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State Space Approach for Modelling of Fuzzy Control System

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Abstract: Fuzzy modelling of dynamical systems provides a practical application in handling real-world problems. In this paper, the state space approach, which is based on the Takagi-Sugeno (TS) fuzzy model, is applied to solve a nonlinear control system. For this purpose, the state-space representation is considered. Fuzzification is employed and the triangular membership functions are introduced. On this basis, the fuzzy inference process is implemented in the system for which the related subsystems are constructed. Accordingly, the TS fuzzy controller is designed to obtained these subsystems. After this, the defuzzification is addressed to give the crisp output. For illustration, the rotary inverted pendulum and Rikitake system are studied. These systems are the nonlinear control examples in engineering studies. From the simulation results, it is noticed that the TS fuzzy controller provides a satisfactory result to the state trajectories for both nonlinear control systems. In conclusion, the efficiency of the TS fuzzy modelling in managing nonlinear control systems is verified apparently.

Keywords: State Space Approach, Takagi-Sugeno Fuzzy Model, Nonlinear Control System, Rotary Inverted Pendulum, Rikitake System

1. Introduction

A fuzzy control system is a control system based on fuzzy logic, which is one of the applications of the fuzzy set theory. Fuzzy logic was introduced by Professor Lotfi A. Zadeh in 1965 [1] and is an active research area nowadays. The concept of fuzzy logic is based on human reasoning regards to imprecise and linguistic information, which show vagueness and uncertainty. On this basis, fuzzy logic was applied to control steam engines for the first time by Mamdani in 1974 [2], and then it is known as fuzzy control. After that, the term fuzzy control and its applications [3] are widely developed.

Fuzzy control, which is also known as fuzzy logic control, is similar to conventional control [4]. Both of these control technologies generate the entire output of a system by referring to the input supplied. Nonetheless, the fuzzy input and fuzzy output are the special features of fuzzy control. In

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addition, fuzzy control consists of three main processes, which are fuzzification, fuzzy inference, and defuzzification [5], enable researchers to handle the complex system with fuzziness, vagueness, and uncertainty. Hence, the outcomes of using fuzzy control, rather than conventional control, are well reported from the perspectives of reliability, efficiency, and robustness.

Fuzzy control is considered simple to be used than conventional control since it does not need any mathematical models, and it can provide an accurate solution even the model is used. Modelling of fuzzy control does not need any complicated model, only having the input-output equation for designing a fuzzy controller [6]. Basically, fuzzy control is divided into two types, which are Mamdani fuzzy controller and Takagi-Sugeno (TS) fuzzy controller [7]. With the fuzzy logic set theory and some related membership functions, these controllers would be designed based on the rule base in the fuzzy logic modelling. By devote of this, the state-space representation is the commonly used approach in the fuzzy control system [8].

In this paper, the state space approach is applied to the fuzzy modelling of nonlinear control systems. To begin, the fuzzy variables are defined for the nonlinear terms in the system and the extreme values of the fuzzy variables are calculated from the domain of the crisp input. With these fuzzy sets, the triangular membership functions are defined, and a set of model rules is then established. Furthermore, the control rules are introduced to design the fuzzy controller. As such, the fuzzy output is converted into the crisp output after the center of area approach is employed. Obviously, fuzzification, fuzzy inference, and defuzzification are carried out properly. Finally, the performance index is computed by using the crisp output. For illustration, the rotary inverted pendulum [9, 10] and Rikitake system [11] are studied.

The rest of the paper is organized as follows. In Section 2, the problem is described, and the fuzzy modelling procedure is discussed. In Section 3, the examples of the rotary inverted pendulum and Rikitake system are studied. The simulation results are obtained and the discussion is given. Finally, a conclusion is made.

2. Materials and Methods

Consider a nonlinear control system given by

$$\dot{x}(t) = f(x(t), u(t)), \quad \text{Eq. 1}$$

$$y(t) = h(x(t), u(t)). \quad \text{Eq. 2}$$

Here, $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the control input and $y \in \Re^p$ is the output vector. While, $f : \Re^n \times \Re^m \to \Re^n$ represents the plant dynamics, $x(0) = x_0$ is the initial state, $\dot{x}$ is the rate of change of the state vector $x \in \Re^n$ and $t \geq 0$. The following performance index

$$J(u) = \phi(x(T)) + \int_0^T L(x(t), u(t))dt \quad \text{Eq. 3}$$

is minimized over the dynamical system in Eq. 1 and Eq. 2, where $\phi : \Re^n \to \Re$ is the terminal cost, $L : \Re^n \times \Re^m \to \Re$ is the operating cost, *J* is the scalar function and *T* is the final time.

Notice that the systems in Eq. 1 and Eq. 2 can be written in the linear-like system [12] as follows,

$$\dot{x}(t) = A(x)x(t) + Bu(t), \quad \text{Eq. 4}$$

$$y(t) = C(x)x(t) + Du(t). \quad \text{Eq. 5}$$

Thus, Eq. 4 and Eq. 5 are known as the state-space representation of the nonlinear system.

2.1 Fuzzy Modelling

In fuzzy modelling, there are three stages to be considered, which are fuzzification, fuzzy inference and defuzzification [13]. Initially, define the fuzzy variables z_j, $j = 1, 2, \ldots, p$, where the number of the fuzzy variables depends on the number of nonlinear terms in the system.

In the fuzzification stage, the numerical value of the crisp input, which is the nonlinear term in the system, is calculated based on a defined interval. As such, the fuzzy input is simply assigned. Furthermore, by considering the fuzzy input, the following triangular membership function

$$M_{ij}(z_j) = \frac{z_j - z_{j,\min}}{z_{j,\max} - z_{j,\min}} \qquad \text{Eq. 6}$$

is defined for $i = 1, 2, \ldots, r$.

In the fuzzy inference stage, a collection of model rules is established for the fuzzy sets. In general, these model rules are constructed as follows,

IF z_1 is M_{i1} and z_2 is M_{i2} and … and z_p is M_{ip},

$$\text{THEN, } \dot{x} = A_i x + B_i u \text{ and } y = C_i x \qquad \text{Eq. 7}$$

for $i = 1, 2, \ldots, r$, where M_{ij} is the membership functions and r is the number of model rules. Note that in the IF-THEN rule, the premise variables are defined by $z_1, z_2, \ldots, z_p$, and the consequent components are the subsystem given by $A_i x + B_i u$. The fuzzy controller is designed by applying the following control rules,

IF z_1 is M_{i1} and z_2 is M_{i2} and … and z_p is M_{ip},

$$\text{THEN, } u = -K_i x \qquad \text{Eq. 8}$$

with K is the feedback gain.

In the defuzzification stage, the fuzzy output, which is produced from the membership functions, is converted into the crisp output after taking the consideration of the model rules. By applying the center of area approach, the final output of the fuzzy model can be inferred as follows,

$$\dot{x} = \sum_{i=1}^{r} \eta_i(z)(A_i x + B_i u), \qquad \text{Eq. 9}$$

$$y = \sum_{i=1}^{r} \eta_i(z)(C_i x + D_i u), \qquad \text{Eq. 10}$$

$$u = \sum_{i=1}^{r} \eta_i(z)(-K_i x), \qquad \text{Eq. 11}$$

where

$$\eta_i(z) = \frac{w_i(z)}{\sum_{i=1}^{r} w_i(z)} \text{ and } w_i(z) = \prod_{j=1}^{p} M_{ij}(z_j), \qquad \text{Eq. 12}$$

with

$$\sum_{i=1}^{r} w_i(z) > 0, w_i(z) \geq 0, \text{ for } i = 1, 2, \ldots, r,$$

$$\sum_{i=1}^{r}\eta_i(z)=1, \eta_i(z)\geq 0, \text{ for } i = 1, 2, \ldots, r.$$

Hence, the performance index in quadratic criterion

$$J(u)=\int_0^T (x^{\mathrm{T}}Qx+u^{\mathrm{T}}Ru)\ dt, \qquad \text{Eq. 13}$$

can be evaluated.

3. Results and Discussion

For illustration, the rotary inverted pendulum [9, 10] and Rikitake system [11] are studied.

3.1 Rotary Inverted Pendulum

Consider the model of the rotary inverted pendulum [9, 10], given by

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{pmatrix}=\begin{pmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 149.2750 & a & c\\ 0 & 261.6090 & b & d\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix}+\begin{pmatrix}0\\ 0\\ 49.7275\\ 49.1493\end{pmatrix}u, \qquad \text{Eq. 14}$$

with the initial state $x(0)=(-0.5\quad -0.1\quad 1.3\quad 1.1)^{\mathrm{T}}$, and

$$a=-1.9860x_2x_4+0.9815x_2x_3-17.0068,$$

$$b=-1.9629x_2x_4+1.7200x_2x_3-16.8090,$$

$$c=-1.3086x_2x_4-49.1493,$$

$$d=-1.2934x_2x_4-86.1356.$$

Here, x_1 is the rotary arm angle and x_2 is the pendulum angle, while x_3 and x_4 are the respective angular velocities of the rotary arm and pendulum, and u is the control input.

Define the fuzzy variables to nonlinear terms,

$$z_1=x_2x_4 \text{ and } z_2=x_2x_3. \qquad \text{Eq. 15}$$

Then, consider the domain for the nonlinear terms of the system as

$$x_1\in[-0.5236, 0.5236],\ x_2\in[-1.9, 1.9],$$
$$x_3\in[-0.1745, 0.1745],\ x_4\in[-1.2, 1.2].$$

Hence, the calculated extreme values of the fuzzy variables are

$$z_{1,\min}=-0.209436,\ z_{1,\max}=0.209436,$$
$$z_{2,\min}=-0.331607,\ z_{2,\max}=0.331607.$$

Thus, the membership functions $M_1(z_1)$ and $M_2(z_1)$ are shown in Figure 1 and the membership functions $N_1(z_2)$ and $N_2(z_2)$ are shown in Figure 2.

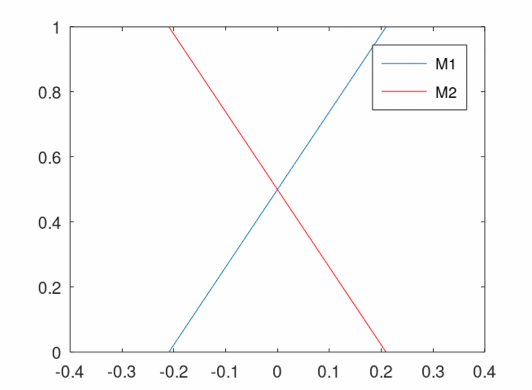

Figure 1: Membership functions $M_1(z_1)$ and $M_2(z_1)$

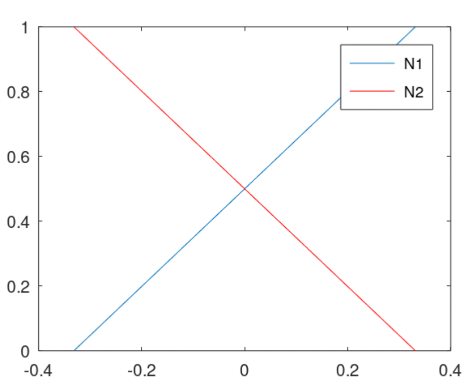

Figure 2: Membership functions $N_1(z_2)$ and $N_2(z_2)$

By using the fuzzy sets from the membership functions, the model rules are defined as follows,

Model Rule 1: IF z_1 is M_1 and z_2 is N_1, THEN $\dot{x}=A_1x+B_1u$.

Model Rule 2: IF z_1 is M_1 and z_2 is N_2, THEN $\dot{x}=A_2x+B_2u$.

Model Rule 3: IF z_1 is M_2 and z_2 is N_1, THEN $\dot{x}=A_3x+B_3u$.

Model Rule 4: IF z_1 is M_2 and z_2 is N_2, THEN $\dot{x}=A_4x+B_4u$.

By substituting extreme values of the fuzzy variables, the following matrices are considered,

$$A_1=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 149.2750 & -16.9163 & -4.6409\\ 0 & 261.6090 & -16.9683 & -8.3427\end{bmatrix},\ A_2=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 149.2750 & -16.2655 & -4.6409\\ 0 & 261.6090 & -15.8277 & -8.3427\end{bmatrix}$$

$$A_3=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 149.2750 & -17.7481 & -5.1890\\ 0 & 261.6090 & -17.7904 & -8.8844\end{bmatrix},\ A_4=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 149.2750 & -17.0973 & -5.1890\\ 0 & 261.6090 & -16.6498 & -8.8844\end{bmatrix}$$

$$B_1=B_2=B_3=B_4=\begin{bmatrix}0\\ 0\\ 49.7275\\ 49.1493\end{bmatrix}$$

Define the control rules,

Model Rule 1: IF z_1 is M_1 and z_2 is N_1, THEN $u=-K_1x$.

Model Rule 2: IF z_1 is M_1 and z_2 is N_2, THEN $u=-K_2x$.

Model Rule 3: IF z_1 is M_2 and z_2 is N_1, THEN $u=-K_3x$.

Model Rule 4: IF z_1 is M_2 and z_2 is N_2, THEN $u = -K_4 x$.

where

$$K_1 = (-3.8730 \quad 261.2691 \quad -4.9894 \quad 8.2550)$$

$$K_2 = (-3.8730 \quad 217.6594 \quad -3.9450 \quad 7.2062)$$

$$K_3 = (-3.8730 \quad 261.7028 \quad -5.0118 \quad 8.2591)$$

$$K_4 = (-3.8730 \quad 218.0201 \quad -3.9656 \quad 7.2085)$$

are the feedback gains that minimizes the performance index given in Eq. 13 with the weighting matrices Q = diag(15, 5, 1, 20) and R = 1. Thus, the state and control trajectories are shown in Figure 3 and Figure 4, respectively, and the performance index of 5366.91 units is minimized.

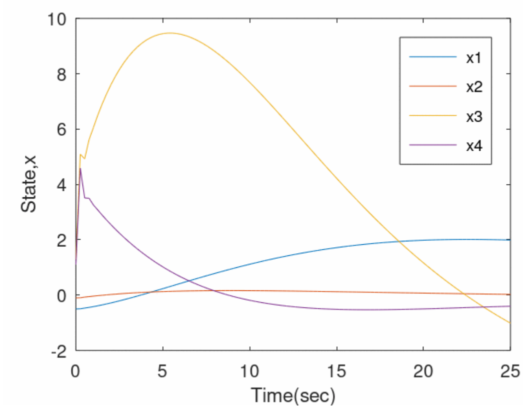

Figure 3: State trajectory of rotary inverted pendulum

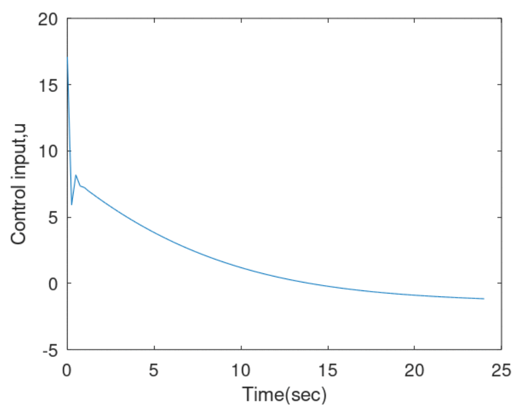

Figure 4: Control trajectory of rotary inverted pendulum

From the graphical solution in Figure 3 and Figure 4, the movement of the rotary inverted pendulum is under control, where the angles of the rotary arm and pendulum are small. The angular velocity of the rotary arm was increased in the first five seconds and then reduced dramatically, while the angular velocity of the pendulum was slightly increased and decreased gradually after one second. The control trajectory performed an effort to ensure that the state trajectories were stabilized.

3.2 Rikitake System

Consider the state equation of the Rikitake system [11],

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -2 & x_3 & 0 \\ x_3 - 5 & -2 & 0 \\ 0 & -x_1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \text{Eq. 16}$$

where x_1 and x_2 are the currents through each disc of dynamo, x_3 is the angular velocity of the discs, and $u = (u_1 \quad u_2 \quad u_3)^{\mathrm{T}}$ is the control input. The initial state is $x(0) = (2 \quad -1 \quad 1)^{\mathrm{T}}$.

The fuzzy variables are defined to the nonlinear terms,

$$z_1 = x_3 \text{ and } z_2 = x_1, \qquad \text{Eq. 17}$$

and consider the following domains,

$$x_1 \in [-1,1],\ x_2 \in [0,0],\ x_3 \in [-1,1].$$

The extreme values of the fuzzy variables are calculated become

$$z_{1,\min} = -1,\ z_{1,\max} = 1,$$
$$z_{2,\min} = -1,\ z_{2,\max} = 1.$$

Figure 5 and Figure 6 show the membership functions for the fuzzy sets.

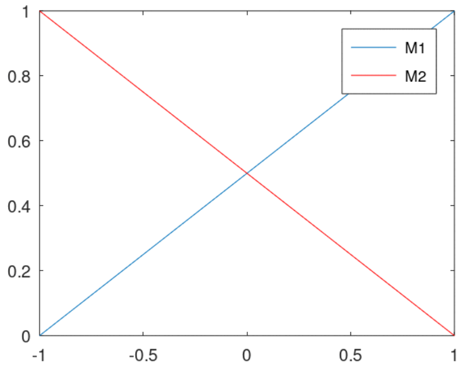

Figure 5: Membership functions $M_1(z_1)$ and $M_2(z_1)$

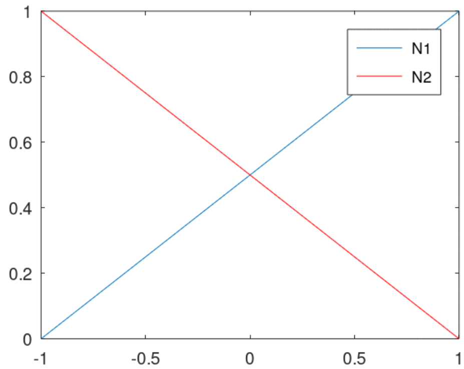

Figure 6: Membership functions $N_1(z_2)$ and $N_2(z_2)$

For the fuzzy inference process, the following model rules are established,

Model Rule 1: IF z_1 is M_1 and z_2 is N_1, THEN $\dot{x} = A_1 x + B_1 u + C_1$.

Model Rule 2: IF z_1 is M_1 and z_2 is N_2, THEN $\dot{x} = A_2 x + B_2 u + C_2$.

Model Rule 3: IF z_1 is M_2 and z_2 is N_1, THEN $\dot{x} = A_3 x + B_3 u + C_3$.

Model Rule 4: IF z_1 is M_2 and z_2 is N_2, THEN $\dot{x} = A_4 x + B_4 u + C_4$.

By substituting extreme values of the fuzzy variables, the following matrices are considered,

$$A_1 = \begin{bmatrix} -2 & 1 & 0 \\ -4 & -2 & 0 \\ 0 & -1 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} -2 & 1 & 0 \\ -4 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix},\ A_3 = \begin{bmatrix} -2 & -1 & 0 \\ -6 & -2 & 0 \\ 0 & -1 & 0 \end{bmatrix},\ A_4 = \begin{bmatrix} -2 & -1 & 0 \\ -6 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B_1 = B_2 = B_3 = B_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ C_1 = C_2 = C_3 = C_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

While, the fuzzy control law in Eq. 8, which has the control rules

Model Rule 1: IF z_1 is M_1 and z_2 is N_1, THEN $u = -K_1 x$.

Model Rule 2: IF z_1 is M_1 and z_2 is N_2, THEN $u = -K_2 x$.

Model Rule 3: IF z_1 is M_2 and z_2 is N_1, THEN $u = -K_3 x$.

Model Rule 4: IF z_1 is M_2 and z_2 is N_2, THEN $u = -K_4 x$.

is designed with the feedback gains

$$K_1 = \begin{bmatrix} 0.4738 & -0.1498 & 0.2376 \\ -0.1498 & 0.2485 & -0.2112 \\ 0.2376 & -0.2112 & 0.9481 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.4738 & -0.1498 & -0.2376 \\ -0.1498 & 0.2485 & 0.2112 \\ -0.2376 & 0.2112 & 0.9481 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 1.7636 & -0.8290 & 0.3087 \\ -0.8290 & 0.5398 & -0.2789 \\ 0.3087 & -0.2789 & 0.9093 \end{bmatrix}, \quad K_4 = \begin{bmatrix} 1.7636 & -0.8290 & -0.3087 \\ -0.8290 & 0.5398 & 0.2789 \\ -0.3087 & 0.2789 & 0.9093 \end{bmatrix}$$

and the performance index of 250.11 units is minimized using the weighting matrices Q = diag(1, 1, 1) and R = diag(1, 1, 1). As a result, the trajectories of state and control are shown in Figure 7 and Figure 8, respectively.

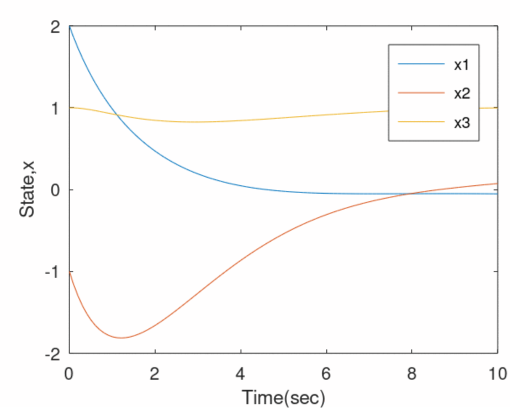

Figure 7: State trajectory, Rikitake system

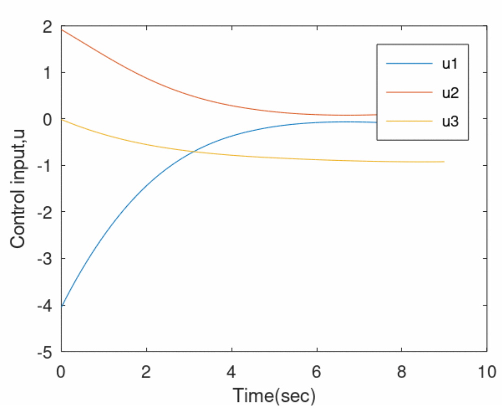

Figure 8: Control trajectory, Rikitake system

The graphical solutions in Figure 7 and Figure 8 show that the current through the discs and the movement of the discs are under control. The control trajectories ensured that the state trajectories towards their steady-state after five seconds. It is noticed that less control effort was taken for stabilizing the movement of the disc, however, more control efforts were needed to stabile the currents through the discs.

4. Conclusion

In this paper, the state space approach for fuzzy modelling of nonlinear control systems was discussed. The nonlinear terms in the system were defined as fuzzy variables and the extreme values of these fuzzy variables were calculated based on the domains of the crisp input. By referring to these fuzzy sets, the triangular membership functions were introduced. The related model rules for the system were established and the fuzzy controller was designed according to the control rules that were constructed. The crisp output was delivered once the center of area approach was applied to the system. For an illustrative example, the rotary inverted pendulum and Rikitake system were studied. The results showed that the trajectories of state and control were satisfactorily obtained. In conclusion, the efficiency of the TS fuzzy modelling to nonlinear control system is verified definitely. For future study, using the TS fuzzy modelling to handle more complex nonlinear control systems would be recommended.

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