph in Fig. 6 shows the Predicted friction factor (Colebrook White equation) against the Measured friction factor. As mentioned previously, thirty equations are going to be considered to find the friction factor in this research. However, only ten equations are shortlisted to be performed in this research due to the relative error of the equation. After conducting the research and doing some data analysis, only five equations give a lower than 5% relative error of the friction factor. The five equations that have been mentioned are Haaland (1983), Achour (2002), Papaevangelou correlation (2010), Fang correlation (2011) and Brkic correlation (2011) for the conditions with air filters and without air filters. Among the five equations, The Fang correlation (2011) offers an easy-to-remember framework that can be solved with a few simple mathematical procedures [3]. Fang correlation (2011) gives the lowest relative error of friction factor, which is 0.18%, 0.22%, 0.35%, and 0.30% for the No filter, Coconut Husk filter, Fibreglass filter and HEPA filter, respectively. This has been proven by the graph above from the graph above which shows that only the Fang correlation (2011) has been intercepted by the straight line; the straight is already set up for a 5% relative error. In the turbulent flow and transition regimes in particular, the Fang correlation (2011) yields more precise estimates of the friction factor than the Colebrook-White equation [4]. Other than that, the implicit Colebrook-White equation is more complicated than the explicit Fang correlation equation.

4. Conclusions

In conclusion, the data show a strong positive correlation between the fan speed (RPM) and the Reynolds number, suggesting that an increase in fan speed causes the airflow to become more turbulent. The degree of turbulence is significantly influenced by the various filter types, with the HEPA filter producing the most restriction and the lowest Reynolds numbers.

The findings demonstrate that, out of the four circumstances studied, No filter, Coconut Husk filter, Fibreglass filter, and HEPA filter, the Fang correlation (2011) produced the lowest relative error. The Fang correlation fared substantially better than the other recognised correlations, such as Haaland (1983), Achour (2002), Papaevangelou (2010), and Brkic (2011), with relative errors ranging from 0.18% to 0.35%. Only the Fang correlation also met the benchmark 5% relative error criterion. Overall, the research finds that, both with and without the use of air filters, the Fang correlation (2011) is the most reliable and appropriate technique for estimating friction factors. system's performance.

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Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of the paper.

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