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Nonlinear Waves Propagation in an Elastic Tube with Stenosis Filled with Newtonian Fluid

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Abstract: The goal of this study is to investigate the nonlinear wave propagation in an elastic tube with stenosis filled with Newtonian fluid. In the present work, the artery is considered as an incompressible, pre-stressed, thin walled stenosed elastic tube and the blood is treated as Newtonian fluid. The dimensional equations of tube and fluid are converted into dimensionless equation by applying non-dimensional quantities. The reductive perturbation method is implemented in this study in order to obtain the various orders of nonlinear differential equations. Next, the forced Korteweg-de Vries (FpKdV) equation with variable coefficient is obtained by solving the various orders of differential equations. Then, the progressive wave solution of forced perturbed Korteweg-de Vries equation with variable coefficients is carried out and discussed. From the graphical output obtained, it shows that the increase of fluid viscosity will cause the radial displacement decreases. Thus, the higher the viscosity of fluid, the fluid is not easy to pass through the stenosis along the tube. Besides, there is resistance on blood flows in the artery due to the presence of viscosity. It is because the radial displacement is decreasing when the time is increasing. The trajectory of the wave is not a straight line while a curve in the plane.

Keywords: Wave Propagation, Forced Perturbed Korteweg-De Vries (Fpkdv) Equation With Variable Coefficient, Newtonian Fluid, Elastic Tube, Stenosis

1. Introduction

The circulatory system provides nutrients and oxygen that are required and transports the harmful chemicals and waste products away from the organs [1]. In previous works of researchers, the researchers studied about nonlinear wave propagation of fluid flow in an elastic tube such as [2], [3], [4], [5], [6], [7] and [8]. Besides, researchers [9] and [10] treated the blood in the artery as non-Newtonian fluid. In human's circulatory system, there is occurring of stenosis condition. Therefore, the tube of this study included with stenosis so that the model is more realistic compare to the previous studies. Thus, the artery is treated as an incompressible, pre-stressed, thin walled elastic tube with a stenosis and the blood as the Newtonian fluid. A Newtonian fluid is one in which the viscous stresses

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generated by its flow are linearly linked to the local strain rate, or the rate of change of its deformation over time at each point [11]. From the mathematical model, the analytical solution is analysed on variation of the wave trajectory, viscous effect of fluid and the radial displacement. Besides, the reductive perturbation method is an efficient way to derive models that may be used to explore nonlinear wave propagation. Normally, it was often applied in the additional assumptions which are including the magnitude of some coefficients [12].

2. Equations of the tube and fluid

According to previous studies the blood is behave like Newtonian fluid when the hematocrit ratio is low while the shear rate is high. The equation of fluid is given as follow [13],

$$\frac{\partial w^*}{\partial t^*}+w^*\frac{\partial w^*}{\partial z^*}+\frac{1}{\rho_f}\frac{\partial p^*}{\partial z^*}-\frac{\mu_v}{\rho_f}\left(\frac{\partial^2 w^*}{\partial z^{*2}}-\frac{8w^*}{r^2 f}\right)=0 , \qquad \text{Eq.1}$$

$$2\frac{\partial u^*}{\partial t^*}+2w^*\left[\frac{\partial r^*}{\partial z^*}+\frac{\partial u^*}{\partial z^*}\right]+\left[r^*(z^*)+u^*\right]\frac{\partial w^*}{\partial z^*}=0 , \qquad \text{Eq.2}$$

where w^* is the averaged axial velocity, f is the stenosis function after the deformation, p^* is the averaged pressure of fluid, ρ_f is the mass density, μ_v is the viscosity of the fluid and $r_f = r^*(z^*)+u^*$ is the final radius after deformation occurred.

The equation of tube could be written as follows [13]:

$$\frac{\partial}{\partial z^*}\left\{\frac{\mu H R^*(z)}{\Lambda}\left(r^{*'}+\frac{\partial u^*}{\partial z^*}\right)\frac{\partial \Sigma}{\partial \lambda_1}\right\}-\frac{\mu H}{\lambda_z}\left[1+\lambda_z^2(R^{*'})^2\right]^{1/2}\frac{\partial \Sigma}{\partial \lambda_2}$$

$$+\Lambda\left(r^*+u^*\right)P_r^*-\rho_0\frac{H}{\lambda_z}R^*(z^*)\left[1+\lambda_z^2(R^{*'})^2\right]^{1/2}\frac{\partial^2 u^*}{\partial t^{*2}}=0 , \qquad \text{Eq.3}$$

where P_r^* is the radial fluid reaction force on the inner surface of the tube, ρ_0 is the mass density of the membrane material, H is the thickness before final deformation, $R^*(z^*)$ is the radius of circularly cylindrical tube, λ_z is the initial axial stretch of the tube, r^* is the variable radius after the static deformation, u^* is the radial displacement, t^* is the time parameter, μ is the shear modulus, z^* is the axial coordinate at the intermediate configuration, Σ is the strain energy density function of the tube material, λ_2 is the stretch ratio along the circumferential curve and λ_1 is the stretch ratios along the meridional curve.

The following non-dimensional quantities are introduced at this stage [13]:

$$t^*=\left(\frac{R_0}{c_0}\right)t, \qquad z^*=R_0 z, \qquad u^*=R_0 u, \qquad w^*=c_0 w, \qquad \frac{\partial t^*}{\partial t}=\frac{R_0}{c_0}, \qquad \frac{\partial z^*}{\partial z}=R_0,$$

$$\frac{\partial u^*}{\partial u}=R_0,\qquad \frac{\partial w^*}{\partial w}=c_0,\qquad r=R_0x,\qquad P^*=\rho_f c_0{}^2 p,\qquad c_0{}^2=\frac{\mu H}{\rho_f R_0},\qquad m=\frac{\rho_0 H}{\rho_f R_0},$$
$$\mu_v=c_0R_0\rho_f\bar{v},\qquad R^*(z^*)=R_0\left[1-F(z)\right],\qquad r^*(z^*)=R_0\left[\lambda_\theta-f(z)\right], \tag{Eq.4}$$

where $\lambda_\theta=\dfrac{r_0}{R_0}$ represents the initial stretch ratio, r_0 is the radius of the origin after finite static deformation, R_0 is the refereance radius and c_0 represents the Moens-Korteweg wave speed. Introducing Eq.4 into Eqs.1, 2 and 3 by applying the chain rule, the following non-dimensional equations are obtained:

$$\frac{\partial w}{\partial t}+w\frac{\partial w}{\partial z}+\frac{\partial p}{\partial z}-\bar{v}\left(\frac{\partial^2 w}{\partial z^2}-\frac{8}{\left[\lambda_\theta-f(z)+u\right]^2}\right)=0$$

$$2\frac{\partial u}{\partial t}+2w\left[-f'(z)+\frac{\partial u}{\partial z}\right]+\left[\lambda_\theta-f(z)+u\right]\frac{\partial w}{\partial z}=0,$$

$$p=-\frac{1}{\lambda_\theta-f(z)+u}\frac{\partial}{\partial z}\left\{\frac{\left[1-F(z)\right]\left(-f'(z)+\frac{\partial u}{\partial z}\right)}{\left[1+\left(-f'(z)+\frac{\partial u}{\partial z}\right)^2\right]^{1/2}}\frac{\partial\Sigma}{\partial\lambda_1}\right\}+\frac{\left[1+\lambda_z{}^2F'(z)^2\right]^{1/2}}{\lambda_z\left[\lambda_\theta-f(z)+u\right]}\frac{\partial\Sigma}{\partial\lambda_2}$$
$$+\frac{4\bar{v}w\left(-f'(z)+\frac{\partial u}{\partial z}\right)}{\lambda_\theta-f(z)+u}+\frac{m\left[1-F(z)\right]\left[1+\lambda^2zF'(z)\right]^{1/2}}{\lambda_z\left[\lambda_\theta-f(z)+u\right]}\frac{\partial^2 u}{\partial t^2}, \tag{Eq.5}$$

where p is the averaged pressure of fluid , w is the averaged axial velocity, u is the radial displacement, $F(z)$ and $f(z)$ are the stenosis functions before and after deformation, z is the axial coordinate at the intermediate configuration and $\bar{v}$ is the effect of viscosity.

3. Nonlinear wave propagation

In this section, the method of reductive perturbation is applied. For the boundary value problem, the following type of stretched coordinates are introduced [13]:

$$\xi=\varepsilon^{1/2}(z-ct),\qquad \tau=\varepsilon^{3/2}z \tag{Eq.6}$$

where ε is a small parameter measuring the weakness of nonlinearity and dispersion and c is a scale parameter to be determined from the solution. From Eq.6, z is solved in term of τ, where

$$z=\varepsilon^{-3/2}\tau \tag{Eq.7}$$

Then, introducing Eq.7 into the expression of $F(z)$ and $f(z)$,the following equations are obtained,

$$f(z)=f\left(\varepsilon^{-3/2}\tau\right)=\varepsilon G(\tau), \qquad F(z)=F\left(\varepsilon^{-3/2}\tau\right)=\varepsilon g(\tau). \qquad \text{Eq.8}$$

The differential relations are introduced in the following form [13]:

$$\frac{\partial}{\partial t}=-\varepsilon^{1/2}c\frac{\partial}{\partial\xi}, \qquad \frac{\partial}{\partial z}=\varepsilon^{1/2}\left(\frac{\partial}{\partial\xi}+\varepsilon\frac{\partial}{\partial\tau}\right). \qquad \text{Eq.9}$$

For the long wave limit, the field quantities u , w, and p are assumed that it can be written as asymptotic series in the following form [13]:

$$u=\varepsilon u_1+\varepsilon^2 u_2+\ldots, \qquad w=\varepsilon w_1+\varepsilon^2 w_2+\ldots,$$

$$p=p_0+\varepsilon p_1+\varepsilon^2 p_2+\ldots, \qquad \text{Eq.10}$$

By introducing Eqs.8 and 9 into Eq.5, the various order equations are obtained:

$O(\varepsilon)$ equations:

$$-2c\frac{\partial u_1}{\partial\xi}+\lambda_\theta\frac{\partial w_1}{\partial\xi}=0, \qquad -c\frac{\partial w_1}{\partial\xi}+\frac{\partial p_1}{\partial\xi}=0, \qquad p_1=\beta_1(u_1-g)+\gamma_1\lambda_\theta G. \qquad \text{Eq.11}$$

$O(\varepsilon^2)$ equations:

$$-2c\frac{\partial u_2}{\partial\xi}+2w_1\frac{\partial u_1}{\partial\xi}+\lambda_\theta\frac{\partial w_2}{\partial\xi}+\left[u_1-g(\tau)\right]\frac{\partial w_1}{\partial\xi}+\lambda_\theta\frac{\partial w_1}{\partial\tau}=0,$$

$$-c\frac{\partial w_2}{\partial\xi}+w_1\frac{\partial w_1}{\partial\xi}+\frac{\partial p_2}{\partial\xi}+\frac{\partial p_1}{\partial\tau}+\frac{8vw_1}{\lambda_\theta^{2}}=0,$$

$$p_2=\left(\frac{mc^2}{\lambda_\theta\lambda_z}-\beta_0\right)\frac{\partial^2 u_1}{\partial\xi^2}+\beta_1 u_2+\beta_2 u_1^{2}+\beta_3(\tau)u_1+\pi(\tau). \qquad \text{Eq.12}$$

Here the coefficients of γ_0, γ_1, γ_2, β_0, β_1 and β_2 are defined by [13]:

$$\gamma_0=\frac{1}{\lambda_\theta\lambda_z}\frac{\partial\Sigma}{\partial\lambda_\theta}, \qquad \gamma_1=\frac{1}{\lambda_\theta\lambda_z}\frac{\partial^2\Sigma}{\partial\lambda_\theta^{2}}, \qquad \gamma_2=\frac{1}{2\lambda_\theta\lambda_z}\frac{\partial^3\Sigma}{\partial\lambda_\theta^{3}},$$

$$\beta_0=\frac{1}{\lambda_\theta}\frac{\partial\Sigma}{\partial\lambda_z}, \qquad \beta_1=\gamma_1-\frac{\gamma_0}{\lambda_\theta}, \qquad \beta_2=\gamma_2-\frac{\beta_1}{\lambda_\theta}. \qquad \text{Eq.13}$$

Solving the Eq.11 and assume $u_1=U(\xi,\tau)$ leads to the following equations:

$$w_1=\frac{2c}{\lambda_\theta}U, \; p_1=\frac{2c^2}{\lambda_\theta}U-\beta_1 g+\gamma_1\lambda_\theta G, \qquad \text{Eq.14}$$

where $U(\xi,\tau)$ is an unknown function whose governing equation will be obtained soon and the condition $\beta_1 = \frac{2c^2}{\lambda_\theta}$ holds true.

Introduce Eq.14 into Eq.12, the following equations are obtained:

$$-2c\frac{\partial u_2}{\partial \xi} + \frac{4c}{\lambda_\theta}U\frac{\partial U}{\partial \xi} + \lambda_\theta\frac{\partial w_2}{\partial \xi} + \frac{2c}{\lambda_\theta}\left[U - g(\tau)\right]\frac{\partial U}{\partial \xi} + 2c\frac{\partial U}{\partial \tau} = 0, \quad \text{Eq.15}$$

$$-c\frac{\partial w_2}{\partial \xi} + \left(\frac{2c}{\lambda_\theta}\right)^2 U\frac{\partial U}{\partial \xi} + \frac{\partial p_2}{\partial \xi} + \frac{2c^2}{\lambda_\theta}\frac{\partial U}{\partial \tau} - \beta_1 g'(\tau) + \gamma_1\lambda_\theta G'(\tau) + \frac{16cv}{\lambda_\theta^{\ 3}}U = 0, \quad \text{Eq.16}$$

$$p_2 = \left(\frac{mc^2}{\lambda_\theta\lambda_z} - \beta_0\right)\frac{\partial^2 U}{\partial \xi^2} + \beta_1 u_2 + \beta_2 U^2 + \beta_3(\tau)U + \pi(\tau). \quad \text{Eq.17}$$

By substituting Eq.17 to Eq.16, obtain that

$$-c\frac{\partial w_2}{\partial \xi} + \left(\frac{2c}{\lambda_\theta}\right)^2 U\frac{\partial U}{\partial \xi} + \left(\frac{mc^2}{\lambda_\theta\lambda_z} - \beta_0\right)\frac{\partial^3 U}{\partial \xi^3} + \beta_1\frac{\partial u_2}{\partial \xi} + 2\beta_2 U\frac{\partial U}{\partial \xi}$$

$$+\beta_3(\tau)\frac{\partial U}{\partial \xi} + \frac{2c^2}{\lambda_\theta}\frac{\partial U}{\partial \tau} - \beta_1 g'(\tau) + \gamma_1\lambda_\theta G'(\tau) + \frac{16cv}{\lambda_\theta^{\ 3}}U = 0 \quad \text{Eq.18}$$

By eliminating w_2, the forced perturbed KdV equation with variable coefficients is achieved as follows:

$$\frac{\partial U}{\partial \tau} + \mu_1 U\frac{\partial U}{\partial \xi} - \mu_2\frac{\partial^3 U}{\partial \xi^3} + \mu_3 U - \mu_4(\tau)\frac{\partial U}{\partial \xi} = \mu(\tau), \quad \text{Eq.19}$$

where

$$\mu_1 = \frac{5}{2\lambda_\theta} + \frac{\beta_2}{\beta_1}, \quad \mu_2 = \frac{\beta_0}{2\beta_1} - \frac{m}{4\lambda_z}, \quad \mu_3 = \frac{4v}{c\lambda_\theta^{\ 2}},$$

$$\mu_4(\tau) = -\frac{\lambda_\theta\gamma_2}{\beta_1}G(\tau) + \left[\frac{\beta_2}{\beta_1} + \frac{1}{2\lambda_\theta}\right]g(\tau), \quad \mu(\tau) = \frac{1}{2}g'(\tau) - \frac{\gamma_1\lambda_\theta}{2\beta_1}G'(\tau). \quad \text{Eq.20}$$

4. Progressive wave solution

A progressive wave solution to the evolution Eq.19 is presented. For that purpose, the following new dependent variable is introduced as [13]

$$U(\xi,\tau) = -V(\xi,\tau) + \exp(-\mu_3\tau)\int_0^\tau \mu(s)\exp(\mu_3 s)ds. \quad \text{Eq.21}$$

Next, Eq.21 is introduced to Eq.19 and obtained as follows:

$$\frac{\partial V}{\partial \tau} - \mu_1 V\frac{\partial V}{\partial \xi} + \mu_1\exp(-\mu_3\tau)\left[\int_0^\tau \mu(s)\exp(\mu_3 s)ds\right]\frac{\partial V}{\partial \xi} + \mu_2\frac{\partial^3 V}{\partial \xi^3} - \mu_3 V + \mu_4(\tau)\frac{\partial V}{\partial \xi} = 0 \quad \text{Eq.22}$$

Introducing the coordinate transformation [13]

$$\tau' = \tau, \qquad \xi' = \xi - \int_0^\tau\left[-\mu_4(q) + \mu_1\exp(-\mu_3 q)\int_0^q \mu(s)\exp(\mu_3 s)ds\right]dq. \quad \text{Eq.23}$$

into Eq.22 and it reduces to the conventional perturbed Korteweg-de vries equation as follows:

$$\frac{\partial V}{\partial \tau'} - \mu_1 V\frac{\partial V}{\partial \xi'} - \mu_2\frac{\partial^3 V}{\partial \xi'^3} + \mu_3 V = 0 \quad \text{Eq.24}$$

According to [14], the solution of Eq.24 may be given by

$$V = a(\tau')\operatorname{sech}^2 \zeta, \qquad \text{Eq.25}$$

where $a(\tau') = a_0 \exp\left(-\frac{4}{3}\mu_3 \tau'\right)$, a_0 is a constant which represent the initial amplitude of the travelling wave and the phase function ζ is

$$\zeta = \left(\frac{\mu_1 a(\tau')}{12\mu_2}\right)^{1/2}\left[\xi' + \frac{\mu_1 a_0}{4\mu_3}\left[1 - \exp\left(-\frac{4}{3}\mu_3 \tau'\right)\right]\right]. \qquad \text{Eq.26}$$

By inserting Eq.25 into Eq.21, the solution of forced perturbed KdV equation with variable coefficients Eq.19 is given by

$$U = -a_0 \exp\left(-\frac{4}{3}\mu_3 \tau\right)\operatorname{sech}^2 \zeta + \exp(-\mu_3 \tau)\int_0^\tau \mu(s)\exp(\mu_3 s)ds. \qquad \text{Eq.27}$$

The phase function ζ of the FpKdV equation with variable coefficient Eq.19 can be expressed as in terms of the coordinates (ξ, τ) as

$$\zeta = \left(\frac{\mu_1 a(\tau)}{12\mu_2}\right)^{1/2}\left\{\begin{array}{l}\xi + \frac{\mu_1 a_0}{4\mu_3}\left[1 - \exp\left(-\frac{4}{3}\mu_3 \tau\right)\right] \\ -\int_0^\tau \left[-\mu_4(q) + \mu_1 \exp(-\mu_3 q)\int_0^q \mu(s)\exp(\mu_3 s)ds\right]dq\end{array}\right\} \qquad \text{Eq.28}$$

5. Results and discussion

The values of γ_0, γ_1, γ_2, β_0, β_1, β_2, c, μ_1, μ_2, μ_3, $\mu_4(\tau)$ and $\mu(\tau)$ are needed for the graphical plotting. According to Demiray [15], the strain energy density function is introduced as follows:

$$\Sigma = \frac{1}{2\alpha}\left\{\exp\left[\alpha\left(\frac{\lambda_z^2 + \lambda_\theta^2 + 1}{\lambda_z^2 \lambda_\theta^2} - 3\right)\right] - 1\right\} \qquad \text{Eq.29}$$

where α is a material constant.

For the coefficients γ_0, γ_1, γ_2, β_0, β_1, β_2, c, μ_1, μ_2, μ_3, $\mu_4(\tau)$ and $\mu(\tau)$ are determined as [14] :

$$\gamma_0 = \frac{1}{\lambda_z \lambda_\theta}\left(\lambda_\theta - \frac{1}{\lambda_z^2 \lambda_\theta^3}\right)F(\lambda_z, \lambda_\theta), \qquad \gamma_1 = \frac{1}{\lambda_z \lambda_\theta}\left[\left(1 + \frac{3}{\lambda_z^2 \lambda_\theta^4}\right) + 2\alpha\left(\lambda_\theta - \frac{1}{\lambda_z^2 \lambda_\theta^3}\right)^2\right]F(\lambda_z, \lambda_\theta),$$

$$\gamma_2 = \frac{1}{2\lambda_z \lambda_\theta}\left[-12\lambda_z^2 \lambda_\theta^5 + 6\alpha\left(\lambda_\theta - \frac{1}{\lambda_z^2 \lambda_\theta^3}\right)\left(1 + \frac{3}{\lambda_z^2 \lambda_\theta^4}\right) + 4\alpha^2\left(\lambda_\theta - \frac{1}{\lambda_z^2 \lambda_\theta^3}\right)^3\right]F(\lambda_z, \lambda_\theta),$$

$$\beta_0 = \frac{1}{\lambda_\theta}\left(\lambda_z - \frac{1}{\lambda_z^3 \lambda_\theta^2}\right)F(\lambda_z, \lambda_\theta), \qquad \beta_1 = \gamma_1 - \frac{\gamma_0}{\lambda_\theta}, \qquad \beta_2 = \gamma_2 - \frac{\beta_1}{\lambda_\theta}, \qquad \text{Eq.30}$$

whereby the function $F(\lambda_z, \lambda_\theta)$ is defined as

$$F(\lambda_z, \lambda_\theta) = \exp\left[\alpha\left(\lambda_z^2 + \lambda_\theta^2 + \frac{1}{\lambda_z^2 \lambda_\theta^2} - 3\right)\right]. \qquad \text{Eq.31}$$

Besides, the trajectory that correspond to ζ =0 is defined as

$$\xi = \frac{\mu_1 a_0}{4\mu_3}\left[-1 + \exp\left(-\frac{4}{3}\mu_3 \tau\right)\right] + \int_0^\tau \left[-\mu_4(q) + \mu_1 \exp(-\mu_3 q)\int_0^q \mu(s)\exp(\mu_3 s)ds\right]dq \qquad \text{Eq.32}$$

In this study, the graphical outputs for wave trajectory, radial displacement and the viscous effect of fluid are illustrated by using MATLAB software. The numerical value of α was found to be 1.948 by Demiray [16]. Besides, the axial stretch, λ_z and the stretch ratio in the circumferential direction, λ_θ are both assumed as 1.6.

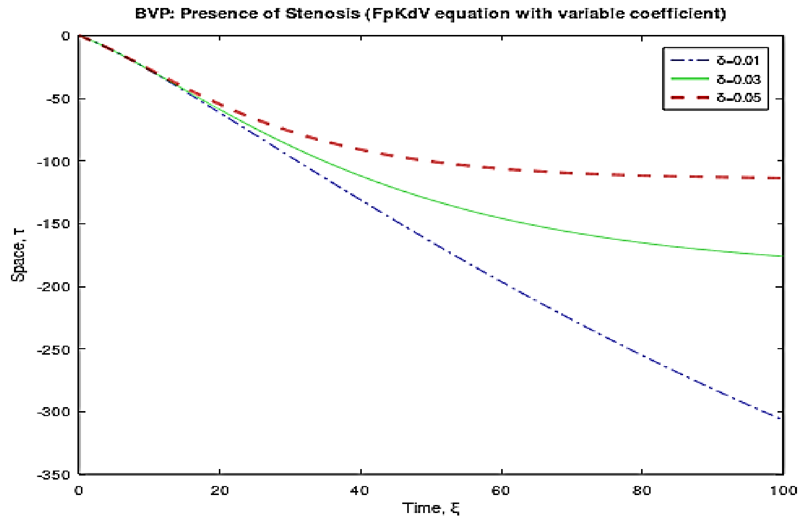


Figure 1: The wave trajectory of the forced perturbed KdV with variable coefficients, time, ξ versus space, τ .

Figure 1 is constructed by using m =0.1 and function G =0. This figure illustrates the trajectory corresponding to ζ =0 of the FpKdV equation with variable coefficient. By choosing different values of δ , it shows that they are rather curves in the (ξ, τ) plane. This is due to the viscous effect of fluid and stenosis in the tube. It reveals the negative gradient. It means that as time increases, the waves will propagate to the left. If the graph shows positive gradient, it means the waves will travel to the right over the time.

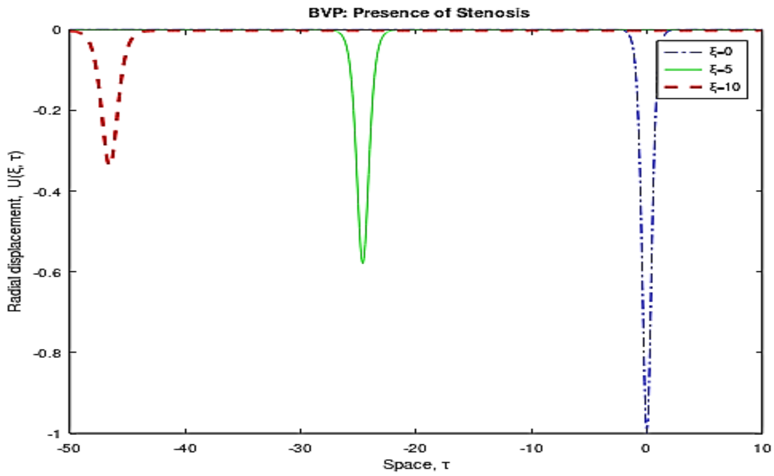
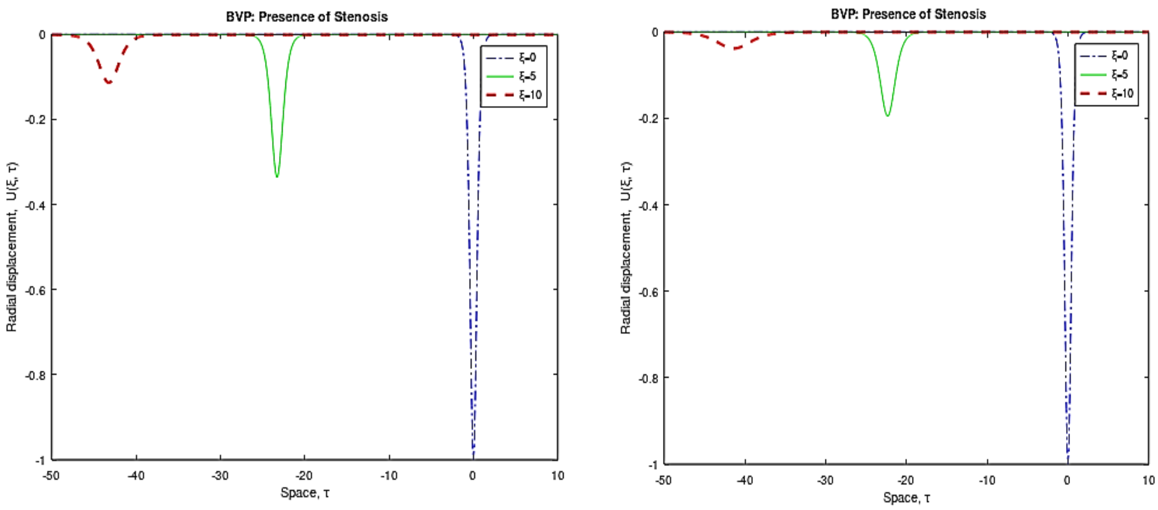
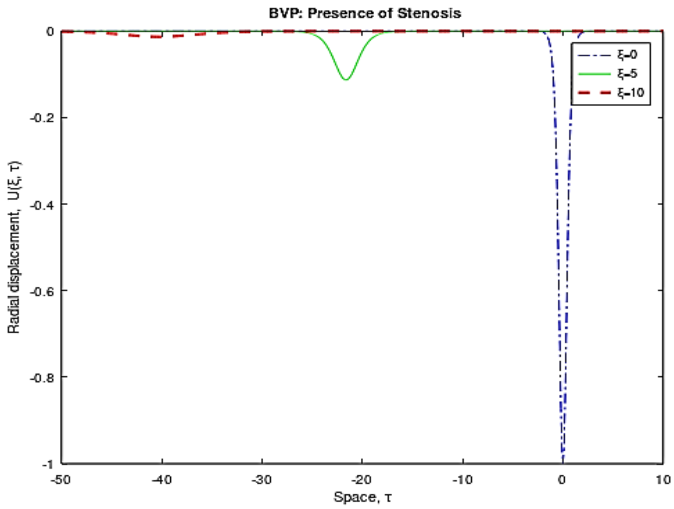


Figure 2: Radial displacement, $U(\xi,\tau)$ versus space, τ for different time, ξ at ν =1 in the presence of stenosis (FpKdV equation with variable coefficient)

Figure 2 displays the variation of the radial displacement versus space, τ in the presence of stenosis when the viscous effect of fluid, ν=1. The bell-shape waves propagate to the left and the amplitude of waves damp as time increases. This is due to the presence of stenosis and fluid viscosity which is to be expected.



(a) (b)



(c)

Figures 3: Radial displacement, $U(\xi,\tau)$ versus space, τ for different time, ξ at (a) ν =2, (b) ν =3, (c) ν =4 in the presence of stenosis (FpKdV equation with variable coefficient)

Next, Figures 3 reveal the variation of the radial displacement with the different fluid viscosity. As might be seen from these figures, the amplitude of waves damp faster as the fluid viscosity increases. This is because the resistance of fluid flow increases when the viscous effect of fluid increases. The resistance of fluid may be caused by tube diameter, tube length, stenosis or the viscosity of fluid. Thus, it can be concluded that as the viscous effect of fluid increases, the solitary wave solutions for the radial displacement to be diminishing to some extent throughout the artery.

6. Conclusion

In this research, the dimensional equations of tube and fluid are converted into dimensionless by applying non-dimensional quantities. The reductive perturbation method is implemented in this study to obtain the various orders of differential equations. Next, the forced perturbed Korteweg-de Vries (FpKdV) equation with variable coefficient is obtained by solving the various orders of differential equations. Then, the progressive wave solution of FpKdV with variable coefficients is solved analytically and discussed. By using MATLAB software, the variation of wave trajectory, radial displacement and effect of viscosity are studied. Finally, from the graphical output that obtained, it shows that the increase of fluid viscosity will cause the radial displacement become smaller. Thus, the higher the viscosity of fluid, the harder for the fluid to propagate in the tube. Besides, there is resistance on blood flows in the artery due to the presence of viscosity. It is because the radial displacement is decreasing when the time, ξ is increasing. Last but not least, the trajectory of the wave is not a straight line while a curve in the (ξ,τ) plane.

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