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Estimation of The Time of Death Using Newton's Law Cooling

Nor Afiza Ramli[1], Cik Sri Mazzura Muhammad Basri[1]*

[1]Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology,
Universiti Tun Hussein Onn Malaysia, 84600 Pagoh, Johor, MALAYSIA

*Corresponding Author Designation

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Abstract: The determination of the time of death is essential to forensic investigation since the time of death represents the presumed time of the offence occurred. Determining the time of death based on investigations of the body still offers great difficulties in forensic science and it is one of the most popular issues especially in the investigation of criminal or suspicious deaths and civil cases. It is relevant in civil law to determine the order of inheritance in case there are multiple deaths occurred among family members at the same time, and it may cause a problem to the police and other law agencies in carrying out the investigations if not knowing the death time. Therefore, this research is to determine the time of death using Newton's law of cooling model. In this study, three different cases taken from previous studies have been considered and used to predict the time of death. All cases are solved using Newton's law of cooling model in order to determine the time of death and cooling period. Then, using MATLAB, the death time calculations and graphical representations of cooling are presented. Newton's law of cooling graph shows that it is inversely proportional with the temperature difference between the body temperature and its surroundings and the time. In the beginning, the temperature drop is relatively fast, but it becomes slower as the body temperature decreases. This study can be concluded that this cooling model provides an approach to estimate the time when a crime or offence is committed.

Keywords: Newton's Law Of Cooling, Body Cooling, MATLAB, Temperature, First Order Differential Equation, Separable Equation.

1. Introduction

Determining the time of death is the main responsibility of forensic investigators since it represents the estimated time the offence occurred. For obvious reasons, the estimations are particularly crucial in the investigation of criminal and undetermined deaths. An exact estimation of time since death is essential for criminal law as it verifies the statements made by the witnesses, restricts the number of suspects and assesses their justifications [1].

Corresponding author: mazzura@uthm.edu.my

Generally, the estimations are especially important in the investigation of criminal or suspicious deaths and civil cases. A study by [2] shows that determining the time of death is sometimes also important regarding civil law since it may affect the sequence of heritage or possible responsibilities resulting from the order of deaths. According to [3], sometimes it is relevant in civil law to determine the order of inheritance in case there are multiple deaths occurred among family members at the same time. Unfortunately, it may be difficult for the police and other law agencies to carry out the investigations if the death time is unknown. Besides, criminal investigations sometimes have problems when a lot of people are arrested in identifying the person involved in the crime.

A wide variety of techniques have been used, and the most popular estimation method is based on body cooling, also known as algor mortis. The evidence provided by the body, information from the environment and historical evidence are three different techniques. The body cooling method has the benefit that it would estimates the time of death with more precise and less time duration, compared to other methods used to estimate the time of death can take a long time. Hence, it will affect the process of investigation that will be conducted. That is the reason determining the time of death based on investigations of the body still offers great difficulties in forensic science and it is one of the most popular issues. Thus, this study aims to determine the time of death using Newton's law of cooling. In order to find the estimated time, the model will be developed using MATLAB software, and then the result will be analyzed.

Newton's law of cooling states that the rate of body heat loss is directly proportional to the difference in body temperature and its surroundings. Besides, this law was generally used to estimate the time passed after death. This law is named after British scientist Isaac Newton, in the late 17th century, who conducted an experiment on the cooling of bodies. Despite the fact, it is shown that the rate of cooling is approximately proportional to the difference of temperatures between the heating body and temperatures of ambient.

In 1868, a Professor at Glasgow, Dr. Henry Rainy, preferred the rate of heat loss of a dead body could be used to estimate the time of death [4]. Rainy used sequential rectal temperature measurements of the dead bodies to find and calculate the rate of heat loss per hour [5]. A study by [2] shows that by repeatedly measuring the temperature in the rectum of a dead body, Rainy succeeded in determining the gradient of the temperature drop curve experimentally over time, corresponding to Newton's cooling coefficient. Furthermore, Rainy stated that skin temperature measurement in the determination of time of death is not precise when applied to human bodies. While [3] presented a verification of the thermodynamic model allowing an estimation of the time of death by calculating the time interval based on a single eyeball temperature measurement at the death scene. The study was performed on 30 cases with a known time of death due to sudden natural or violent death using pin probes associated with a high accuracy electronic thermometer (Dostmann-electronic).

Also, [3] shows that the post-mortem interval was determined based on Newton's law of cooling model, recently tested on pigs and effectively tried through investigation on three human cases. Therefore, a significant decrease in eye temperature is more rapid, as well as almost no influence of body mass and time of death in human death could be assessed with good accuracy. Moreover, [6] states that the main principle of death time estimations is the calculation from a measured value along the curve depending on the time of return to the starting point. The characteristics of the curve such as the slope and the initial value are affected by internal and external, antemortem and post-mortem conditions. Comparison of various techniques requires research of the same post-mortem interval. For practical purposes, it must be concluded that the measure of writing on estimating the time of death has an inverse correlation with its importance in practice.

2. Methodology

A mathematical model differential equation which is Newton's law of cooling will be used in this study. Generally, various type of problems arises in the differential equation and death time estimation is a part of it. The study of differential equations was full of unexpected connections since it provides critical tools for analysing a wide range of relevant occurrences, starting with basic physical principles [7]. In addition, Newton's law of cooling model is solved using a separable equation. In this study, a separable equation of Newton's law of cooling will be used to solve and estimate the time of death.

2.1 Separable Equation

This type of equation is usually in the form:

$$f(y)dy + g(x)dx = 0 \text{ or } \frac{dy}{dx} = \frac{f(x)}{g(y)} \qquad \text{Eq. 1}$$

and the general solution is:

$$\int f(y)dy + \int g(x)dx = c \qquad \text{Eq. 2}$$

According to [7], a first order differential equation can be solved using integration if it is possible to gather all y terms with dy and all x terms with dx.

2.2 Model Assumptions

There had some assumptions of the model that needed to be considered before calculating the estimated time of death. These assumptions were important to help this study. The following are the assumptions of the model [8]:

- The surrounding temperature, T_s is assumed to remain unchanged.
- The body temperature is the same as its surface temperature that assumed as normal body temperature or uniform cooling.

2.3 Newton's Law of Cooling Model

According to the law, the rate of change of the temperature $\frac{dT}{dt}$, is (by Newton's law of cooling) proportional to the difference between the temperature of the body T(t) and the surrounding temperature, T_s. It would vary as:

$$\frac{dT}{dt} \propto (T - T_s)$$

That implies:

$$\frac{dT}{dt} = -k(T - T_s) \qquad \text{Eq. 3}$$

The negative sign indicates that as time increases, the temperature will decrease. If the body temperature is hotter than the environment temperature $T(t) - T_s > 0$, the body temperature will decrease and the derivative $\frac{dy}{dx}$ should be negative [1].

2.4 Solution to The Model

From Eq. 1, used the technique of separable equation to get the solution which is,

$$T(t) = T_s + (T_0 - T_s)e^{-kt} \qquad \text{Eq. 4}$$

where,

$T(t)$ is the body temperature at time t

T_s is the surrounding constant temperature (ambient temperature)

T_0 is the initial temperature of the body
t is the time in preferred units (minutes or hours)
k is a constant to be found

2.5 Develop the Model using MATLAB Software

In this research, Newton's law of cooling model is developed using MATLAB in order to estimate the time of death. Furthermore, the results of cooling constant, k and the time of death, t also can be obtained. In this study, three different cases taken from previous studies have been considered and used to predict the time of death. Thus, using MATLAB, the death time calculations and graphical representations of cooling for each case are presented.

3. Results and Discussion

Data for analyzing the model were taken from three different cases. The data for Case 1 was obtained from Imperial Journal Of Interdisciplinary Reseach (IJIR), Volume 3 by M. Suneeta Rani, and for Case 2, it was taken from International Invention, Innovative & Creative (InIIC) Conference, Volume 2 by Rahmah Shahril, Maznah Banu, Mohamed Habiboo Raman and Rasidah Buang. In addition, the data for Case 3 was obtained from International Journal of Management, IT and Engineering, Volume 3 by Boniface O. Kwach, Omolo Ongati, Nyakinda J. O and Rachael Nyang'inja. It had been considered and used in predicting the time of death to test computer solutions and it was obtained from the following sources in the literature:
Case 1:

The body is located in a room that is kept at a constant temperature of 68.0°F. The lieutenant arrived at 9:40 P.M. and measured the body temperature as 94.4°F at that time. Another measurement of the body temperature at 11:00 P.M. is 89.2°F. Find the estimated time of death and normal body temperature is 98.6°F [9].

Case 2:

A man was killed in his apartment and the body was discovered at 7:00 A.M. The coroner noted that the room temperature was kept the same level all the time at 70°F. The temperature of the body was 72.5°F the moment it was found, and after 60 minutes, the temperature dropped to 72°F. Let T(t) be the temperature t in minutes of the body and the normal body temperature is 98.6°F [10].

Case 3:

The police officer arrived at a crime scene at 10:40 P.M. and the body temperature was 94.4°F. The body was located in a room where the room temperature is kept at a constant 70°F. Not long after the death, the body temperature decreased, and assumed that the victim's temperature was 98.6°F at the time of death. After 90 minutes, the body temperature was recorded at 89°F [11].

3.1 Results and Discussion

Analysis of three cases obtained from the sources above is presented. An application of the first order ordinary differential equation, Newton's law of cooling model was used to find the estimation of the time of death occurred. Table 4.1 below summarizes all the three cases together with the results of the cooling constant, k and the time of death, t that had been obtained by MATLAB software.

Table 1: Analysis of result

Case	1	2	3
Initial temperature, T_0, (°F)	94.4	72.5	94.4
Surrounding temperature, T_s, (°F)	68	70	70
Body temperature at time *t*, *T*(*t*), (°F)	89.2	72	89
Time since the first reading, *t* (minutes)	80	60	90
Normal Temperature at time *t* (°F)	98.6	98.6	98.6
Cooling constant, *k*	0.16452 /hour	0.22314 /hour	0.16676 /hour
Time of death, *t*	8:47 P.M.	8:05 P.M.	9:43 P.M.

Table 1 shows the information gathered in three cases with the result of cooling constant, *k* and when the time of death occurred, *t* that had been obtained and generated using MATLAB software. Besides, the software performs several calculations to display the results, in order to find *k*, the temperature of initial, surrounding, body temperature and also the time since the first reading were used. Then, the time of death was determined by using the *k* and normal temperature at 98.6°F.

Next, using MATLAB software, the death time calculations and graphical representations of cooling are illustrated in Figure 1, Figure 2 and Figure 3 below. Then, Figure 4 shows the combined plots for the determination of the time of death for Case 1, Case 2, and Case 3.

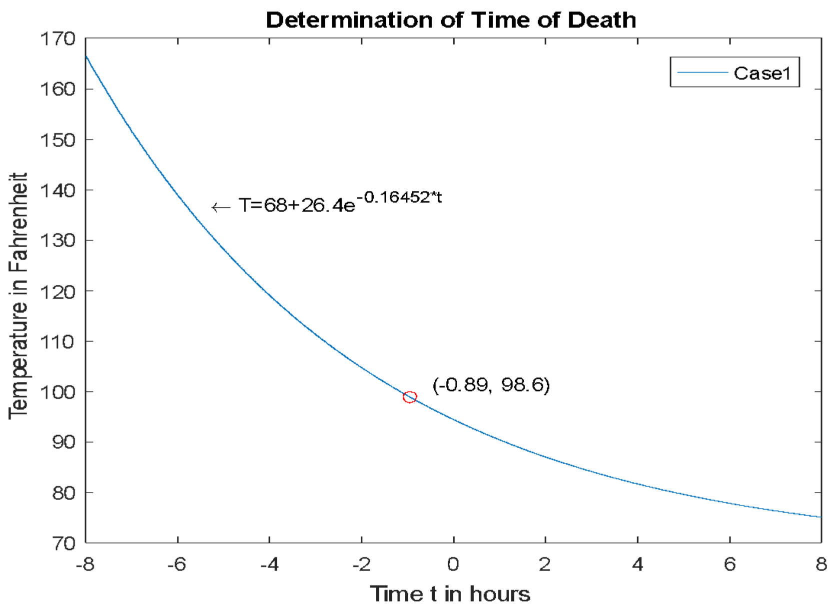

Figure 1: Determination of time of death for Case 1

Figure 1 shows the graph for the determination of time of death for Case 1. Based on the cooling curve above, an exact solution for Case 1, $T = 68 + 26.4e^{-0.16452t}$ used to plot the graph and locate the time of death occurred at normal temperature, $T = 98.6$°F. The lieutenant arrived at the scene at 9:40 P.M. and the temperature of the dead body recorded was 94.4°F. After 80 minutes, the temperature was recorded again that showed 89.2°F. Table 1 showed that the actual time of death was at 8:47 P.M., which was 0.89 hours or 53 minutes before it was discovered.

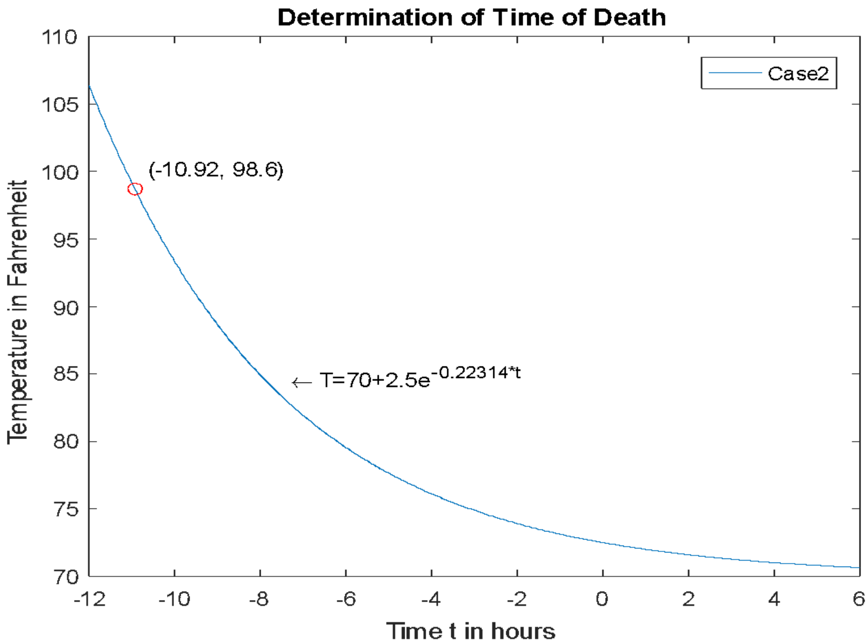

Figure 2: Determination of time of death for Case 2

The result presented in Figure 2 is the determination of death time for Case 2. The solution for Case 2, T = 70 + $2.5e^{-0.22314t}$ used to plot the graph and the red point, (-10.92, 98.6) in the figure above is the locate time of death occurred at normal temperature. The body was found at 7:00 A.M. At this time, the temperature recorded was 72.5°F. After an hour, the body temperature dropped to 72°F. The death occurred approximately 10.92 hours before it was retrieved and the actual time of death was at 8:05 P.M.

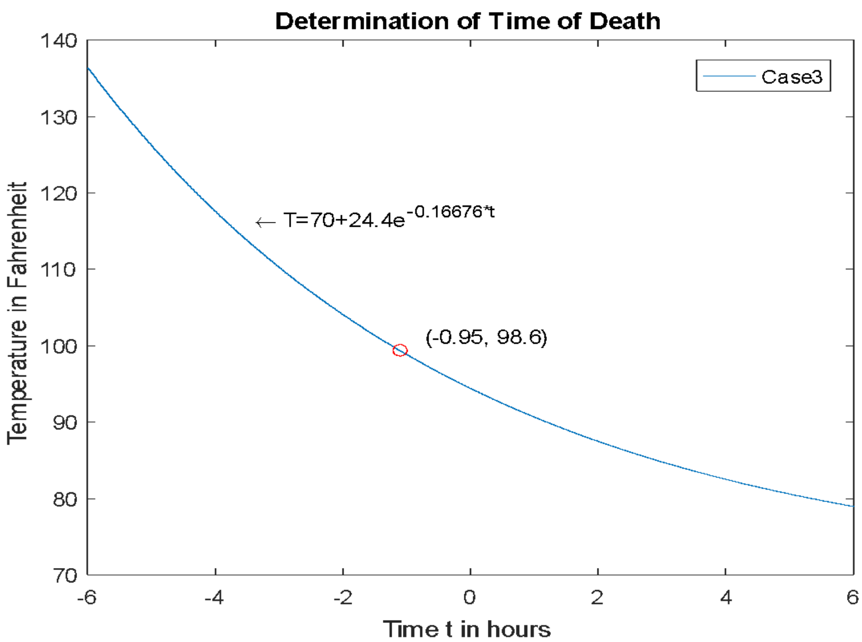

Figure 3: Determination of time of death for Case 3

Figure 3 illustrates the determination of death time for Case 3. The solution for Case 3 is T = 70 + $24.4e^{-0.16676t}$ used to plot the graph and locate the time of death occurring at normal temperature, T = 98.6°F. The police officer arrived at a crime scene and found a dead body at 10:40 P.M. The temperature was measured and recorded 94.4°F. After 90 minutes, the body temperature was recorded, and it fell to 89°F. The actual time of death was at 9:43 P.M, which was the death occurred approximately 0.95 hours or 57 minutes before the body was discovered.

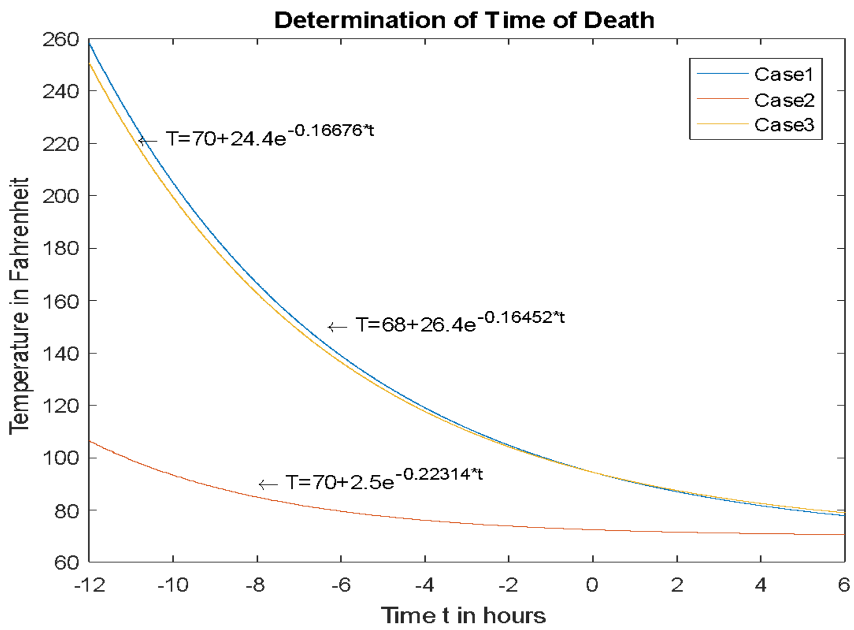

Figure 4: Combined plots for the determination of time of death

Figure 4 shows the combined plots for the determination of time of death for Case 1, Case 2 and Case 3. The graph above clearly shows that all the plots were inversely proportional between the temperature of the body heat loss and the time of death. That means when the temperature of the body heat loss is decreased, the time estimation will be increased. Besides, the cooling of the body depends upon the difference between its temperature and its surroundings. The rate of cooling is faster initially, then slows as the body temperature decreases.

Furthermore, Table 1 clearly shows the temperature of a dead body at time *t* decreases from the initial temperature. However, the temperature does not go below the ambient or room temperature since the system in the body quit working and there is no mechanism to make it happen. Newton's law of cooling describes the rate at which an object changes its temperature when it is exposed to radiation. After death, there is no heat produced in the body. Radiation is the process of transferring heat from one object to another without performing any physical contact. Due to the temperature difference between the body and its environment, the body will lose heat. In addition, the body temperature of the corpse decreases as the time taken to conduct the investigation increases. Therefore, the cooling rate is faster initially, then slows as the body temperature decreases, as shown in the Figure 4.

4. Conclusion

In conclusion, the time of death can be modelled using Newton's law of cooling since the body no longer maintains this temperature after death. The cooling of body depends upon the difference between its temperature and its surroundings, and then the body will lose heat. Thus, it cools and equilibrates to ambient temperature. From the data visualized using MATLAB software, we can conclude that the graph obtained was inversely proportional between the temperature of the body heat loss and the time of death. The cooling rate is faster initially, then slows as the body temperature decreases.

Furthermore, the body heat loss should be by radiation only. Newton's law of cooling explains the rate at which an object or entity changes its temperature when it is exposed to radiation. However, the temperature difference between the body and its surrounding must be small. For extremely high temperatures, Newton's law of cooling is invalid. This model states that the rate of energy flow is proportional to the difference in temperature between the body and its surroundings.

The body cooling method has the benefit that it would be able to estimate the time of death with more precision and less time duration. Therefore, MATLAB software is recommended to calculate Newton's model laws of cooling in investigating certain crimes, when death occurs. This model is important in conducting investigations of crimes and unknown death in order to avoid any mistakes and unrelated decisions. Besides, the variables such as body position, body size, clothing or covering, and environment should be accounted for when solving the model to improve current models to account for the long autopsy period and estimate the time of death more accurately.

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