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Analysis of Fourier Series in Acoustic

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Abstract: There is a relationship between sounds and the Fourier series. Some researchers found that the Fourier series is one of the applications in music that calculates the frequency by using the amplitude of the wave. Hence, this research is mainly related to kids' hearing and hearing is important because it is one of the five senses, and it is vital for warning and communication from afar, so there is a problem statement for this situation. Nowadays, most of the sounds that have been exposed to the kids are more random and do not have any range that the guardian put on a limit that can damage their hearing Therefore, the aims of the research are to study the method of Fourier series in acoustics and develop the simulation using MATLAB. The Fourier series and partial differential equation has been used because the variable and formula are suitable for sound with periodic function. Then, the wave motion from the sound is analysed using the platform of the graph. Three data have been used in this research to compare and make a conclusion from the difference in the pattern of the graphs shown. The analysis from the research shows that when the frequency increase, the number of waves also increases. Therefore, we can conclude that the higher frequency can affect the kids' hearing because the motion of waves that the kids receive is high.

Keywords: Fourier Series, Acoustic, Partial Differential Equation, Frequency, Kids' Hearing

1. Introduction

Nowadays, many teaching tools for children use music to get their attention and learn new things easily. According to many reports, using music to encourage interaction between two hemispheres leads to more innovative thought [1]. Some television programmes have songs that are trending among children, such as Didi and Friends. Some cases include the children in the Netherlands that 14.2% of age-school children were high-frequency hearing loss because of the noises [2].

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Hearing loss on the children will have the potential to lack learning in their growth, including independent learning and language learning [3]. Therefore, the future learning of the kids depends on their hearing at an early age because most of the things that they hear are one of their experiences in their lives.

However, this method has to note the frequency of the sound used for children to prevent hearing loss. There are some facts that children with hearing loss will make it difficult to understand people's surroundings and affect the learning process because the communication between the children and others is limited [4].

Therefore, some action should be taken to prevent the children from hearing loss, so people that are on duty to take care of the children must use the suitable frequency by new technology. In order to do that, this research has two aims which are studying the method of Fourier series in acoustics and developing a simulation of Fourier series in acoustics by using MATLAB. The expected outcome is the graphs of each in frequency must be recorded, and each data must follow the concept that the frequency of sounds cannot over limited from the frequency guidelines for the kids' hearing.

At first, the main topic for this research is the partial differential equation (PDE) which means partial derivatives in the equation [5]. The difference between ordinary differential equation (ODE) and PDE is depends on the number of the independent variables in the equation. ODE has only one independent variable needed in the differential equation, but PDE depends on two or more independent variables. Therefore, PDE will be more complicated and implicit in solving.

In PDE, one-dimensional waves are the only formula that has been found with the travelling wave at once instant of time *t*. However, there is some experiment of the formula by using product solution, so the other formula and concept have been created. Therefore, the other knowledge about the one-dimensional waves has been changed to Fourier series by adding some condition that is the summation on the final formula of one-dimensional waves.

After that, this research focused on wave equations related to the Fourier series, which can form a frequency of music for children. Hence, the research target is children because the frequency of sound for children is more sensitive and prevents early hearing loss compared to the adult.

The relation between the Fourier series and sound is one of the reasonable applications of the Fourier series in this research. The amplitude and frequency of sound can be found with the Fourier series method [6].

In conclusion, PDE is suitable for this research because of the one-dimensional waves produced by the Fourier series. The Fourier series is wave motion that includes the repetition, time, and combination in trigonometric function which is sine and cosine. Therefore, the number of waves and the pattern of waves shown in the analysis then and each repetition will make the different pattern based on the frequency.

Therefore, the PDE and Fourier series are needed to create a solution in detecting the suitable sound for children and prevent unsuitable frequency that will affect their learning process by getting hearing loss. The pitch and frequency of sounds in children's toys are the purposes of the study conducted to ensure the hearing health of kids and make sure their growth is the best state so they can have a normal life and a great experience in getting experience in their young age.

2. Methodology

Kids' hearing is one concern in the research that involved the frequency of sound in kids' toys. Since there is a relation between the Fourier series and sound, then this research applied the PDE and the Fourier series as the theories to solve the problem.

2.1 Partial Differential Equation

One-dimensional (1-D) wave behaviour on the vibration of a string is mathematically described by the 1-D wave equation [7]:

$$\frac{\partial^2\psi(x,t)}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2\psi(x,t)}{\partial t^2} = 0 \qquad \text{Eq. 1}$$

where $\psi(x,t)$ is the instantaneous transverse displacement amplitude of the string at the point x at the time t. We can rewrite this as

$$\frac{\partial^2\psi(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi(x,t)}{\partial t^2} \qquad \text{Eq. 2}$$

From the formula, the left-hand side (LHS) of the Eq.2 has only space-derivatives of the one-dimensional variable and x associated with it. Meanwhile, the right-hand side (RHS) of the equation has only time-derivatives associated with it. There is a suggestion of technique for the wave equation, which is a product solution that can produce $\psi(x,t) = U(x)T(t)$ where $U(x)$ contains only spatially *x*-dependent terms and $T(t)$ contains only temporally time-dependent terms.

As the reference, it insert $\psi(x,t) = U(x)T(t)$ into the above differential equation, and then explicitly carry out the differentiation:

$$\frac{\partial^2 U(x)T(t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 U(x)T(t)}{\partial t^2} \qquad \text{Eq. 3}$$

It explicitly used the fact that $T(t)$ and $U(x)$ are show as functions of time which is *t* and space which is *x* only, respectively.

$$T(t)\frac{\partial^2 U(x)}{\partial x^2} = \frac{1}{v^2}U(x)\frac{\partial^2 T(t)}{\partial t^2} \qquad \text{Eq. 4}$$

Since $T(t)$ and $U(x)$ are shows as functions of time, *t* and space, *x*, it converts the partial derivatives of *t* and *x* to total derivatives of *t* and *x*.

$$T(t)\frac{d^2 U(x)}{dx^2} = \frac{1}{v^2}U(x)\frac{d^2 T(t)}{dt^2} \qquad \text{Eq. 5}$$

Last but not least, from Eq. 5 it divides both sides of the equation with $\psi(x,t) = U(x)T(t)$ and then it considers as the function of LHS is entirely function of $U(x)$ and RHS is entirely function of $T(t)$.

$$\frac{1}{U(x)}\frac{d^2 U(x)}{dx^2} = \frac{1}{v^2}\frac{1}{T(t)}\frac{d^2 T(t)}{dt^2} = -k^2 \qquad \text{Eq. 6}$$

The equation must be satisfied for all possible values of *x* and *t*, so it needs to be equal to a constant which is $-k^2$ for both sides of the latter equation. The separation constant is one of the separations of variables technique used because of the constant that arises.

By using the separation of variables technique, it comes out with two wave equations:

$$\frac{d^2U(x)}{dx^2} = -k^2U(x) \qquad \text{Eq. 7}$$

$$\frac{1}{v^2}\frac{d^2T(t)}{dt^2} = -k^2T(t) \qquad \text{Eq. 8}$$

Now $v = f\lambda = (\omega/2\pi)(2\pi/k) = \omega/k$, so it can conclude $vk = \omega$. Insert and rewrite the two equations above:

$$\frac{d^2U(x)}{dx^2} + k^2U(x) = 0 \qquad \text{Eq. 9}$$

$$\frac{d^2T(t)}{dt^2} + \omega^2T(t) = 0 \qquad \text{Eq. 10}$$

The coordinate system such that the *x*-axis coincides with the equilibrium shape of the string, one end of the string at *x* = 0, and the other end at *x* = *L*. Mathematically, the so-called boundary condition at the ends of the string is the displacement amplitude is zero at the ends of the string include $\psi(x=0,t) = \psi(x=L,t) = 0$ for fixed ends, independent of time, *t*.

Thus, the boundary condition is only relevant to the *U*(*x*)-wave equation and the boundary condition for fixed ends is $U(x=0) = U(x=L) = 0$. In general, there are only two allowed spatially periodic standing wave solutions to the wave equation. Either sin *kx*, or cos *kx*. The boundary condition above for fixed ends allows only the sin *kx*-type solutions because sin 0 = 0, and sin *kL* = 0 if and only if $kL = n\pi$, *n* = 1, 2, 3, 4, … . Denoting $k_n = n\pi/L$, we see that the allowed standing wave solution to the *U*(*x*)-wave equation are of the form:

$$U_n(x) = A_n\sin(k_nx) = A_n\sin(n\pi x/L) \qquad \text{Eq. 11}$$

We also see that the boundary condition(s) on the spatial $U(x)$ standing wave solutions to the $U(x)$-wave equation determine the frequencies of the eigen-modes/normal modes of vibration. Since $\lambda = 2\pi/k$, then $\lambda_n = 2\pi/k_n = 2\pi L/n\pi = 2L/n$, *n* = 1, 2, 3, 4, …Since $f = v/\lambda$, then $f_n = v/\lambda_n$ and thus $f_n = v/\lambda_n = nv/2L$, *n* = 1, 2, 3, 4, …

Also, since spatial wavelengths and temporal frequencies are intimately related to each other via $f = v/\lambda$ and hence also the eigen-frequencies, $f_n = v/\lambda_n = nv/2L$, ® $\omega_N = v_n = n\pi v/L$, then the allowed solutions to the temporal Helmholtz equation, $T(t)$□$T_n(t)$, with both $\sin\omega_nt$ and $\cos\omega_nt$ type solutions allowed. Note that the eigen-frequencies for standing waves of a string with fixed endpoints are integer-multiples of the lowest mode of vibration, $f_n = nf_1$, n = 1, 2, 3, 4, 5, …. with $f_1 = v/2L$ and $\lambda_n = \lambda_1/n$ with $\lambda_1 = 2L$.

In reality, the detailed shape of the vibrating string at some point in time known as the initial conditions determines the relative amount of the allowed $\sin\omega_n t$ and $\cos\omega_n t$ type solutions because this simply specifies, for each eigen-mode, what its phase is or how far along in its oscillation cycle it is at time t = 0.

Temporal phase information is encoded into the time and space Helmholtz equation's allowable eigen-solutions, $T_n(t)$, as follows:

$$T_n(t) = b_n \sin\omega_n t + c_n \cos\omega_n t \qquad \text{Eq.12}$$

With the following conditions on the coefficients b_n and c_n:

$$-1 \le b_n \le 1$$

$$-1 \le c_n \le 1$$

$$\sqrt{b_n^2 + c_n^2} = 1$$

One could equivalently write the allowed eigen-solutions of the temporal Helmholtz equation simply as one or the other of the following forms:

$$T_n(t) = \sin(\omega_n t + \delta_n);\ \ \delta_n = \tan^{-1}(c_n/b_n) = \cot^{-1}(b_n/c_n)$$

$$T_n(t) = \cos(\omega_n t + \varphi_n);\ \varphi_n = \tan^{-1}(b_n/c_n) = \cot^{-1}(c_n/b_n)$$

$$\delta_n = \varphi_n + \frac{\pi}{2}$$

These relations can be obtained directly from the above $T_n(t) = b_n \sin\omega_n t + c_n \cos\omega_n t$ relation using the trigonometric identities for $\sin(A+B)$ and $\cos(A+B)$, respectively.

It can be written as the allowed eigen-solutions of the temporal Helmholtz equation in yet another, equivalent form, using complex notation:

$$T_n(t) = e^{i(\omega_n t + \varphi_n)} \qquad \text{Eq. 13}$$

The complete eigen-function solution $\psi(x,t) = U(x)T(t)$ for standing waves on a vibrating string with fixed ends are of the form:

$$\begin{aligned}\psi(x,t) &= U(x)T(t)\\ &= A_n \sin(k_n x)[b_n \sin \omega_n t + c_n \cos\omega_n t] = A_n \sin(n\pi x/L)[b_n \sin(n\pi vt/L) + c_n \cos(n\pi vt/L)]\\ &= A_n \sin(k_n x)\sin(\omega_n t + \delta_n) = A_n \sin(n\pi x/L)\sin[(n\pi vt/L) + \delta_n]\\ &= A_n \sin(k_n x)\cos(\omega_n t + \varphi_n) = A_n \sin(n\pi x/L)\cos[(n\pi vt/L) + \varphi_n]\\ &= A_n \sin(k_n x)e^{i(\omega_n t + \varphi_n)} = A_n \sin(n\pi x/L)e^{i(\omega_n t + \varphi_n)}\end{aligned} \qquad \text{Eq. 14}$$

From the final of the PDE it shows that the Fourier series is made from the PDE with the other condition to make a new concept in order to fulfil the analysis of sounds.

2.2 Fourier Series

Given a function *f*(*x*) in the form

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos nx + b_n \sin nx] \qquad \text{Eq. 15}$$

Fourier series representation of *f*(*x*) defined on [0, 2*π*] when it exists, is given by Eq. 15 with Fourier coefficients

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx dx, \quad n = 0, 1, 2, \ldots \qquad \text{Eq. 16}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx dx, \quad n = 1, 2, \ldots \qquad \text{Eq. 17}$$

These expressions for the Fourier coefficients are obtained by considering special integrations of the Fourier series.

2.3 The Application of Fourier Series in Sound

There is a natural appearance of such sums over sinusoidal functions in music [8]. A pure note can be represented as

$$y(t) = A\sin(2\pi ft) \qquad \text{Eq. 18}$$

where A = amplitude, f = frequency in hertz (Hz), and t = time in seconds.

When a function is periodic with period T if $f(t+T) = f(t)$ for all t and the smallest such positive number T is called period.

In general, if $y(t) = A\sin(2\pi ft)$, the period is found as $T = \frac{2\pi}{2\pi f} = \frac{1}{f}$. We have to add shifted function to get the Fourier analysis, so we get $y(t) = A\sin(2\pi ft + \varphi)$.

Phase of sine function is $2\pi ft + \varphi$ and φ is called phase shift. Trigonometric identity can be used for the sine of the sum of two angles to obtain

$$\begin{aligned} y(t) &= A\sin(2\pi ft + \varphi)\\ &= A\sin(\varphi)\cos(2\pi ft) + A\cos(\varphi)\sin(2\pi ft)\end{aligned} \qquad \text{Eq. 19}$$

So, defining $a = A\sin(\varphi)$ and $b = A\cos(\varphi)$ we can rewrite this as $y(t) = a\cos(2\pi ft) + b\sin(2\pi ft)$.

As shown in Eq. 18 deriving to get the Fourier series which the sum of cosine and sine. The value of *A* and φ can be found by the formula $A = \sqrt{a^2 + b^2}$, $\tan\varphi = \frac{b}{a}$.

2.4 Developing a simulation using MATLAB

In this research, a simulation for the Fourier series in acoustics in developed using MATLAB. Then, analyze the comparison of the graph pattern using different frequencies in Hertz in order to find the suitable frequency of toys or songs for kids.

3. Results and Discussion

Three different frequencies have been used to analyze the pattern of wave motion using MATLAB. The graphs of the frequency in 10 Hz, 20 Hz, 30 Hz are shown in Figure 1, Figure 2, and Figure 3 below.

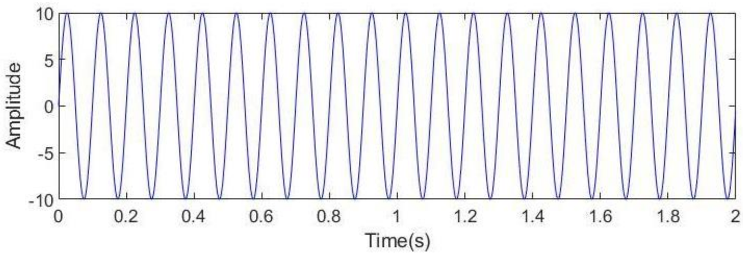

Figure 1: Time(s) versus Amplitude by Using Frequency in 10 Hz

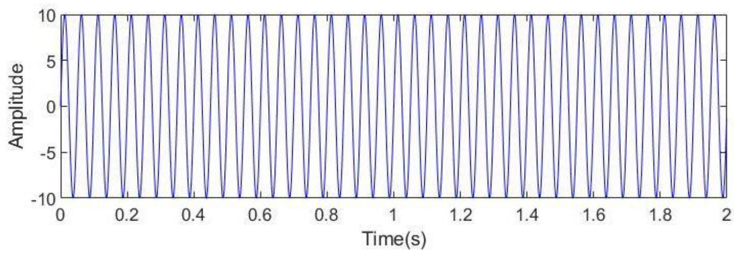

Figure 2: Time(s) versus Amplitude by Using Frequency in 20 Hz

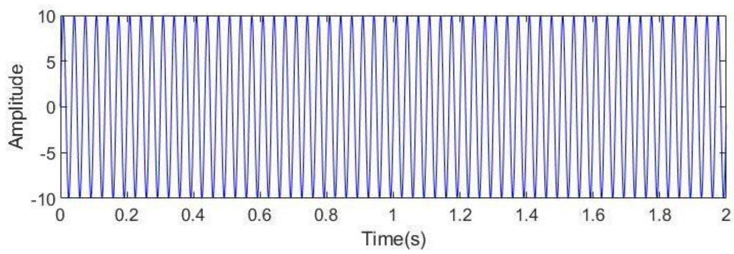

Figure 3: Time(s) versus Amplitude by Using Frequency in 30 Hz

As we can see in Figure 1, Figure 2 and Figure 3, the pattern of waves depends on the dependent variable while the independent variable is only the variable that is constantly the same to carry out the analysis. Thus, it can figure out the dependent variable and independent variable based on the number of waves produced from the graph, so it can conclude that the dependent variable is frequency, but the independent variable is time.

The comparison of the graph can be seen through the pattern of each frequency used in this research. As the results shown, the frequency of 10 Hz is the least waves, and the 30 Hz is the most waves, so the frequency of 20 Hz has the middle number of the waves between 10 Hz and 30 Hz. Other than that, the distance between the waves is difference which is 10 Hz has the longest distance between waves while 30 Hz has the shortest distance between waves. The graph frequency of 20 Hz has a middle distance between 10 Hz and 30 Hz.

Therefore, the comparison of each frequency shows that from the frequency of 10 Hz to 30 Hz, it can conclude as when the frequency increases, the number of waves also increases. Moreover, the analysis can be made by the distance between the waves are getting closer and shorter wavelengths as the frequency increase. Thus, we can conclude that the dependent variable which is frequency is important since it can give an impact to the pattern of the graphs.

Next, the other analysis of these waves shows that the pitch is also increasing because more waves are produced. The pitch is related to the level of sounds received by each individual, especially the kids. That mean, the higher frequency can cause harm to kids to hear that sounds. Based on the analysis of other research, the highest frequency to the kids can adapt of sounds for a normal hearing is 500 Hz [9].

However, the analysis also can be interrupted by the pressure of the instrument that produced the sounds. This means that the higher pressure, the more harm the kids' hearing. Other than that, the length of the sounds to the kids can also be one factor that can be harmful. The distance between the other object, like humans and the source of a sound, provides the different levels of sound received [10]. The lower volume sound received if the object is farther from sound sources. It shows that the volume of the toys or songs that the kids received is related to the pressure of the distance from the sound sources.

The analysis can be conclude as when the higher frequency, the higher pitch that can be produced. Thus, the effect of the high frequency is causing the loss of hearing for the humans especially the kids. Thus, the kids must be controlled by the guardian because there is a limit for the kids to listen to any sounds. The guardian should identify the safe toys to give to the kids hear to either the toys or songs for a long time cause harm to the kids hearing.

Even though this analysis is a success, there is some disadvantage which is the distance of the sound source that is not involved in this analysis but it is important to measure the level of sound received by the kids. Therefore, the analysis has the limit to get accurate in hearing control.

4. Conclusion

The conclusion is each sound has its own frequency, especially kids' toys. Therefore, knowledge about the frequency and pressure of the sounds is important for the kids to ensure their health and hearing. Then, the guardian can control the kids from hearing the unsafe frequency since there is a limit for the kids to listen to any sounds.

Many techniques have been recorded for the other research, but in this research, we used partial differential equation (PDE) and Fourier series as the method. PDE can be one technique in mathematics to include in order to find the suitable frequency for kids. The Fourier series method in this research is simple but suitable for the frequency as it shows the formula has the results of the graph.

By developing the model using MATLAB, the pattern of the graph using different frequencies can be shown. As the result of the analysis, we can conclude that the higher frequency, the number of waves increases. Thus, it can affect the hearing of kids. The data that has been used only the random selection to make a better understanding for the reader and easy to apply for real life.

However, the analysis has limit and it can be interrupted by the pressure of the toys or songs and the distance of the kids and the source of sound because the hearing of the children damaged by those reasons.

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