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Nanofluids Flow Over a Stretching Sheet with Convective Boundary Condition

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Abstract It has been proved that nanofluids improve the thermal conduction and convection heat transfer capabilities of base liquids. Nanofluids are a novel type of heat transfer fluid that consists of both a base fluid and nanoparticles. As there has been a surge of interest in studying nanofluid flows in the last few years, this research is regarding a study on the nanofluids flow over stretching sheets with boundary conditions. In this research, the governing equations are transformed from partial differential equations into a set of nonlinear ordinary differential equations using similarity variables. Afterward, Runge-Kutta-Fehlberg method (RKF45) is adapted to solve a set of similarity equations that approach the boundary conditions. Five parameters influence the transport of momentum, energy, and concentration of nanoparticles in their respective boundary layers which are the parameters of Brownian motion *Nb*, thermophoresis *Nt*, Prandtl number *Pr*, Lewis number *Le*, and convection *Bi*. The impacts of all the parameters on boundary layers for thermal and concentration are depicted graphically. *Nb*, *Nt,* and *Bi* heating all contribute to the thickening of the thermal boundary layer. The concentration layer thickens as *Bi* enhances, but as *Le* increases, the concentration layer becomes thinner. The impact of Lewis number on the temperature distribution is minor but *Nb*, *Nt,* and *Bi* causes the local temperature to rise.

Keywords: Nanofluids, Stretching Sheet, Convective Boundary Condition

1. Introduction

Today's scientists and engineers are creating a large variety of nanoscale materials to take advantage of their exceptional properties, such as higher strength, lighter weight, better light spectrum regulation, and higher chemical reactivity compared to their larger-scale counterparts [1]. Nanofluids take the spotlight as it offers intriguing new possibilities for improving heat transfer effectiveness in comparison to pure liquids and it is considered as the future generation of heat transfer fluids. Nanofluids are colloidal suspensions of nanometre-sized particles, called nanoparticles in a base fluid [2]. Examples of nanoparticles used in nanofluids are created from metals, oxides, carbides, or carbon

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nanotubes while typical base fluids include water, ethylene glycol and oil [3]. The fluid flow over a stretching surface has numerous uses for example in extrusion, wire drawing, metal spinning and hot rolling [4]. Crane [5] first introduced the idea of a stretching sheet and developed a closed form solution for viscous fluid flow over a stretching surface. Later, it was generalised to a three-dimensional case [6]. Aziz [7] suggested the use of convective boundary conditions for the first time in analysing the Blasius flow as the convective surface boundary condition appeared to be more appropriate than the steady surface boundary condition.

After that, various studies finished with the steady surface boundary condition were reconsidered with the convective surface boundary condition. Using Runge-Kutta method with shooting technique to aid the issue numerically, it is discovered that increasing the values of buoyancy variables induces an elevation in the velocity distribution while decreasing the micro-rotation, thermal, and nanoparticle concentration in the study of Rehman et al., [8]. Next, it is prevailed that when the porous parameter is enhanced, the dimensionless velocity drops while temperature and concentration increase in Williamson nanofluid over a stretching cylinder by Ibrahim & Negera [9]. While Hayat et al., [10] studied the effect of electrically conducting second-grade nanofluid flow across intensify heated stretching sheet. The formulation of the problem incorporates Brownian movement, viscous dissipation, and thermophoretic features. When the flowing stream and wedge travel in opposite directions, numerical evidence shows the presence of a non-unique solution. Moreover, another method used is the Keller box approach by Faisal et al., [11] to explain the hydromagnetic nanofluid flow due to an unstable bi-directional stretching surface that was not equally heated. In an industrial process, enhancing heating or cooling can save energy, save time, increase temperature, and extend the equipment's working life. A great deal of work has been done to obtain a better understanding of heat transfer performance to apply it to the process of increasing the effectiveness of heat transfer.

Hence, this research is a study on the nanofluids flow over stretching sheets with boundary conditions. The objective for this research is to study the influence of a convective boundary condition on boundary layer flow, heat transfer and nanoparticle fraction over a stretching sheet.

2. Mathematical Formulation

Consider a stable state of two-dimensional flow of nanofluid boundary layer (*x*, *y*) through a stretching sheet with linear velocity contradistinction of distance x i.e. $U_w = ax$ where a is a real positive number for stretching sheet and *x* is the coordination from the position when sheet velocity is zero as displayed in Figure 1 below. The sheet surface temperature, T_w is the consequence of a convection heating process with a temperature T_f and a heat transfer coefficient h while uniform nanofluid volume fraction at the outer side of the sheet is C_w. Next, the steady temperature and the constant nanofluid volume fraction distant from the outer side of the sheet are T_∞ and C_∞. For this problem, the Boungiorno [12] model is modified to follow the continuity, momentum, energy, and volume fraction equations [13].

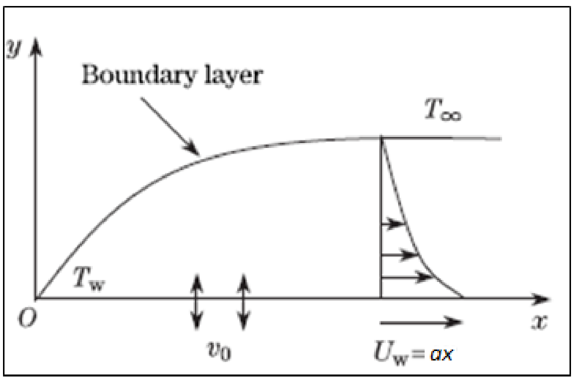

Figure 1: Geometry of problem

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad Eq.\ 1$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial x}+v\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \qquad Eq.\ 2$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial y}+v\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \qquad Eq.\ 3$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\tau\left\{D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]\right\}, \qquad Eq.\ 4$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \qquad Eq.\ 5$$

In *x* and *y* directions, u and *v* are for velocities, ρ is the fluid pressure, ρ_f is a density of base fluid, T is local temperature, v stands for kinematic viscosity of basic fluid, α represents the thermal conduction of fluid, while D_B and D_T are Brownian diffusion and thermophoresis diffusion factor respectively. Moreover, $\tau=(\rho c)_p/(\rho c)_f$ is the ratio of the heat capacity of nanoparticles to the heat capacity of fluid. The boundary conditions are stated as:

$$y=0, \quad u=ax, \quad v=0, \quad -k\frac{\partial T}{\partial y}=h\left(T_f-T\right), \quad C=C_w. \qquad Eq.\ 6$$

$$y\to\infty, \quad u=0, \quad v=0, \quad T=T_\infty, \quad C=C_\infty. \qquad Eq.\ 7$$

A simpler dimensionless form is introduced

$$\psi=(av)^{\frac{1}{2}}xf(\eta), \quad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}, \quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}, \quad \eta=\sqrt{\frac{a}{v}}y \qquad Eq.\ 8$$

where η is similarity factor, and ψ as stream function clarified as $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$, that satisfies *Eq*. 1. While *Eq*. 2 to *Eq* 5 is reduced to the following ordinary differential equations by using boundary layer estimations and similarity values *Eq*. 8 as follows:

$$f'''+ff''-f'^2=0, \qquad Eq.\ 9$$

$$\theta''+Pr\,f\theta'+Pr\,Nb\phi'\theta'+Pr\,Nt\theta'^2=0, \qquad Eq.\ 10$$

$$\phi''+Le\,f\phi'+\frac{Nt}{Nb}\theta''=0, \qquad Eq.\ 11$$

approach to the inquiry boundary conditions

$$f(0)=0, \quad f'(0)=1, \quad \theta'(0)=-Bi[1-\theta(0)], \quad \phi(0)=1, \qquad Eq.\ 12$$

$$f'(\infty)=0, \quad \theta(\infty)=0, \quad \phi(\infty)=0, \qquad Eq.\ 13$$

where primes symbolize derivative with respect to η. Further, Brownian motion denotes as *Nb*, thermophoresis *Nt*, Prandtl number *Pr*, Lewis number *Le* and convection *Bi* which are defined as

$$Pr=\frac{v}{a}, \quad Le=\frac{v}{D_b}, \quad Nb=\frac{(\rho c)_p D_B(C_w-C_\infty)}{(\rho c)_f v},$$

$$Nt=\frac{(\rho c)_p D_T\left(T_f-T_\infty\right)}{(\rho c)_f v T_\infty}, \quad Bi=\frac{h\left(\sqrt{\frac{v}{a}}\right)}{k}. \qquad Eq.\ 14$$

Due to the pressure effects, heat transport is not exist caused of nanoparticle concentration gradients when *Nb* = 0. The reduced Nusselt number *Nur* is calculated using the dimensionless temperature on sheet surface $\theta'(0)$

Eq. 15

$Nur = Re_x^{-1/2}\, Nu = -\theta'(0),$

where,

$$Nu = \frac{q_w x}{k(T_w - T_\infty)}, \qquad Re_x = \frac{u_w(x)x}{v}, \qquad Eq.\ 16$$

where q_w is the surface thermal flux and q_m is the surface mass flux.

3. Results and Discussion

After transforming the partial differential equations to the first order ordinary differential equations by utilising the similarity variables has been proven, then Runge-Kutta Fehlberg method or RKF45 is adapted to obtain the numerical results by using Maple software. Results are presented graphically to give more understanding of the subject following the inquiry and has been compared with the results obtained by Makinde and Aziz [14]. Comparisons demonstrate that for each value of the relevant parameters, there is a high level of agreement. Therefore, it showed that the present results are accurate.

3.1 Temperature profiles

Figure 2 illustrate the impact of Brownian motion and thermophoresis parameters on the temperature distribution. When both *Nt* and *Nb* are set to 0.1, the line graph starts at below which indicates a low temperature. As there is an increment in the values for *Nb* and *Nt* to 0.2,0.3 and 0.5, a gradual slope is shown suggesting a decrement in the reduced Nusselt number. It has been discovered that as *Nt* and *Nb* levels rise, so does the temperature, thus leading to quicker random mobility of nanoparticles in fluid flow and further resulting in the thickening of thermal boundary layer.

As noticed in Figure 3, the influence of Lewis number on temperature distribution is visible when near the sheet as the slopes start to assimilate further away from the sheet. When the values are set from lower to higher for *Le* which are 5,10,15, and 20, the graph shows a decrement in the temperature. Also, there will be a significant reduction for thickness in the boundary layer where there is an escalation in Lewis number.

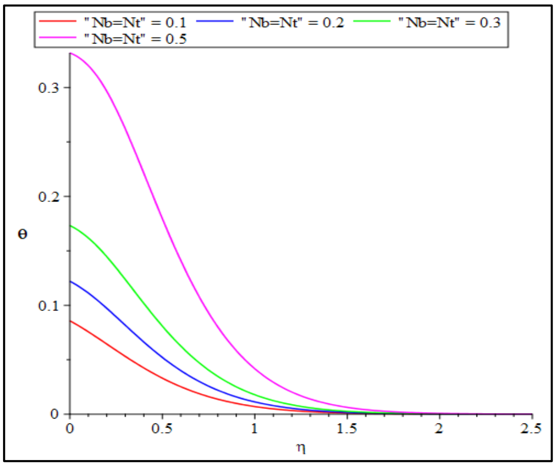

Figure 2: **Effect of *Nt* and *Nb* on temperature**
Le = 5, *Pr* = 5, *Bi* = 0.1

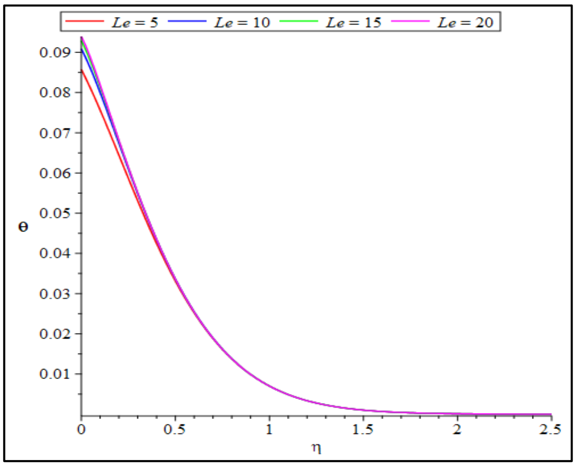

Figure 3: Effect of *Le* on temperature
Nt = *Nb* = 0.1, *Pr* = 5, *B*i = 0.1

Figure 4 depicts the effect of the Biot number on the thermal boundary layer. Apparently, thermal expansion infiltrates further into the quiescent fluid due to higher surface temperatures result from more convection which are set to 0.1, 1.0, 5.0 and 10. Next is the temperature profiles shown in Figure 5 demonstrate that when the Prandtl number escalates, the thickness of the thermal boundary layer becomes thinner as the slopes progressively elevate. As a result, the reduced Nusselt number, which is proportionate to the preliminary curve, is increasing.

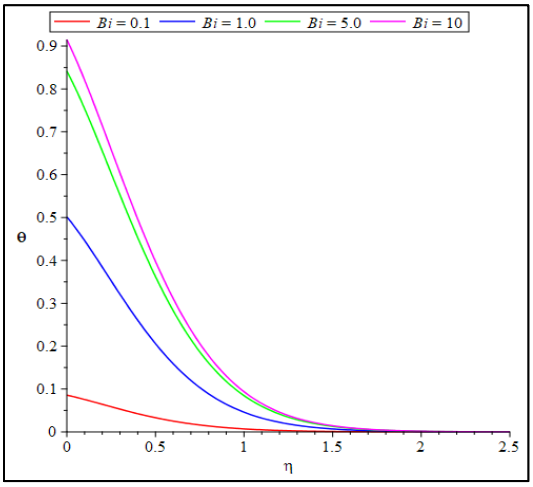

Figure 4: Effect of *Bi* on temperature
Nt = *Nb* = 0.1, *Pr* = *Le* = 5

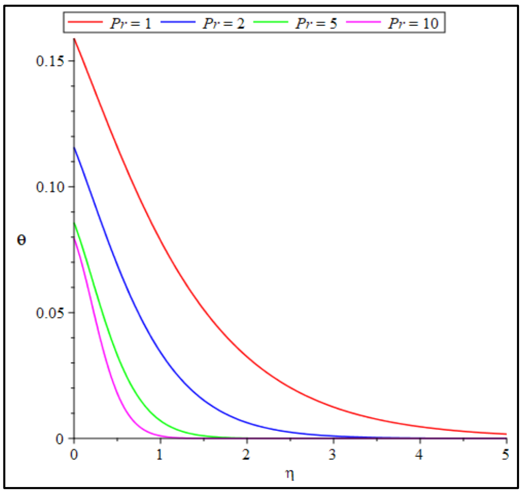

Figure 5: Effect of *Pr* on temperature
Nt = *Nb* = *Bi* = 0.1, *Le* = 5

3.2 Concentration profiles

Figure 6 depicts the nanoparticle concentration corresponding to Figure 2. The concentration profiles, unlike the temperature profiles, are very marginally impacted when there is an increment for Brownian motion and thermophoresis. A contrast for Figure 3 and 7 reveals that the Lewis number has a large influence on the concentration distribution as shown in Figure 7 but only has minimal impact on the temperature profile shown in Figure 3. A greater Lewis indicates a decrease Brownian diffusion coefficient D_B, relate to *Eq*. 14 for a basic fluid of a specific kinematic viscosity *v*, which must result in a reduced penetration deepness for the concentration boundary layer [3].

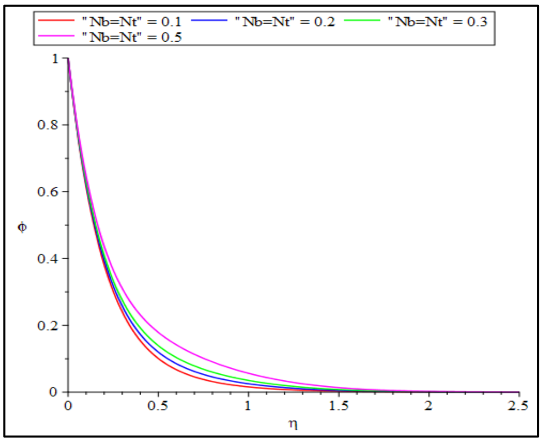

Figure 6: Effect of *Nt* and *Nb* on concentration
Le = 5, *Pr* = 5, *B*i = 0.1

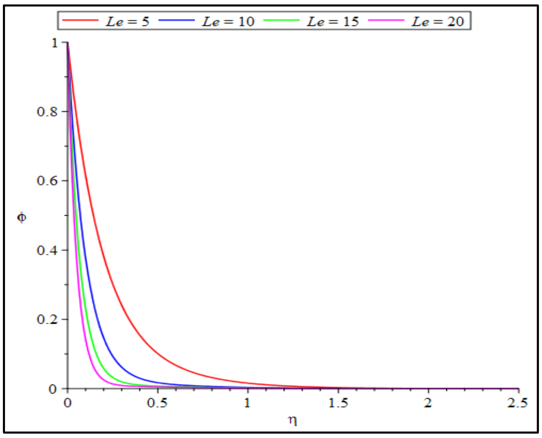

Figure 7: Effect of *Le* on concentration
Nt = *Nb* = 0.1, *Pr* = 5, *Bi* = 0.1

The thermal penetration depth increases when the convection heating of the sheet increases, as seen in Figure 8. Due to the concentration profile being influenced by the temperature field, a greater convection is expected to facilitate deeper into concentration. This prediction is achieved in Figure 8, which forecasts larger concentrations with increasing Biot numbers. Figure 9 is the influence of Prandtl number on concentration profiles, but we can barely see any changes in concentration graph for the input of difference values *Pr* for values 1, 2, 5, and 10.

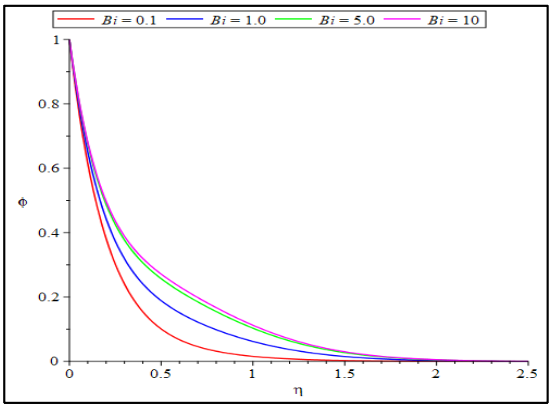

Figure 8: Effect of *Bi* on concentration
Nt = *Nb* = 0.1, *Pr* = *Le* = 5

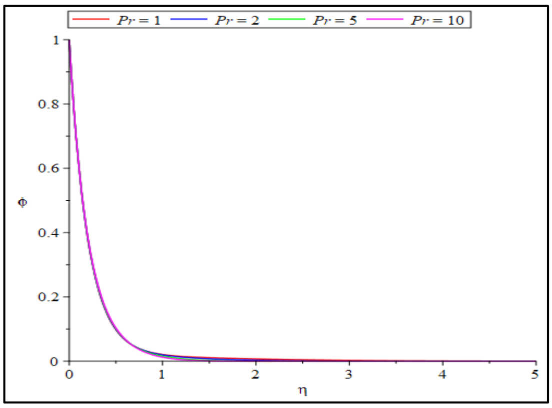

Figure 9: Effect of *Pr* on concentration
Nt = *Nb* = *Bi* = 0.1, *Le* = 5

Theinfluence Brownian motion and thermophoresis on the local Nusselt number against the convection *Bi* is shown in Figure 10. The degree of convective heating upon the sheet surface intense as the variable *Bi* grows, resulting in a rising transfer of heat from one place to another (lower surface to top surface) by the movement of nanofluids. This leads the pattern of the graph for local Nusselt number grows as the convection rises thus resulting in thickening the thermal boundary layer.

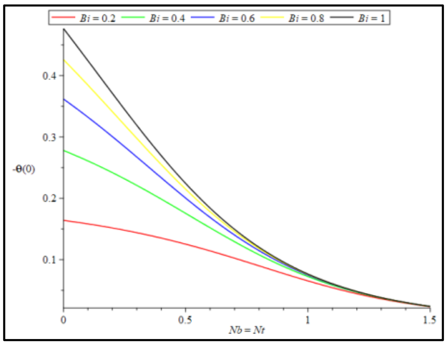

Figure 10: Local Nusselt number graph for

4. Conclusion

In this study, nanofluids flow over a stretching sheet is considered. The derivation of governing equations from partial derivative equations to a set of nonlinear ordinary derivative equations through similarity variables has been proven. Numerical solutions are obtained by employing the Runge-Kutta Fehlberg method in attempts to reproduce the results by Makinde and Aziz. Results are depicted visually to illustrate the influence of the five parameters. Comparisons demonstrate that for each value of the relevant parameters, there is a high level of agreement. Therefore, it showed that the current results are reliable. Five parameters influence the transport of momentum, energy, and concentration of nanoparticles in their respective boundary layers which are the parameters of Brownian motion *Nb*, thermophoresis *Nt*, Prandtl number *Pr*, Lewis number *Le*, and the Biot number. Based on the results, Brownian motion, thermophoresis, and convective heating all contribute to the thickening of the thermal boundary layer. The concentration layer thickens as *Bi* increases, but as *Le* increases, the concentration layer becomes thinner. Although the influence on the temperature distribution cause by Lewis number is minor, the local temperature rises as Brownian motion, thermophoresis, and convective heating all enhance. Moreover, local Nusselt number is spotted to grow while convection rises.

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