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Solving Thermal Radiation On Boundary Layer Flow Using BVP4 CTechnique

Aleya Ashima Mohd Kamil[1], **Fazlina Aman**[2*]

[1 2]Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology,
Universiti Tun Hussein Onn Malaysia, Pagoh Campus, 84600 Pagoh, Muar, Johor, MALAYSIA

*Corresponding Author Designation

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Abstract: The purpose of this research is to analyse the effect of thermal radiation and Newtonian heating on boundary layer flow. A similarity transformation is used to transform the governing equations from partial differential equations to a system of ordinary differential equations. The governing equations are then solved numerically by using shooting method with bvp4c. The numerical results for temperature and velocity profile are obtained by using MATLAB R2020b. The results depend on four parameters which are Prandtl number, suction/blowing parameter, thermal radiation parameter and conjugate parameter. It is found that the value of wall temperature increases with the presence of thermal radiation and blowing parameter while it shows different behavior for the suction parameter effect.

Keywords: Thermal Radiation, Boundary Layer Flow, BVP4C Technique

1. Introduction

Knowledge on laminar boundary layer flow and heat thermal is noteworthy in various works that related to engineering and science filed. Laminar boundary layer is the most broadly perceived boundary condition. Laminar boundary layer is a boundary at which stream flow takes place in layer form and move in the continuous path or streamline without mixing with the other layer in the adjacent path. The study on the boundary layer flow in the stationary plate was first studied by Blasius. Mohamed *et al*

**Corresponding author: fazlina@uthm.edu.my*

[1], who studied thermal radiation effect on laminar boundary layer flow over a permeable flat plate in Newtonian heating (NH) and the problem was successfully solved by using the RKF45 method with MAPLE. Permeable flat plate has gotten a lot of thought inside the field of logical and engineering application. Related papers that use the permeable flat plates in their research in [2] and [3].

Heat transfer is clarified as the energy exchange between two medium and the transmission, and generally called as thermal energy. This process happened indirectly when there is the presence of complexity among temperatures, and this process will proceed until both two mediums reach a similar temperature. Radiation is one of the heats transfer process besides conduction and convection. Thermal radiation is a medium fit for radiate thermal energy as electromagnetic waves. The heat transfer by radiation does not require the presence of an intervening medium. Thermal radiation effects are extremely important in the content of the flow process involving high temperature. PUTLEY [5], the first scientist that study thermal radiation is Della Porta with optical experiments at the end of the 16 centuries. Siddiqa *et al.* [8], studied on micropolar fluid along a vertical surface on the impact of thermal radiation on conjugate common convection flow. In this study, when the thermal radiation decreases, the coefficient of skin friction and the coefficient of couple stress lessen.

Newtonian heating is portrayed as the process in which internal opposition is insignificant when contrasted with surface obstruction which have numerous applications for instance exchangers, conjugate heat transfers along and solar radiation. Hayat *et al.* [4] studied on the stagnation point flow of a Burgers fluid in the impacts of Newtonian heating where it has been indicated that the temperature rises altogether close to the wall with a thermal slip due to the Newtonian heating phenomenon.

Shampine *et al.* [7] defines bvp4c as an effective method to solve the boundary value problem, but it is not appropriate for high accuracies or problem with the sharp changes in the solution based on the underlying method and computing environment. Bvp4c is the MATLAB software boundary value problem solvers and will accept multi-point of boundary value problem directly. Raymond J [6] state that bvp4c function uses Simpson formula as basic discretization and three-point Lobatto method of order 4.

Therefore, the aim of this present paper is to analyse numerically the effect of the thermal radiation on boundary layer flow using shooting method with bvp4c technique and validate the results by comparing with the results obtained by Mohamed *et al.* [1].

2. Materials and Methods

A permeable flat plate in a steady incompressible viscous fluid of ambient temperature T_∞ and free stream velocity U_∞ are being considered as shown in Figure 1.

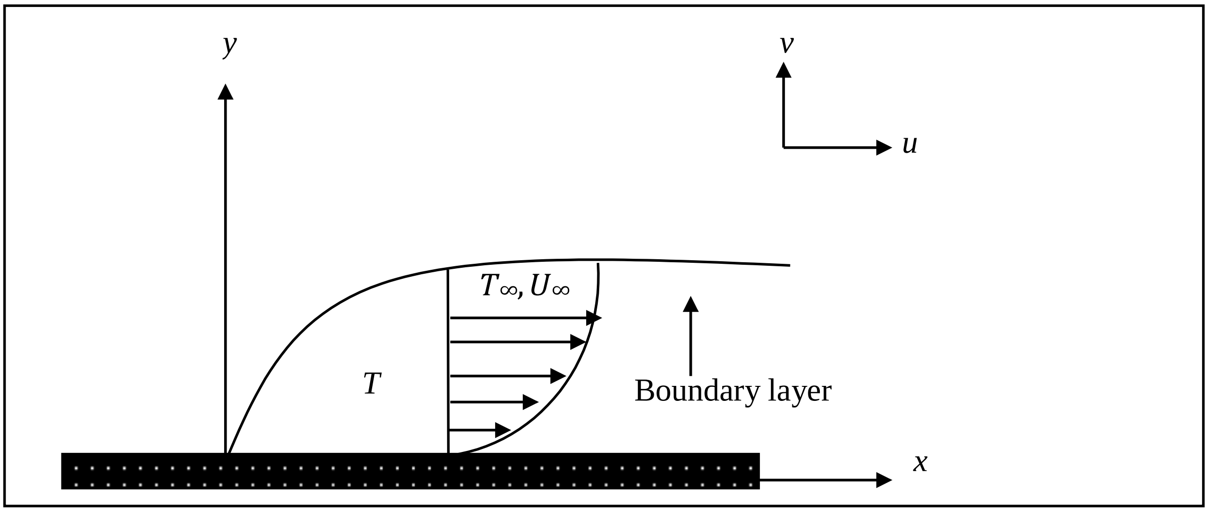

Figure 1:Physical model and coordinate system

Under these assumptions the governing boundary layer equation are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=v\frac{\partial^2 v}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{pC_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{pC_p}\frac{\partial q_r}{\partial y} \tag{3}$$

subject to the boundary conditions that can be expressed as:

$$u=0, v=v_w, -\frac{\partial T}{\partial y}=h_s T \text{ and } y=0, \quad u=U_{oo}, T \square T oo \text{ as } y \square \text{ oo}, \tag{4}$$

where v is the kinematic viscosity, ρ is the fluid density, k is thermal conductivity, C_ρ is specific capacity at constant pressure, v_w is mass transfer velocity at the surface and h_s is heat transfer coefficient. These governing equations are then transformed from partial differential equation to ordinary differential equation using similarity transformation:

$$\eta=\left(\frac{U_{oo}}{2vx}\right)^{1/2} y, \psi=\left(2U_{oo}vx\right)^{1/2} f(\eta), \theta(\eta)=\frac{T-T_{oo}}{T_{oo}}, \tag{5}$$

and the equations $(1)-(4)$ become

$$f'+ff'=0, \tag{6}$$

$$\frac{1}{\Pr}\left(1+\frac{4}{3}N_R\right)\theta'+f\theta'=0. \tag{7}$$

where Pr is Prandtl number and

N_R is radiation parameter with corresponding boundary conditions

$$f(0)=\lambda, f'(0)=0, \theta'(0)=-\gamma[1+\theta(0)], \quad f'(\eta)\square 1, \theta(\eta)\square 0, \text{ as } y \square \text{ oo}, \tag{8}$$

where

$$\gamma=-a\left(\frac{2v}{U_{oo}}\right)^{1/2} \tag{9}$$

is the conjugate parameter and λ is measure the permeability or transpiration rate at the plate surface. The transformed governing equations are solved by using shooting method in MATLAB R2002b with bvp4c.

3. Results and Discussion

The values of wall temperature and skin friction coefficient for different values of Pr when thermal radiation is zero is presented in Table 1. Tables 2 and 3 show values of wall temperature and skin friction coefficient when $N_R = 0.5$ and 1 respectively for different values of suction/blowing parameter. It can be observed that the comparison values of both wall temperature in Newtonian heating (NH) and skin friction coefficient with Mohamed *et al.* [1] are in good agreement throughout all tables. From Table 1, it can be seen that in Newtonian heating the value of $\theta(0)$ decreases as Pr increases. Meanwhile, changes of Prandtl number does not give any effect on the value of $f''(0)$. This is because Pr does not have any relation with the momentum equation of the problem.

Tables 2 and 3 shows the increase of suction ($\lambda > 0$), will reduce the values of wall temperature and enhance the skin friction coefficient. While increase in blowing ($\lambda < 0$), will reduce the skin friction coefficient and increase the wall temperature. Other than that, the present of thermal radiation will increase the value of $\theta(0)$ while it does not give any effects on $f''(0)$.

Next, the effects of temperature profiles with various thermal radiation and suction/blowing parameters with fixed values of Pr, N_R and γ, can be seen in Figures 2-3 respectively. It is observed that increment of N_R enhanced the temperature profiles and its boundary layer thicknesses. This is because of thermal radiation effects added amount of heat on the surface. The heat spread away from the surface which thickening the boundary layer. While the increase of λ in suction case gives a reduction in thermal boundary layer thickens. The opposite trend can be seen for blowing case where the thermal boundary layer thickness increases rapidly. Figure 4 shows the velocity profile for various values of suction/blowing parameter with fixed values of Pr, N_R and γ. It is clear that blowing effect reduces the skin friction coefficient as shown in Tables 2 and 3 and it also thickens the velocity boundary layer thickness.

Table 1: Comparison of the present solution with previously published result for various of Pr when $\lambda = N_R = 0$ and $\gamma = 0.1$

Pr	Wall Temperature $\theta(0)$		Skin Friction Coefficient $f''(0)$	
	Mohamed *et al.*[1]	Present	Mohamed *et al.*[1]	Present
0.7	0.318560	0.318560	0.469600	0.469600
5	0.139750	0.139750	0.469600	0.469600
10	0.107556	0.107556	0.469600	0.469600
30	0.072081	0.072081	0.469600	0.469600
50	0.060098	0.060099	0.469600	0.469600
75	0.052097	0.052097	0.469600	0.469600
100	0.047105	0.047105	0.469600	0.469600

Table 2: Comparison of values of wall temperature $\theta(0)$ and skin friction coefficient $f''(0)$ for various λ when Pr = 0.7, $N_R = 0.5$ and $\gamma = 0.1$.

λ	$N_R = 0.5$			
	Mohamed *et al.*,[1]		Present	
	θ(0)	$f''(0)$	θ(0)	$f''(0)$

-0.70	46.1390	0.0539	46.1390	0.0531
-0.65	4.71501	0.0743	4.7149	0.0743
-0.60	2.5272	0.0975	2.5272	0.0975
-0.55	1.7404	0.1222	1.7418	0.1222
-0.50	1.3331	0.1485	1.3337	0.1485
-0.30	0.6975	0.2658	0.6975	0.2658
-0.10	0.4750	0.3986	0.4750	0.3986
0	0.4097	0.4696	0.4097	0.4696
0.10	0.3601	0.5432	0.3601	0.5432
0.30	0.2896	0.6970	0.2896	0.6970
0.5	0.2418	0,8579	0.2418	0.8579
1.0	0.1701	1.2836	0.1701	1.2836

Table 3: Comparison of values of wall temperature $\theta(0)$ and skin friction coefficient $f''(0)$ for various λ when Pr = 0.7, $N_R = 1$ and $\gamma = 0.1$.

λ	$N_R = 1$			
	Mohamed *et al.*,[1]		Present	
	θ(0)	$f''(0)$	θ(0)	$f''(0)$
-0.70	8.1393	0.0539	7.6840	0.0531
-0.65	3.4727	0.0743	3.4125	0.0743
-0.60	2.2564	0.0975	2.2376	0.0975
-0.55	1.6909	0.1222	1.6829	0.1222
-0.50	1.3575	0.1485	1.3575	0.1485
-0.30	0.7842	0.2658	0.7842	0.2658
-0.10	0.5596	0.3986	0.5596	0.3986
0	0.4908	0.4696	0.4908	0.4696
0.10	0.4374	0.5432	0.4374	0.5432

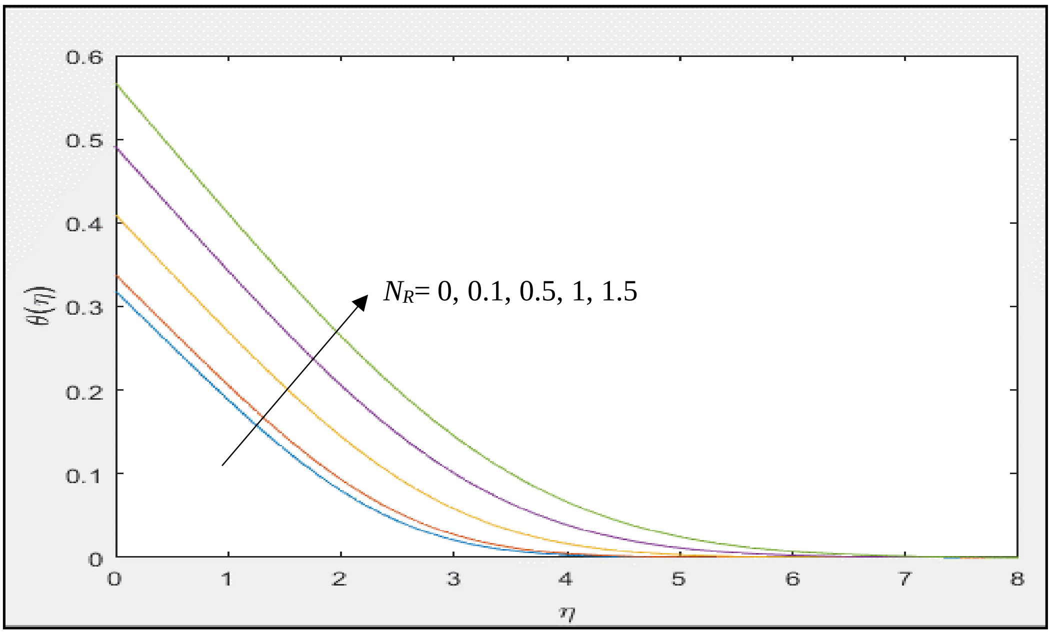

Figure 2: Temperature profiles $\theta(\eta)$ for various values for N_R when Pr = 0.7, λ= 0 and γ=0.1.

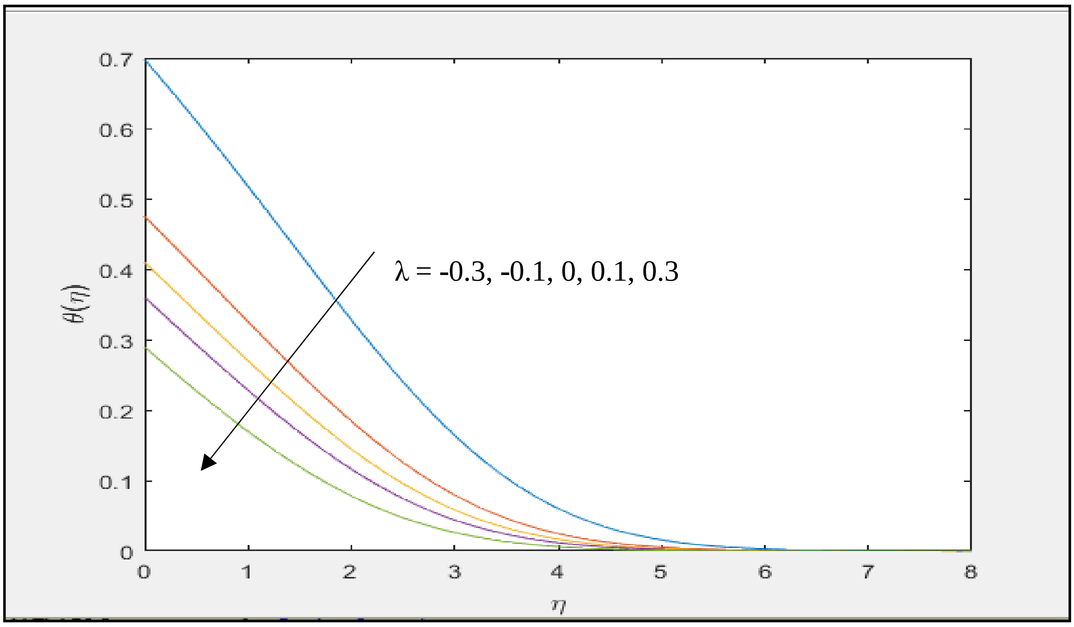

Figure 3: Temperature profiles $\theta(\eta)$ for various values of λ when Pr = 0.7; N_R = 0.5 and γ =0.1.

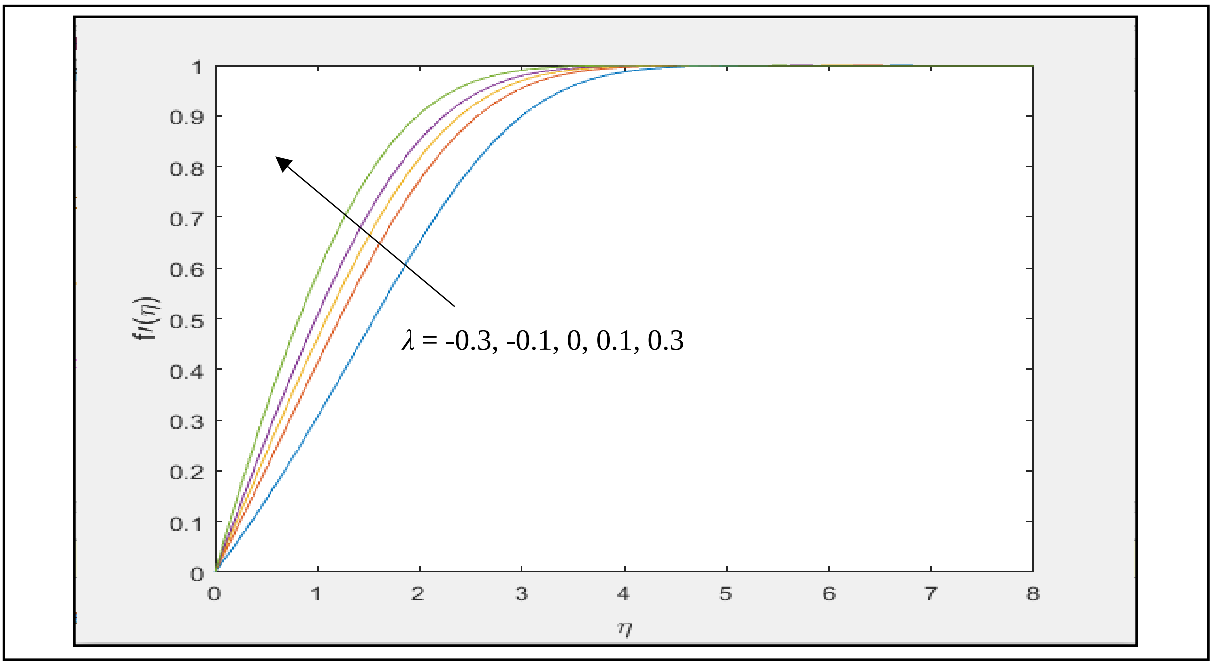

Figure 4: Velocity profiles $f'(\eta)$ for various values of λ when Pr = 0.7; N_R = 0.5 and γ=0.1.

4. Conclusion

The effect of thermal radiation on boundary layer flow over a permeable plate with Newtonian heating has been studied. In order to validate the accuracy of the present studies, the results are compared with Mohamed *et al.* [1]. It can be concluded that

- The increasing thermal radiation parameter and conjugate parameter enhance the wall temperature.
- The increasing Prandtl number reduces wall temperature.
- Skin friction coefficient is not affected by thermal radiation parameter, conjugate parameter and Prandtl number.
- Blowing ($\lambda < 0$) has lower skin friction coefficient and higher wall temperature, higher thermal and velocity boundary layer thickness compared to suction ($\lambda > 0$).

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