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# A Mathematical Optimization Model for the Multi-Objective Sweep Based M-VRPSTW

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**Abstract**: Good transportation systems are often described as those satisfying several quality factors, such as minimum cost, minimum time and/or minimum distance. However, there are many issues that affect the efficiency of transportation of goods. In the case of vehicle routing with hard time windows (VRPHTW or simply VRPTW), service may not be able to take place if the vehicles arrive at customers beyond their latest time for service. Thus, introducing soft time windows where hard time windows for service may be violated with some penalty costs may provide an alternative that is more suitable in terms of applicability, practicality and costs savings. In past VRPSTW studies, unlimited number of vehicles is assumed when solving. However, in real practice, companies only have limited number of vehicles. Thus, in this study, limited number of vehicles is introduced on the VRPSTW (m-VRPSTW). The objectives of this study are to formulate new bounds for the time windows for depot's and customers' service, to define the penalty costs function models for early and late arrivals, to formulate a 0-1 Integer Programming model for the m-VRPSTW and to solve the model by using the Sweep Heuristics and Genetic Algorithm approach. The Binary Integer Programming model is formulated to solve m-VRPSTW with three objectives functions which are to maximize the total number of customers served, to minimize the number of vehicles used and to minimize the total traveling cost. Solomon benchmark instances has been used in the computational experiments for tests using different number of vehicles. The model was solved by using MATLAB Optimization Tool Box. Feasible solutions are found for them-VRPSTW model. Results based on the m-VRPSTW model are presented and compared with the VRPSTW model of unlimited number of vehicles. The best results are selected based on the following criteria: minimum total schedule time (TST), minimum average waiting time and minimum average penalty cost incurred. Further refinement on model and solution method is recommended to improve the solution quality.

Keywords: Optimization, vehicle routing, VRPSTW

#### 1. Introduction

The Vehicle Routing Problem (VRP) has drawn enormous interests from many researchers during the last decades because of its vital role in planning of distribution systems and logistics in many sectors of wide applications such as the distribution of cash amounts among bank branches, disposal of garbage and industrial wastes, distribution of fuel to and among fuel stations, school transportation services and the like. VRP is a route optimization initiative consisting in finding minimal cost set of delivery routes to completely service all customer requests without violating operational constraints.

Vehicle Routing Problem with Time Windows (VRPTW) is one of the well-known classes of VRP featuring the requirement of satisfying time windows of services. Due to this time windows adherence requirement, the traditional VRPTW is also known as the Vehicle Routing Problem with Hard Time Windows (VRPHTW). These time windows constraints are hard constraints such that a route is not feasible if the service of a customer either starts before the earliest time  $(a_i)$  or ends after the latest time  $(b_i)$  specified for the customer which is also known as the time window for any customer *i*. If a vehicle arrives at a station (or node) to pick up a customer earlier than the lower bound of the customer's time window, the vehicle must wait until the service is possible. Also, if a vehicle arrives later than the upper bound of the customer's time window, the vehicle must starts and ends its route at the depot. The VRPTW considers the following three cases:

Case 1: If vehicles arrives within  $[a_i, b_i]$ , then it is on time.

Case 2: If vehicles arrives before  $a_i$ , then it is an early arrival where the vehicle has to wait until  $a_i$ . Service cannot start prior to  $a_i$ .

Case 3: Any vehicle cannot arrive after  $b_i$ . This is a delayed arrival and also considered as a violation to the customer's time window. Delayed arrival is not allowed.

The aim of a typical VRPTW is to design least cost routes from one central depot to a set of geographically scattered points, so called clients or customers where each node is visited only once by a single vehicle within a specific time interval called the customer's time window. Each route must be completed within a total route time that is within the depot's service time window. Figure 1 shows a typical solution of a VRPTW problem.



Figure 1: An Example of VRPTW's Solution

As shown in Figure 1, at each customer *i*, the start of the service must be within a given time interval named a time window,  $[a_i, b_i]$ ,  $i \in C$ . *C* denotes the set of customers,  $C = 1, 2, \dots, n$  where *n* is the number of customers. Vehicles must also leave the depot within the time window and return during its time window in which  $[a_0, b_0]$  is the depot's service begin and finish time windows. A vehicle is permitted to arrive before the opening of the time window, and wait at no cost until service becomes possible, but it is not permitted to arrive after the latest time window.

The VRPTW with strict compliance to the time windows such that vehicles must service all customers strictly within time windows is difficult in practice, and it is likely to receive very high costs due to use of many vehicles (Qureshi, Taniguchi and Yamada, 2010) [1]. Thus, VRPTW would have

problem with practicality in applications in distribution and logistics due to the rising importance of just-in-time (JIT) production systems and the increasingly tight coordination of supply chain operations. These are among the reasons for the application of the Vehicle Routing Problem with Soft Time Windows (VRPSTW). VRPSTW is an extension of VRPHTW in which some or all customers' time window requirements are not strictly adhered to and can be violated by paying appropriate penalties. Thus, there are two variants of the VRPTW defined by introducing either hard or soft time windows at the customer locations. The VRPHTW refers to problem in which the time windows must be strictly followed whereas the VRPSTW allows service to be done before the earliest time or after the latest time, but penalty costs must be added. For each customer i, certain penalty functions can be introduced to calculate the penalty payable if the vehicle arrives before  $a_i$  or after  $b_i$  in the case of VRPSTW. If a certain customer's time window cannot be violated (as in VRPHTW), the penalty payable to that customer for any violation is set to infinity.

There are many good reasons for allowing the time windows to be soft, as stated in Koskosidis, Powell and Solomon (1992) [2], Taillard et al. (1997) [3], and Chiang and Russell (2004) [4]. For example, according to Chiang and Russell (2004), the VRPSTW, which is a relaxation of the VRPHTW, has many practical applications such as: (1) relaxing time windows can result in lower total costs without hurting customer satisfaction significantly; (2) many applications do not require hard time windows -e.g. the delivery of fuel/gas to service stations, (3) travel times cannot be accurately known in many practical applications, and (4) VRPSTW approaches can be used to solve VPRHTW if the penalties are modified appropriately. In addition, VRPSTW solutions provide a workable alternative plan of action when the problem with hard time windows is infeasible. Most of these VRP and VRPTW problems can be represented using mathematical programming models. Linear Programming (LP), Non-Linear Programming (NLP), Mixed Integer programming (MIP), Goal programming (GP) or any other type of mathematical programming model will be developed based on the nature of the problems, types of decision variables, the constraints or the objective functions involved. Exact methods such as branch and bound, branch and price, column generation, Lagrangean Relaxation and others can be employed to solve the models. These approaches, combined with certain heuristics, would normally guarantee optimal or exact solutions, however, at large expense of computing time [4].

Many researches have been performed on VRP due to its importance for applications in transportation, distribution and logistics. Transportation refers to the movement of products from one location to another which is it makes its way from the beginning of the supply chain to the customer's handle. In Malaysia, transport logistics is an important domain in human activity. It supports and contributes to the increase in economic and makes possible most other social activity. Hence, transportation of goods via different types of vehicles underlines the importance of routing and scheduling problems. Nowadays, routing problems are becoming the main concern of organizations in most developing countries including Malaysia.

#### 2. Materials and Methods

The materials and methods of the study consists of the following phases:

#### Phase 1: Data Collection and Preliminary Modelling.

- i. Investigate on current issues and approaches pertaining to VRPTW, VRPSTW and m-VRPSTW.
- ii. Determine scope, model and methods to be pursued in this study.
- iii. Analyse benchmark data instances.
- iv. Identify assumptions, objective functions, parameters, input and decision variables, constraints and other related components for the Integer Programming model.

#### Phase 2: Development of the Mathematical Model

i. Establish the road network involved by plotting nodes (customer's and depot locations) based on the data.

- ii. Set the number of vehicles available (m-VRPSTW). The fleet of vehicles consists of homogeneous capacitated vehicles.
- iii. Formulate the programming model and sub-models (model for soft time windows intervals and also penalty costs function model) to be developed.
- iv. Refine and adjust the model.

#### Phase 3: Development of Mathematical Solution Approach

- i. Conduct SWOT analysis on various exact solution methods employed in past researches for integer programming involving large-scale problems.
- ii. Select the most suitable mathematical solution approach for the developed model.
- iii. Develop an enhanced solution method through adaptation, modification and introduction of new step on the identified existing solution method.
- iv. Do coding and programming.
- v. Conduct a pilot run of computational experiment.
- vi. Perform validation and verification of model based on pilot experiment results.
- vii. Refinement of model, solution method and algorithm.

#### Phase 4: Solving the Model

- i. Conduct full-scale computational experiments.
- ii. Analyse the results.
- iii. Develop a GUI interface for effective presentation of solutions, model and solution method used.
- iv. Analyse comparison with results of past models (where appropriate).
- 2.1 Soft Time Windows Model's Principles

New bounds for the time windows for depot's and customers' service are formulated. Let  $[a_i, b_i]$  be the hard time window at customer *i* and thus,  $[l_i, u_i]$  will be the soft time window at customer *i* as in Figure 2. In this study, let  $[a_i - s, b_i + s]$ , which is s = 30 time units be the lower soft time windows and upper soft time windows respectively. Arrival time of vehicles can be delayed about 30 time units is reasonable because traveling time also considered time delays due to traffic congestion or disruption during the traveling time. If the vehicle violates the customer's hard time window, no service is allowed. If the vehicle violates the customer's soft time window, service is allowed at a penalty cost. Penalty can be defined in five cases.

l	i a <sub>i</sub>		b <sub>i</sub> u	i
Has to wait for service. Penalty cost for waiting	Penalty cost for early arrival	Original hard time windows	◆ Penalty cost ↓ for late arrival	No service

Figure 2: Hard and Soft Time Windows

- **Case 1:** If arrival time at customer  $i(r_i)$  is lower than lower soft time window  $(l_i)$ , then the vehicle must wait until it reaches the lower soft time window to serve customer i. The vehicle is not allowed to serve the customer and needed to wait until it reaches the lower soft time window,  $l_i$ . Penalty cost (p) will be incurred for waiting.
- **Case 2:** If arrivals time at customer  $i(r_i)$  is greater than lower soft time window  $(l_i)$  but smaller than ready time for the customer  $(a_i)$ , the vehicle can serve the customer but certain penalty cost (p) will be incurred.

- **Case 3:** If arrival time at customer  $i(r_i)$  is in between time window  $[a_i, b_i]$ , and the total time for the arrival time  $(r_i)$  and service time at customer  $i(s_i)$  is less than the due time of the customer  $i(b_i)$ , the vehicle can service customer i with no penalty incurred.
- **Case 4:** If arrivals time at customer  $i(r_i)$  is greater than due time of the customer  $i(b_i)$ , the vehicle can serve the customer but certain penalty cost will be incurred.
- **Case 5:** If arrival time at customer  $i(r_i)$  is greater than upper soft time window  $(u_i)$ , no service for the customer.

2.2 Penalty Costs Function Models

Penalty cost is defined in five cases. The weightage of the penalty described based on the maximum penalty,  $p_{max} = 10$ .

```
Case 1
```

```
If r_i < l_i
Then
          Waiting time (w_i) = l_i - r_i
          Penalty cost (p_i) = (0.5)(p_{max})
Case 2
If l_i \leq r_i < a_i
Then Case 2a
                    If r_i \le (0.2s + l_i)
                    Then w_i = 0
                            p_i = (0.5)(p_{max})
          Case 2b
                    If (0.2 \ s + l_i) < r_i < \left(\frac{a_i - l_i}{2} + l_i\right)
                    Then w_i = 0
                            p_i = (0.3)(p_{max})
          Case 2c
                    If \left(\frac{a_i - l_i}{2} + l_i\right) \le r_i < a_i
Then w_i = 0
                           p_i = (0.2)(p_{max})
Case 3
If a_i \leq r_i \leq b_i
Then w_i = 0
         p_i = 0
Case 4
If b_i < r_i \le u_i
Then
          Case 4a
                    If r_i \leq (0.2s + b_i)
                    Then w_i = 0
                            p_i = (0.2)(p_{max})
          Case 4b
                    If (0.2s + b_i) < r_i < \left(\frac{u_i - b_i}{2} + b_i\right)
                    Then w_i = 0
                            p_i = (0.3)(p_{max})
          Case 4c
                    If \left(\frac{u_i - b_i}{2} + b_i\right) \le r_i \le u_i
Then w_i = 0
                            p_i = (0.5)(p_{max})
Case 5
```

If  $r_i > u_i$ Then "No Service"

#### 2.3 Formulate a 0-1 Integer Programming Model for the m-VRPSTW

In this study, a m-VRPSTW as a Binary Integer Programming problem is proposed. In the objective function, for any violation of the time windows, there will be penalties incurred. These penalties weight the effect of not satisfying the customers' preferences on the time interval during which the delivery should have taken place. Besides, additional cost will be incurred for waiting in case of parking fee or undesirable condition such as vehicle blocking the traffic due to no space for parking.

Binary Integer Programming model by Tas et al. (2013) [5] and Calvete et al. (2007) [6] were adopted and adapted accordingly to suit the data and problem involved. The notation and principles to be followed by the model are represented as the following:

#### 2.4 Model's Notation

The notations are adopted from various past studies on the VRPSTW. Additional variables and parameters will be added as necessary when specific formulation based on the soft time windows formulated and development of penalty costs functions are completed. The notations used in the model of this study are as followed:

of nodes (each 1									
	representing the	central dep	pot or a cu	stomer's					
$N = \{1, \cdots, n\}.$									
f directed arcs (eacl	h representing a o	direct connec	tion). $i = 1$	refers to					
al depot, while ind	ices $i = 2 \tan n$	refer to the c	customer $A =$	:{(i,j):					
n <i>i</i> to <i>j</i> .									
The travel time associated with going from node $i$ to node $j$ through arc $(i, j)$									
of customer <i>i</i> .									
ime of customer <i>i</i> .									
ime at customer <i>i</i> .									
e time (service finis	hing) from custor	mer <i>i</i> . $d_i = d_i$	$l_i + s_i$ and $d_i$	$\geq b_i$ .					
time of customer <i>i</i> .	$r_i \le l_i . w_i = l_i - l_i $	$-r_i$ .							
cost for customer <i>i</i> .		·							
e time	(starting	time)	from	depot.					
	-			_					
e windows of custor	mer <i>i</i>								
e windows of custo of time of the day i	mer $i, i \neq 0$ , ind n which goods sl	licating prefe hould be sup	erences regar plied or servi	ding the ce to be					
e windows of sched	uling for depot.								
windows of schedu	ling for depot.								
m vehicles represer	ting the fleet of	vehicles avai	lable to be us	sed. $V =$					
<i>m</i> }									
m total number of v	ehicles available.								
city of the vehicle.									
mber of customers s	erved.								
node									
vehicle									
hicle <i>k</i> travels dire erwise	ctly from node <i>i</i>	to nodej;							
	). $N = \{1, \dots, n\}$ . of directed arcs (eacl ral depot, while ind $\}$ . m <i>i</i> to <i>j</i> . el time associated w l of customer <i>i</i> . time of customer <i>i</i> . time of customer <i>i</i> . re time (service finis time of customer <i>i</i> . re time (service finis time of customer <i>i</i> . re time). he windows of custome windows of custome windows of custome windows of schedu <i>m</i> vehicles represent <i>m</i> } in total number of va acity of the vehicle. imber of customers so or node or vehicle ehicle <i>k</i> travels dire erwise	). $N = \{1, \dots, n\}$ . of directed arcs (each representing a d ral depot, while indices $i = 2 \text{ ton } n$ i = 2  ton  n k = 1  time associated with going from notl of customer $i$ . time of customer $i$ . time of customer $i$ . re time (service finishing) from custo time of customer $i$ . $r_i \leq l_i \cdot w_i = l_i - 1$ cost for customer $i$ . re time (starting ). ne windows of customer $i$ ie windows of customer $i$ , $i \neq 0$ , ind of time of the day in which goods sh d. ne windows of scheduling for depot. e windows of scheduling for depot. m vehicles representing the fleet of m im total number of vehicles available. acity of the vehicle. imber of customers served. or node or vehicle ehicle $k$ travels directly from node $i$ erwise	). $N = \{1, \dots, n\}.$ of directed arcs (each representing a direct connect ral depot, while indices $i = 2 \text{ ton refer to the } c$ }. m <i>i</i> to <i>j</i> . el time associated with going from node <i>i</i> to node l of customer <i>i</i> . time of customer <i>i</i> . time of customer <i>i</i> . re time (service finishing) from customer <i>i</i> . $d_i =$ time of customer <i>i</i> . $r_i \leq l_i \cdot w_i = l_i - r_i$ . cost for customer <i>i</i> . re time (starting time) ). ne windows of customer <i>i</i> windows of customer <i>i</i> , $i \neq 0$ , indicating prefe of time of the day in which goods should be sup d. ne windows of scheduling for depot. e windows of scheduling for depot. m vehicles representing the fleet of vehicles avai $m_i$ im total number of vehicles available. acity of the vehicle. imber of customers served. or node or vehicle ehicle <i>k</i> travels directly from node <i>i</i> to node <i>j</i> ; erwise	). $N = \{1, \dots, n\}$ . of directed arcs (each representing a direct connection). $i = 1$ ral depot, while indices $i = 2$ ton refer to the customer $A = \}$ . m <i>i</i> to <i>j</i> . el time associated with going from node <i>i</i> to node <i>j</i> through arc l of customer <i>i</i> . time of customer <i>i</i> . time at customer <i>i</i> . re time (service finishing) from customer <i>i</i> . $d_i = l_i + s_i$ and $d_i$ . time of customer <i>i</i> . $r_i \le l_i \cdot w_i = l_i - r_i$ . cost for customer <i>i</i> . re time (starting time) from the windows of customer <i>i</i> , $i \ne 0$ , indicating preferences regar of time of the day in which goods should be supplied or servi d. the windows of scheduling for depot. e windows of scheduling for depot. for wehicles representing the fleet of vehicles available to be us $m_i$ un total number of vehicles available. acity of the vehicle. unber of customers served. or node or vehicle ehicle <i>k</i> travels directly from node <i>i</i> to node <i>j</i> ; erwise					

$Z_v$	$:= \{1, \text{ if vehicle } v \text{ is actually used}; \}$
	(0, otherwise
C <sub>t</sub>	: Cost paid for one unit of distance.
$d_{ij}$	: Distance along the arc.
$p_i$	: Penalty cost incurred at customer <i>i</i>
$e_{iv}$	1, if there is time window violation at node i served by vehicle v;
	- (0, otherwise

**Decision Variables:** 

$$x_{ijv} = \begin{cases} 1, \text{ if vehicle } v \text{ travels directly from node } i \text{ to node } j; \\ 0, \text{ otherwise} \end{cases}$$

$$z_v = \begin{cases} 1, \text{ if vehicle } v \text{ is actually used}; \\ 0, \text{ otherwise} \end{cases}$$

$$(3.1)$$

#### 2.5 Sweep Solution Method

In this section, the solution method used in this study is elaborated. Sweep Heuristics approach is used in this study. The Sweep Heuristics is a method of bunching customers into groups. The customers who are in the same group are physically and geographically close together and can be served by the same vehicle. The Sweep Heuristics is used in this study since the "cluster first and route second" method is employed in determining routes for solving the m-VRPSTW. The idea behind the heuristic is to first cluster the customers into routes having regard for the vehicle capacity and customers' demand. As mentioned by Han and Tabata (2002) [7], Sweep Algorithm is used for clustering customers that are geographically close together in the same group so they can be served by the same vehicle. In order to construct the route, the process sweeps customers by increasing polar angle into the current cluster and the sweep is stopped when adding the next customer would violate the maximum vehicle capacity. Then, the process is followed by the route construction approach with or without the route improvement heuristics to achieve a route with minimum travel distance.

The process of sweeping and routing is continued with the customer that violated the previous vehicle's capacity as the starting point for the next route and the process is completed when all points have been swept and included in a route. Thus, Sweep heuristics is maximizing the demand catered by each vehicle aside from minimizing the distance during each of the sweeps. Thus, the Sweep Algorithm is a good example of the "cluster first, route second" approach.

#### 2.6 Computational Tools

This study used Genetic Algorithm (GA) to solve the model. The algorithm was coded using MATLAB version 2009b. For this study, MATLAB Optimization Tool Box was used as software to compute the optimal or approximate solution to the m-VRPSTW model formulated. The software provides widely used algorithms for standard and large-scale optimization, which solve constrained and unconstrained continuous and discrete problems. The toolbox is capable of solving problems formulated using various mathematical programming such as linear programming, integer programming, nonlinear programming, quadratic programming and multi-objective programming where exact solution or near optimal solutions can be found. MATLAB Optimization Tool Box is an interactive tool, which is user-friendly and not too complicated.

GA is a class of adaptive stochastic optimization algorithms involving search and optimization introduced by Holland (1975) [8]. GA approaches are simple in concept and can be described as follows:

- generate a **population** of possible answers to the problem at hand,
- choose the best **individuals** from the population (using methods inspired by survival of the fittest),
- produce a new **generation** by combining these best ones (using techniques inspired by reproduction crossover, mutation, recombination, etc.)
- **stop** when the best individual of a generation is good enough (or you run out of time).

#### 3. Results and Discussion

Sample data in Table 1 displays narrow (short) time windows. For example, Customer 1 in data set R101 of data set R, as shown in Table 1, is available to be served the earliest at 161 unit time and the service must finish the latest at 171 unit time, which makes the service time window for Customer 1 as [161, 171]. Note that, in VRPTW and VRPSTW, no time window violation occurs if vehicle arrives or serves customer within the time windows. The benchmark data set R1 can be found in Appendix A. Complete benchmark instances for all six data sets are available at <u>http://w.cba.neu.edu/~msolomon/problems.htm</u> (2013) [9].

Table 1. Sample Data Bet K101											
Customer	Х	У	Demand Ready Time		Due Time	Service Time					
1	35	35	0	0	230	0					
2	41	49	10	161	171	10					
3	35	17	7	50	60	10					
4	55	45	13	116	126	10					
5	55	20	19	149	159	10					
6	15	30	26	34	44	10					
7	25	30	3	99	109	10					
8	20	50	5	81	91	10					
9	10	43	9	95	105	10					
10	55	60	16	97	107	10					

Table 1: Sample Data from Data Set R101

Table 2 shows the results of solving the VRPSTW by assuming unlimited number of vehicles and also a set of limited number of vehicles (m-VRPSTW) for data set R101. Table 2 presents the analysis of results from data set R101. Based on Table 2, it can be observed that limiting the number of vehicles has caused not all of the customers can be served within the constraints given (vehicle capacity, time depot's time window, customers' time windows, etc). Aside from that, it is noted that the lower the number of vehicles available, the lower are the number of customers served. Thus, reduction in the total scheduling time (TST) and total distances occurred due to less number of customers can be served. Based on results in Table 2, limiting the number of vehicles, m, to 33, for example, still allows for 99 out of 100 customers to be served. Not much difference observed in terms of average penalty cost per vehicle and also average waiting time per vehicle. Limiting the number of vehicles to 16 or below is not advisable since it results in only 50% or less number of customers can be catered by the vehicles available. m = 24 is recommended due to its lowest TST and waiting time.

Table 2. Results 101 Data Set R101																		
Criteria	Solution when assuming unlimited number	m-VRPSTW																
	of vehicles (Fixed Number of Vehicles)																	
Number of Vehicles	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17
Number of Customers Served	100	99	96	93	89	87	84	81	78	75	72	69	66	63	60	65	54	51
Total TST (unit) for all vehicles	6293	6203	5913	5762	5612	5449	5208	5000	4624	4461	4016	4001	3889	3671	3694	3366	3183	3004
Average TST per Vehicle (unit)	185	188	185	186	187	188	186	185	178	178	167	174	177	175	185	177	177	177
Total Distance (unit) for All Vehicles (unit)	3898	3876	3713	3639	3556	3425	3345	3258	3097	2966	2817	2795	2636	2499	2328	2256	2139	1985
Average Total Distance per Vehicles (unit)	115	117	116	117	119	118	119	121	119	119	117	122	120	119	116	119	119	117
Total Penalty (unit)	299	280	271	262	273	240	231	222	195	204	205	186	177	168	171	163	154	149
Average Penalty per Vehicle (unit)	9	8	8	8	9	8	8	8	8	8	9	8	8	8	9	9	9	9
Total Waiting Time for All Vehicles (unit)	1395	1337	1240	1193	1156	1154	1023	932	747	745	479	516	593	542	766	460	504	509
Average Waiting Time per Vehicle (unit)	41	41	39	38	39	40	37	35	29	30	20	22	27	26	38	24	28	30

Table 2: Results for Data Set R101

#### 4. Conclusion

The importance of the practical implementation of the solution to the VRPSTW in logistics has made this area of study crucial. However, the literature or research in VRPSTW is still considered lacking. In this study, a new formulation of mathematical model was presented which introduces the m-VRPSTW, a variant of the VRPSTW in which a limited number of vehicles is available. The model is a multi-objective model where three objectives are considered which are minimizing the total schedule time (TST), minimizing the total number of vehicles used and maximizing the total number of customers served.

This study has few contributions. Firstly, the introduction of the new penalty cost function and the m-VRPSTW model can be considered as an academic contribution. The findings, although may still need further improvement, have provided a platform for this m-VRPSTW to be improved and produce better solution. The results found also provide some useful understanding of the VRPSTW with limited number of vehicles aside from showing that the m-VRPSTW model formulated produce feasible solutions even though they may not be the optimal or utmost best yet. In addition, in this study, the programming mathematical model helps in modelling the demand distribution.

In this study, a variant of VRPSTW constrained by a limited vehicle fleet, the m-VRPSTW is considered which seems to be more realistic in logistics since some companies may have limitations to have a large number of vehicles. Further improvements to the model and the solution method could possibly be achieved by investigating possible factors that may influence the TST, number of vehicles and number of customers to be catered. Then, refinement of the penalty cost function as well as directly putting the penalty cost as one of the objective functions of the model may also be considered.

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### Appendix A

The appendix A show the sample of data set R101.

VEHICLE 200
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### Source: http://web.cba.neu.edu/~msolomon/r101.htm

CUST	UST YCOOPD YCOOPD		DEMAND	READY	DUE	SERVICE		
NO.	ACCOLD	TCOOLD	DEFEND	TIME	TIME	TIME		
1	35.00	35.00	0.00	0.00	230.00	0.00		
2	41.00	49.00	10.00	161.00	171.00	10.00		
3	35.00	17.00	7.00	50.00	60.00	10.00		
4	55.00	45.00	13.00	116.00	126.00	10.00		
5	55.00	20.00	19.00	149.00	159.00	10.00		
6	15.00	30.00	26.00	34.00	44.00	10.00		
7	25.00	30.00	3.00	99.00	109.00	10.00		
8	20.00	50.00	5.00	81.00	91.00	10.00		
9	10.00	43.00	9.00	95.00	105.00	10.00		
10	55.00	60.00	16.00	97.00	107.00	10.00		
11	30.00	60.00	16.00	124.00	134.00	10.00		
12	20.00	65.00	12.00	67.00	77.00	10.00		
13	50.00	35.00	19.00	63.00	73.00	10.00		
14	30.00	25.00	23.00	159.00	169.00	10.00		
15	15.00	10.00	20.00	32.00	42.00	10.00		
16	30.00	5.00	8.00	61.00	71.00	10.00		
17	10.00	20.00	19.00	75.00	85.00	10.00		
18	5.00	30.00	2.00	157.00	167.00	10.00		
19	20.00	40.00	12.00	87.00	97.00	10.00		
20	15.00	60.00	17.00	76.00	86.00	10.00		
21	45.00	65.00	9.00	126.00	136.00	10.00		
22	45.00	20.00	11.00	62.00	72.00	10.00		
23	45.00	10.00	18.00	97.00	107.00	10.00		
24	55.00	5.00	29.00	152 00	162.00	10.00		
25	65.00	35.00	5.00	172 00	103.00	10.00		
20	45 00	20.00	17 00	132 00	142.00	10.00		
28	35 00	40 00	16 00	37 00	47 00	10.00		
20	41 00	37 00	16.00	39 00	49 00	10.00		
30	64 00	42 00	9 00	63 00	73 00	10.00		
31	40.00	60.00	21.00	71.00	81.00	10.00		
32	31.00	52.00	27.00	50.00	60.00	10.00		
33	35.00	69.00	23.00	141.00	151.00	10.00		
34	53.00	52.00	11.00	37.00	47.00	10.00		
35	65.00	55.00	14.00	117.00	127.00	10.00		
36	63.00	65.00	8.00	143.00	153.00	10.00		
37	2.00	60.00	5.00	41.00	51.00	10.00		
38	20.00	20.00	8.00	134.00	144.00	10.00		
39	5.00	5.00	16.00	83.00	93.00	10.00		
40	60.00	12.00	31.00	44.00	54.00	10.00		
41	40.00	25.00	9.00	85.00	95.00	10.00		
42	42.00	7.00	5.00	97.00	107.00	10.00		
43	24.00	12.00	5.00	31.00	41.00	10.00		

## <u>R101</u>

44	23.00	3.00	7.00	132.00	142.00	10.00
45	11.00	14.00	18.00	69.00	79.00	10.00
46	6.00	38.00	16.00	32.00	42.00	10.00
47	2.00	48.00	1.00	117.00	127.00	10.00
48	8.00	56.00	27.00	51.00	61.00	10.00
49	13.00	52.00	36.00	165.00	175.00	10.00
50	6.00	68.00	30.00	108.00	118.00	10.00
51	47.00	47.00	13.00	124.00	134.00	10.00
52	49.00	58.00	10.00	88.00	98.00	10.00
53	27.00	43.00	9.00	52.00	62.00	10.00
54	37.00	31.00	14.00	95.00	105.00	10.00
55	57.00	29.00	18.00	140.00	150.00	10.00
56	63.00	23.00	2.00	136.00	146.00	10.00
57	53.00	12.00	6.00	130.00	140.00	10.00
58	32.00	12.00	7.00	101.00	111.00	10.00
59	36.00	26.00	18.00	200.00	210.00	10.00
60	21.00	24.00	28.00	18.00	28.00	10.00
61	17.00	34.00	3.00	162.00	172.00	10.00
62	12.00	24.00	13.00	76.00	86.00	10.00
63	24.00	58.00	19.00	58.00	68.00	10.00
64 CE	27.00	69.00	10.00	34.00	44.00	10.00
65	15.00	77.00	9.00	73.00	83.00	10.00
60	62.00	77.00	20.00	51.00	127 00	10.00
60	49.00	75.00	25.00	127.00	137.00	10.00
60	67.00 56.00	30.00	25.00	142 00	95.00	10.00
70	37 00	39.00	50.00	50 00	60 00	10.00
70	37.00	56 00	5.00	182 00	192 00	10.00
72	57.00	68 00	15 00	77 00	87 00	10.00
73	47 00	16 00	25 00	35 00	45 00	10.00
74	44 00	17 00	9 00	78 00	88 00	10.00
75	46.00	13.00	8.00	149.00	159.00	10.00
76	49.00	11.00	18.00	69.00	79.00	10.00
77	49.00	42.00	13.00	73.00	83.00	10.00
78	53.00	43.00	14.00	179.00	189.00	10.00
79	61.00	52.00	3.00	96.00	106.00	10.00
80	57.00	48.00	23.00	92.00	102.00	10.00
81	56.00	37.00	6.00	182.00	192.00	10.00
82	55.00	54.00	26.00	94.00	104.00	10.00
83	15.00	47.00	16.00	55.00	65.00	10.00
84	14.00	37.00	11.00	44.00	54.00	10.00
85	11.00	31.00	7.00	101.00	111.00	10.00
86	16.00	22.00	41.00	91.00	101.00	10.00
87	4.00	18.00	35.00	94.00	104.00	10.00
88	28.00	18.00	26.00	93.00	103.00	10.00
89	26.00	52.00	9.00	74.00	84.00	10.00
90	26.00	35.00	15.00	176.00	186.00	10.00
91	31.00	67.00	3.00	95.00	105.00	10.00
92	15.00	19.00	1.00	160.00	170.00	10.00
93	22.00	22.00	2.00	18.00	28.00	10.00
94	18.00	24.00	22.00	188.00	198.00	10.00
95	26.00	27.00	27.00	100.00	110.00	10.00
96	25.00	24.00	20.00	39.00	49.00	10.00
97	22.00	27.00	11.00	135.00	145.00	10.00
98	25.00	21.00	12.00	133.00	143.00	10.00
99	19.00	21.00	10.00	58.00	68.00	10.00
100	20.00	26.00	9.00	83.00	93.00	10.00
101	18.00	18.00	17.00	185.00	195.00	10.00