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# A Study of Discrete Insurance Risk Model and its Ruin Probability

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**Abstract**: This study reflects on the risk management perspective especially in terms of insurance risk models. From the scenery of the insurance company, the main idea of the risk model looks into how risks affects the probability of ruin. Probability of ruin is determined using different risk model such as Poisson and Binomial distributions. Therefore, the main objectives of this study is to study the discrete time risk model and to minimize the probability of ruin which the main scope being the probability of ruin being calculated using the discrete time model. From the study, we found that there are several different ways to reduce the probability of ruin such as reinsurance and reinvestment. From the expected findings made, there are 3 different ways in computing the probability of ruin using the discrete time risk model which are the Poisson compound, Monte Carlo simulation and Pareto claims. They take individual forms of reinsurance for the portfolio and identifies the optimal reinvestment levels for the insurance companies.

**Keywords**: Discrete-Time, Insurance, Reinsurance, Insurer, Insured, Claims.

## 1. Introduction

Risk management is acknowledged as the most important topic for the insurance industry and the occurrence of certain losses as a prerequisite for a claim from the insurer [1]. If the particular claim is known to be larger than the estimated insurance premium, the insurer could be facing loads of losses. Generally, three different risks can be covered by insurance, such as personal risk, property risk, and liability risk. Personal risk refers to a type of risk where it can affect the health and safety of an individual due to illness or accidents; property risk is the risk where is borne by a person's property, such as theft or fire or anything by the "Acts of God"; and finally yet importantly is liability risk where a person or business associated found to be liable due to their negligence or willful acts that caused to another person's property.

The purpose of an insurance company is mainly reducing or completely removing or taking over risks from the customers towards the insurance company. Insurers will usually consider every

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quantifiable factor that is necessary to develop different high profiles against low risked insurance as the level of risk determines the insurance premiums. Insurance companies that might have to experience ruins when their stacks have become negative makes it impossible for companies to meet their financial obligations such as claims [2]. Therefore, many different insurance risk models are used to calculate the risk against the profits that an insurance company would be able to earn at a particular time. Some of the common ways of computing the probability of ruin is using the continuous time risk model with Poisson compound and the Binomial distributions. This can be observed from the research of Shuanming, Lu & Garrido (2009) [3] where the authors applied both of these models to portray and compute the probability of ruin. Discrete time insurance model is a model where any surplus available at the beginning of the year will be reinvested with an interest rate [4]. Even though the discrete time insurance model has got its own several unique features, however it is not the main option compared to the continuous risk model as there is lack of literature and research to support the statement on discrete time insurance model being much more efficient. This study helps to identify the ruin probability in a discrete insurance risk model. From this research, the outcome from the discrete time insurance model will be tested on its efficiency being a suitable and optimal method to reduce the probability of ruin.

## 2. Equations

Equation [1] is basically the discussion for the probability of ruin using the discrete time risk model which was introduced by Constantinescu, Kozubowski & Qian (2019) [5] and will be highly discussed within the context throughout the entire study.

$$\psi(u) = \frac{1-q}{p}\left(\frac{1-p}{q}\right)^{u+1} \qquad \text{Eq. 1}$$

The problems are having huge association with ruins and the calculation of ruin probabilities are closely related to quantities, which is also known to be the Sparre Andersen risk model [6] and the financial and insurance risks [7], [8] that came up with the theoretical implementation that is given in a nontrivial collective risk model. The consideration of the binomial risk model is given by

$$U_t = u + t - \sum_{i=1}^{t} X_i \qquad \text{Eq. 2}$$

for $t \in \mathbb{N} \cup \{0\}$ with the probability of ruin to be

$$\psi(u) = P[\exists t, U_t < 0 : U_0 = u] \qquad \text{Eq. 3}$$

The probability of ruin is then noted as $p \geq 1 - q$ and the observation from Eq.1 holds if and only if

$$\frac{1-q}{p}\left(\frac{1-p}{q}\right)^{u+1} \leq 1 \qquad \text{Eq. 4}$$

Nevertheless the different proposition from different authors as stated from above which helped in gathering the accurate methods for the probability of ruin.

## 3. Methodology

The results and discussion section presents data and analysis of the study. Within the insurance risk model, one of the few pre-requisites in order for claims to be present with a catastrophic event taking place. Under the insurance risk model, several conditions have to be followed accordingly which is stated as below:

(a) Losses made will be deemed as the random variable

(b) Circumstances of the losses have to be subject to the definition

(c) Lack of over-exposure towards risks

(d) The risk estimation has to be consistent with the premium and the insurance market.

In this research, the focus is placed on the discrete-time risk models and under the compound binomial model, with the assumption of the premium of the income period is one and the number of claims which is period is governed by the binomial process.

With the surplus of the insurance company at the time period of , the model can be described as

$$U(t) = u + t - \sum_{i=1}^{N(t)} X_i \qquad \text{Eq. 5}$$

Further from here, the compound binomial model can be rewritten into multiple different theorems depending on the conditions and the variables.

This follows on the model formulation which was suggested by many different researchers. Liang and Young (2018) basically discusses on minimizing the probability of ruin [9]. In general, many different insurers have definitely employed on an integrated investment and reinsurance strategies to ensure that they are able to gain more profits. Some of the few methods include the model with pareto claims and model with geometric claims. In order to formulate the different models within the discrete time insurance risk, some of the methods to reduce the risk is then formulated which is through reinsurance, reinvestment and a combination of both. To reduce the probability of risk, it is best to invest all the surplus to a non-risky market with taking the ruin probability as part of the risk measure [10].

3.1 Model with pareto claims

The parameter $\alpha$ controls the tails of ZMDP distributions, where it follows a power law just as from the case of the DP distribution. To prove using the ZMDP survival function, if $X \sim ZMDP(\alpha, \lambda, q)$ then

$$P(X > x) \sim (1-q)\lambda^a x^{-a} \qquad \text{Eq. 6}$$

as x → ∞. The second argument is that if the parameter is $\lambda > 0$ has control over the size of the ZMDP random variable even though the scale of the parameter is usual. The basic model of the stochastic representation of the ZMDP distribution to the DP model is $x \sim ZMDP(a, \lambda, q)$ then $xdef \ = \ IN$ where the variables I and N are independent, where I has a Bernoulli distribution with parameter of $1 - q$ and $N \sim DP(a, \lambda)$.

3.2 Model with geometric claims

Considering that the compound binomial risk model where $\theta = \vartheta$, the $\{X_t\}_{t\geq 0}$ has the ZMG distribution with a success probability. When comparing the ZMP and the ZMG models, there is a same claim expectation where both the PMF models which have the same value of q when zero claims were made. However, the PMF drops quicker compared to the ZMG model.

The binomial risk model which can be considered as:

$$U_t = u + t - \sum_{i=1}^{t} X_i, t \in N_0 = \{0,1,\ldots.\}, \qquad \text{Eq. 7}$$

This was introduced earlier with the probability of ruin being,

$$\psi(u) = \wp(U_0 = u) \qquad \text{Eq. 8}$$

This model admits that there is a strong form of claim amounts being $\{X_i\}$ which has got *zero-modified geometric* (ZMG) distribution ZMG (q,p).

The original authors of this formula has extended a particular formula by using the mixing approach as from Eq. 7 and another formula which can be extended as below:

$$f(x) = \lambda e^{-\lambda x}, x \in R_+ \qquad \text{Eq. 9}$$

assuming that there is a form of 'mixing' a random variable on the $R_+$, where the amount of claim listed as $\{X_i\}$ is independent and is identically distributed as a form of zero modified geometric ZMG (q,p) with the probability of success being

$$\rho = e^{-\theta} \qquad \text{Eq. 10}$$

In order to obtain a strict mathematical formula from a particular problem, the assumption that all of the stochastic quantities are to be defined as probability space of $(\Omega, F, \{F_t\}_{t \in R+}, P)$ which satisfies the filtration of $\{F_t\}_{t \in R+}$ which represents the information available at the time of *t* and their basis for all decision-making. The process risk which is considered in this paper is formed based on 2 important processes which is the insurance process and the investment generating process. When there is an absence for reinsurance, the surplus process $\{P_t\}_{t \in R+}$ is given by the model

$$P_t = ct + \sigma_1 W_{1,t} - S_t, \; t \geq 0, \tag{Eq. 11}$$

where $S_t$, defined as

$$S_t = \{\frac{\sum_{i=1}^{N_t} \quad X_i}{0} \; if \; N_t > 0 \; if \; N_t = 0 \;, \tag{Eq. 12}$$

is a compound Poisson process which represents the aggregate claims which is made by the policymakers.

3.3 Diffusion Perturbed Risk Model with Reinsurance

Zhang, Jin, Qian, and Wang (2018) states that quota-share reinsurance is when the reinsurer accepts a fixed share of their liabilities which is determined by the main insurer when they are under the arrangement of the original contract of the insurance [11]. When entering the reinsurance market, insurance companies would prefer to find the best treaty which turns back to the concept of optimal reinsurance.

As the insurance risk being controlled by the insurer, it takes on Quota Share (QS) proportional reinsurance where the retention level k ∈ [0,1] with the insurance process will be present of the QS reinsurance is now at:

$$Pk\, t \; = c^k t + k\sigma_1 dW_{1,t} - kS_t \tag{Eq. 13}$$

with the dynamics as:

$$dPk\, t \; = c^k dt + k\sigma_1 dW_{1,t} - kdS_t \tag{Eq. 14}$$

When a cedent enters into a quota sharing reinsurance treaty with the reinsurer, the claims and the premiums will be shared according to the level of retention $k \in [0,1]$.

Every claim of X at the time of occurrence, when the surplus is prior to the claim payment of u, the cedent will pay kX while the reinsurer will have to pay (1-k)X. For every premium amounted to c is received by the insurer, $c^R = (1-k)$c is to be paid to the reinsurer and $c^k = c - c^R$ is to be retained by the cedent. If k=0, then there is a chance for a full reinsurance where the entire portfolio will be ceded to the insurer, whereas k=1 would indicate no possible reinsurance. When insurance companies is able to take into full reinsurance and to receive a possible return without any risks, it is undesirable under the reinsurer's standpoint. Therefore, if $c^R = (1-k)(1+\theta)\lambda\mu$ is the reinsurance premium which is to be paid for the QS reinsurance then the insurance premium rate is going to be $c^k = c - c^R = [k(1+\theta) - (\theta - \eta)]\lambda\mu$, where $\theta \in (\eta, \infty)$ is going to be the reinsurer's safety loading. In order for the net profit condition is going to be fulfilled it is going to be,

$$[k(1+\theta) - (\theta - \eta)]\lambda\mu - k\lambda\mu > 0,$$

we need

$$k > \underline{k} = 1 - \frac{\eta}{\theta}, \tag{Eq. 15}$$

otherwise the ruin will be certain for any initial capital $u > 0$.

3.4 Diffusion Perturbed Risk Model with Investment

When an investor chooses to invest part of its surplus, into a risk-free asset and a risky asset, the return on investment process will be:

$$R_t = rt + \sigma_2 W_{2,t}, t \geq 0, R_0 = 0, \quad \text{Eq. 16}$$

where the r is the risk free interest rate so that the $R_t = rt$ which implies that one unit of investment will be worth as $e^{rt}$at the time when t. $W_2$ is another one-dimensional Brownian motion which is independent against the surplus-generating process P and $\sigma_2$is the volatility of the stock price thus the term diffused $\sigma_2 W_2$ indicates a random fluctuation within the investment returns.

3.3 Reinsurance and reinvestment

The risk process is made up of combinations from the surplus-generating process which is compounded by the proportional of reinsurance as observed in Equation 3 and the investment generating process in Equation 6. This leads to the insurance portfolio represented by the risk surplus process $U^k = \{Uk\ t\}_{t \in R+}$ which has got the dynamics of

$$dUk\ t = dPk\ t + Uk\ t - dR_t \quad \text{Eq. 17}$$

The reinsurance strategy labelled $k$ is admissible only if it is $F_{t-}$ is progressively measurable and it takes the values from the set [0,1].

3.6 Hamilton-Jacobi-Bellman Approach (HJB) and Integrodifferential

The implementation of the Hamilton-Jacobi-Bellman Approach (HJB) is to understand the approach applied and especially to understand the connection towards the Markov process theory. The modification approach towards the continuous time can be connected to the strategy $U$ for the value of $X\frac{U}{T} = x$ when the time recorded was $t$ < T is

$$V^u(t,x) = E[\int_t^T e^{-\delta(s-t)} r(X\frac{U}{s}, U_s)ds + e^{-\delta(T-t)} r_{T(X\frac{U}{T})} | F_t]. \quad \text{Eq. 18}$$

T is considered to be the stopping time. The presentation of the discussion can be simplified by considering the value functions which is not going to be dependent on t so that the partial derivative can be skipped in respect of t,

$$V^u(x) = E[\int_t^T e^{-\delta(s-t)} r(X\frac{U}{s}, U_s)ds + e^{-\delta(T-t)} r_{T(X\frac{U}{T})} | X_t = x] \quad \text{Eq. 19}$$

The entrance times such as the time of ruin are the stopping times which yields the value functions which is not going to be time-dependent. Under the law of process $\{X_s\}$, there is a dependent on $F_t$ via $X_t$ and $\{U_s: 0 \leq s \leq t\}$ only. Time-dependent value functions which can be treated from the process $\{t, X_t\}$. This is one of the most common trick within the Markov process theory since it is homogenous Markov process which is obtained.

By using Ito's formula, the infinitesimal generator from the process U$\frac{k}{t}$ from equation 9 is given by the integrodifferential operator:

$$Ag(u) = \frac{1}{2}(\sigma\frac{2}{2}u^2 + k^2\sigma\frac{2}{1})g''(u) + (ru + c^k)g'(u) + \lambda \int_0^\infty (g(u - kx) - g(u))dF(x) \quad \text{Eq. 20}$$

Since the investment generating process $R_t$ is governed by Equation 6, it follows under a weak assumptions that the ruin probability $\psi(u)$ is differentiable twice continuously on (0,∞) and the solution to

$$A\psi(u) = -\lambda \underline{F}(u) \quad \text{Eq. 21}$$

The integrodifferential operator from Equation 14 does not easily give rise to closed-form solutions which comes up to the need for the use of numerical methods which proves the Equation 16 as below:

$$\phi(u) = 0 \; on \; u < 0,$$
$$\phi(0) = 0 \; if \; \sigma\frac{2}{1} > 0, \qquad \text{Eq. 22}$$
$$\phi(u) = 1.$$

3.7 Optimal reinsurance

The diffusion approximation can be considered as a classical risk process as it claims are reinsured by the proportional of reinsurance with the level of retention being $b \in [0,1]$. Therefore the premium income $c = (1+\eta)\lambda\mu$ where the assumption is $\eta > 0$. For a claim $Y_i$, the cedent pays b$Y_i$, and the reinsurer pays (1-b) $Y_i$. The reinsurer on the other hand is expected to use an expected value principle for the reinsurance premium calculation. The premium rate for reinsurance will then be:

$$(1+\theta)\lambda E[(1-b)Y_i] = (1+\theta)(1-b)\lambda\mu \qquad \text{Eq. 23}$$

The premium rate available for the insurer will then be c(b) = (b(1+ $\theta$)-( $\theta$- $\eta$)) $\lambda\mu$. The diffusion approximation will then read

$$x + (b\theta - (\theta-\eta))\lambda\mu + b\sqrt{\lambda\mu_2 W_t} \qquad \text{Eq. 24}$$

where W is the standard Brownian motion.

3.8 Optimal investment

The diffusion approximation to the surplus process is defined as

$$X\frac{0}{t} = x + \eta t + \sigma_S W\frac{S}{t,} \qquad \text{Eq. 25}$$

where $\eta, \sigma_S > 0$. The insurer will now have an increased possibility of increasing their investment in a risky asset such as the modelled Black-Scholes model

$$Z_t = exp\{(m - \frac{\sigma\frac{2}{1}}{2})t + \sigma_1 W\frac{I}{t}\} \qquad \text{Eq. 26}$$

where $m, \sigma_I > 0$ or equivalently,

$$dZ_t = mZ_t\, dt + \sigma_I Z_t dW\frac{I}{t,} \quad Z_0 = 1 \qquad \text{Eq. 27}$$

The Brownian motions $W^S$ and $W^I$ are supposed to be independent . The insurer will have the option to choose from the amount $A_t$ when a given time of t. For instance, if the investment were to opt for strategy A, the surplus process fulfils,

$$dX\frac{A}{t} = (\eta + A_t m)\, dt + \sigma_S dW\frac{S}{t} + \sigma_I A_t dW\frac{I}{t}, \quad X\frac{A}{0} = x \qquad \text{Eq. 28}$$

3.9 Optimal investment and reinsurance

By considering both situations, both the investments and the reinsurance is possible. The controlled process will then fulfil the stochastic differential equation

$$dX\frac{Ab}{t} = (b_t\theta - (\theta-\eta) + mA_t)dt + \sigma_S b_t\, dW\frac{S}{t} + \sigma_1 A_t\, dW\frac{I}{t}, \quad X\frac{Ab}{0} = x \qquad \text{Eq. 29}$$

The HJB equation will then correspond to

$$\frac{sup}{A,b}\{\frac{1}{2}(A^2\sigma\frac{2}{I} + b^2\sigma\frac{2}{S})f"(x) + (mA + b\theta - (\theta-\eta))f'(x)\} = 0 \qquad \text{Eq. 30}$$

Since we are looking towards an increasing , twice continuously differentiable solution with f(0) = 0 and f(∞)=1. With the strategies are well admissible, it adapts a cadlag processes (A,b) with $b_t \in [0,1]$ such that $\{X\frac{A,b}{t}\}$ will be defined well. The filtration $\{F_t\}$ generates into the process $\{(W\frac{S}{t}, W\frac{I}{t})\}$.

By fulfilling the continuously differentiable strictly increasing solution by fulfilling f''(x) < 0. It yields to

$$A^*(x) = -\frac{mf\prime(x)}{\sigma_I^2 f''(x)} \qquad B^*(x) = -\frac{\theta f\prime(x)}{\sigma_S^2 f''(x)}$$

The equation can be solved into

$$-\frac{1}{2}\left(\frac{m^2}{\sigma_I^2} + \frac{\theta^2}{\sigma_S^2}\right)\frac{f\prime(x)^2}{f''(x)} - (\theta - \eta)f'(x) = 0 \qquad \text{Eq. 31}$$

## 4. Results

From the methodology, we get to identify the different methods used to achieve the probability of ruin through the discrete time method. The probability of ruin can be identified through one of the few ways which are Poisson compound, Monte Carlo simulation and Pareto claims.

### 4.1 Poisson compound

There are 3 different ways that was introduced in order to achieve the most accurate results. The first was using a Poisson compound with the continuous time *t* can be defined as :

$$U(t) = u + ct - s(t) \qquad \text{Eq. 32}$$

where u is the initial capital, c being the premium income rate per unit of time. The premium rate c is then calculated by using the premium principle of

$$c = (1 + \theta)\lambda p_1 \qquad \text{Eq. 33}$$

where □ $> 0$, when the security loading ins relative. With the risk of insolvency, the surplus becomes less than zero with the initial capital u or the probability of time ruin over an undefined time.

$$\psi(u) = Pr(U(t) < 0, for\ some\ t > 0 | U(0) = u \qquad \text{Eq. 34}$$

The probability of ruin when there is a surplus will be based on the Poisson compound of the Poisson aggregate claims as the distribution amount will be exponential such as $□_1 = 1/□$ □□ □□□□ (□) as it gains the form of

$$\psi(u) = \frac{1}{1+\theta} exp - \theta\beta u 1 + \theta \qquad \text{Eq. 35}$$

claim amount distributive was summarized $\psi_{□□}(□)$ similar to the exact ruin probability.

With the non-ruin probability of $□(□) = 1 - □(□)$ then it will go through a transformation with the function g(t) as 0<t<1 by $□(□) = □^{-1}(□)$. It can be derived as $□'(□) = \frac{□}{□□}□^{-1}(□) = \frac{1}{□'(□^{-1}(□))}$

### 4.2 Monte Carlo

Another way was using the Monte Carlo method. By using the Monte Carlo simulation, to obtain the approximation $□\frac{*}{□}$. A number of iterations is also added *n,m,* and the element of *d* to be large numbers. A sequence is then generated $\{□_1, □_2, □_3 \ldots\ldots □_□$ using an i.i.d of exp (□) and $□_1, □_2, \ldots\ldots □_0$ with $□_□$.

The amount analysed is then obtained of $\{Y\frac{\square}{2,\square}, \square\frac{\square}{3,\square}, \dots, \}$. The steps discussed earlier is then checked again by going through the steps. It is repeated based on *m* times.

4.3 Pareto claims

The third method is by using the pareto claims method. When using pareto claims with the initial surplus of *u* and a retention level of $\square \in [0,1]$ the ruin probability will be given to $\psi_{\square}(\square)$. Therefore when there are large claims the asymptotic values of the probability of ruin is given to:

$$\psi_k(u) = \frac{1}{k\theta-(\theta-\eta)}\frac{k}{1+u/k} \qquad \text{Eq. 36}$$

The probability of ruin is then minimized when $k^P = 2(\theta-\eta)u/(\theta u-(\theta-\eta))$. The pareto distribution claims will then be assuming $\theta = \eta = 1$, as it accounts to $\psi_k(u) = k/(k+u)$ where $k^P = 0$. The insurance company will have to reinsure their portfolio entirely in terms of risks.

By bringing in different values of *k* it is clear that the probability of ruin becomes smaller as k -> 0 which indicates an asymptotically optimal retention level which has to be $k^P = 0$.

From the perspectives of the exponential claims, the optimal choice for the quota-share retention *k* will maximize the adjustments of the coefficient *p(k)* which can be expressed as:

$$k^P = min\,\{(1-\frac{\eta}{\theta})(1+\frac{1}{\sqrt{1+\theta)}}, 1\} \qquad \text{Eq. 37}$$

where $\theta$ and $\eta$ are the safety loadings of the reinsurer and the insurer. By maximizing the adjustments for the coefficient yields, it becomes the asymptotically best strategy as can be expected from the retention level which is optimal $k^*$ which can be tend to $k^P$. This study from the very beginning observes a cheap reinsurance idea where $\theta = \eta$ which gives rise to the fact that $k^P = 0$. This is because of the fact that the optimal level for insurance companies to reinsure their entire portfolio itself or to take a fully proportional reinsurance.

**5. Conclusion**

In conclusion, this study focuses on the discrete insurance risk model and its ruin probability. There were multiple different models and claims which was portrayed in this research relevant to the topic in hand. In this research, the main idea was from the perspective of the insurance company and the insured. This study highlights on the risks involved from the insurer especially when there is a claim made from the insured. There were several different previous researchers which laid down their idea towards the probability of ruin from different perspectives such as the discrete time models. From this model, we get to understand behind ideas laid down especially on determining the risks that can be gathered from the insurance policies. Moving on study discusses on the models which is involved within the context of the claims made. As we understood from the literature review, the idea of zero claims from the insured would definitely be impossible and that there is bound to be a claim. Therefore the calculation or the assumption of the models related to the claims are crucial to ensure that the insurance company would be able to predict the claims from their insurance company. There were many different researchers which suggested different models which took into account different areas of the risks involved as well as the suggested. Based on the different models, the insurance risk models can be revisited again in determining the risk values or levels which is to be expected via claims.

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