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A Predator-Prey Model For Stock Market

Nur Afira Zakuan[1], Kavikumar Jacob[1,*]

[1]Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology,
Universiti Tun Hussein Onn Malaysia, Johor, MALAYSIA

*Corresponding Author Designation

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Abstract: As companies are continuously competing, they are affected because their interaction influences the availability of resources in their development. Besides, this model uses numerical techniques as tools in order to calculate the coefficient of the model. The predator-prey model for stock market was studied numerically and presented. This study focused on applying the biological mathematical model to the evaluation of behaviour between two companies using a predator-prey model as the basis. Moreover, in this research, equilibrium point, stability, and their periodic solution by the use of Hopf Bifurcation were analysed in order to determine the requirement that will ensure the coexistence of both companies or the failure of them or both. Lastly, this study will discuss the two competing companies in the real economy to discover the use of predator-prey models in the stock market specifically and studied the values of the parameters that will impact whether the system is a stable or experienced bifurcation.

Keywords: Lotka-Volterra Models, Predator, Prey, Stock Market

1. Introduction

Predator-prey model or also known as Lotka-Volterra model which consists of a pair of differential equations, is often used to represent the biological system interaction in which different species act as predators and another as prey. This model is similar to the traditional biological concept of the predator targeting a prey for its food source [1]. Biologically, prey populations have an impact on the size of predator populations. According to [2], a strategic concept for prey growth with the absence of predators has been developed and its term of predation can be interpreted also as functional predatory behaviour in line with a reproduction rate due to prey density.

This model used whether predatory interactions could be created to design a basic financial market structure which comprised about the competing prey companies and investment company that reacts as predators. A study by [3] explained that Lotka–Volterra model can be adapted to a market environment where the population reflects customers or market share. It is also said that two species may be

**Corresponding author: kavi@uthm.edu.my*

competing technologies, services, networks, or even two competing species in the business environment. The Lotka-Volterra model are used to study the competitive market dynamic which lead the company to seek to have a strong market share and it is important to continually develop its own scientific and technological innovation and market innovation.

The aim of this research is to study the predator-prey model for stock market system. The following objectives is to determine the dynamical behaviour such as stability analysis of predator-prey model for stock market system. Lastly, Hopf Bifurcation analysis is conducted to identify the critical point where a system stability and also periodic solutions.

2. Materials and Methods

2.1 Formulation of Mathematical model

This research considered predator-prey model of Lotka-Volterra system to study the interaction between two species act as predator and the other as prey. This predator-prey model is served to demonstrate a financial market system which comprises of two competing companies. Meanwhile, the method of Hopf Bifurcation is considered in this research to analyse the stability behaviour in stock market system. Analysis of stability is carried out and numerical simulations for the parameter values are used to analyse the dynamic behaviour in the system.

In the stock market, a predator company is keen to purchase shares in order to combine with one of the two prey-competitors as this company has chances for its advantage between the prey. The predator-prey model is proposed as,

$$\dot{X}_1 = s_1 X_1\left(1-\frac{X_1}{K_1}\right) - m_{12}X_1X_2 - v_1X_1Yr_1(X_1,X_2,Y)$$

$$\dot{X}_2 = s_2 X_2\left(1-\frac{X_2}{K_2}\right) - m_{21}X_1X_2 - v_2X_2Yr_2(X_1,X_2,Y) \qquad \textit{Eq. 1}$$

$$\dot{Y}_1 = -\mu Y + c_1v_1X_1Yr_1(X_1,X_2,Y) + c_2v_2X_2Yr_2(X_1,X_2,Y)$$

where,

$$r_1(X_1,X_2,Y) = \frac{1}{1+\left(\frac{a_2X_2+b_2Y}{X_1}\right)^n} \quad \text{and} \quad r_2(X_1,X_2,Y) = \frac{1}{1+\left(\frac{a_1X_1+b_1Y}{X_2}\right)^n} \qquad \textit{Eq. 2}$$

for $i=1,2$. X_1 and X_2 are relative share prices of competing prey companies, Y is the relative price of predator company per share, X_1 and $X_2, Y \geq 0$, $n=1,2,3,....$

2.2 Equilibrium criteria

Using the idea in [2], there are interior equilibrium point which is attained by figuring a positive solution for the following equations from two intersecting isoclines which are observed at a unique point $(X_1^\bullet, X_2^\bullet)$. First and foremost, put

$$\dot{X}_1 = \dot{X}_2 = \dot{Y} = 0 \qquad \textit{Eq. 3}$$

where X_1, X_2 and Y all not equal to zero. Let the equilibrium point be $\left(\dot{X}_1, \dot{X}_2, \dot{Y}\right)$. Then it yields,

$$s_1 \dot{X}_1\left(1-\frac{\dot{X}_1}{K_1}\right)-m_{12}\dot{X}_1\dot{X}_2-\frac{v_1\dot{X}_1\dot{Y}}{1+\left(\frac{a_2X_2^*+b_2Y^*}{X_1^*}\right)^n}=0$$

$$s_2 \dot{X}_2\left(1-\frac{\dot{X}_2}{K_2}\right)-m_{21}\dot{X}_1\dot{X}_2-\frac{v_2\dot{X}_2\dot{Y}}{1+\left(\frac{a_1X_1^*+b_1Y^*}{X_2^*}\right)^n}=0 \qquad \textit{Eq. 4}$$

$$\{-\mu+\frac{c_1v_1X_1^{\bullet}}{1+\left(\frac{a_2X_2^*+b_2Y^*}{X_1^*}\right)}+\frac{c_2v_2X_2^{\bullet}}{1+\left(\frac{a_1X_1^*+b_1Y^*}{X_2^*}\right)}\}Y^{\bullet}=0$$

By solving the system equation in *Eq. 4*, we will obtain this function

$$f(X_1^{\bullet},X_2^{\bullet})\equiv\frac{\left(s_1\left(1-\frac{X_1^*}{K_1}\right)-m_{12}X_2^{\bullet}\right)\left(X_1^{\bullet}+a_2X_2^{\bullet}\right)}{v_1X_1^{\bullet}+b_2\left(m_{12}X_2^{\bullet}-s_1\left(1-\frac{X_1^*}{K_1}\right)\right)}-$$

$$(\frac{1}{\mu})\left\{c_1\left(s_1X_1^{\bullet}\left(1-\frac{X_1^{\bullet}}{K_1}\right)-m_{12}X_1^{\bullet}X_2^{\bullet}\right)+c_2\left(s_2X_2^{\bullet}\left(1-\frac{X_2^{\bullet}}{K_2}\right)-m_{21}X_1^{\bullet}X_2^{\bullet}\right)\right\}=0 \qquad \textit{Eq. 5}$$

$$g(X_1^{\bullet},X_2^{\bullet})\equiv\frac{\left(s_2\left(1-\frac{X_2^*}{K_2}\right)-m_{21}X_1^{\bullet}\right)\left(X_2^{\bullet}+a_1X_1^{\bullet}\right)}{v_2X_2^{\bullet}+b_1\left(m_{21}X_1^{\bullet}-s_2\left(1-\frac{X_2^*}{K_2}\right)\right)}-$$

$$(\frac{1}{\mu})\left\{c_1\left(s_1X_1^{\bullet}\left(1-\frac{X_1^{\bullet}}{K_1}\right)-m_{12}X_1^{\bullet}X_2^{\bullet}\right)+c_2\left(s_2X_2^{\bullet}\left(1-\frac{X_2^{\bullet}}{K_2}\right)-m_{21}X_1^{\bullet}X_2^{\bullet}\right)\right\}=0 \qquad \textit{Eq. 6}$$

By using *Eq. 5* and *Eq. 6*, it can be solved for $X_1^{\bullet}$ and $X_2^{\bullet}$, which satisfied the condition $X_1^{\bullet}>0$, $X_2^{\bullet}>0$. Based on *Eq. 5*, when $X_1^{\bullet}\to 0,$ then $X_2^{\bullet}\to X_{2a}^{\bullet}$ where

$$X_{2a}^{\bullet}=\frac{c_2s_2b_1+\mu}{c_2\left(\frac{s_2b_1}{K_2}+v_2\right)} \qquad \textit{Eq. 7}$$

This has always been positive, because all parameters in the model are positive since all parameters are positive. In the same way for *Eq. 6* when $X_1^{\bullet}\to 0,$, then $X_2^{\bullet}\to X_{2b}^{\bullet}$ where

$$X_{2b}^{\bullet}=\frac{a_1\mu+c_1s_1b_1}{\frac{c_1s_1b_1}{K_1}} \qquad \textit{Eq. 8}$$

It is valid for all positive parameter values in the model. The $X_{2a}^{\bullet}$ and $X_{2b}^{\bullet}$ points define the points where the isoclines f and g in equations *Eq. 5* and *Eq. 6* meet the X_2 axis.

2.3 Stability Analysis

The Jacobian Matrix is used to analyze the stability in the system at equilibria. The stability of the point of equilibrium $(X_1^{\bullet}, X_2^{\bullet}, Y^{\bullet})$ evaluated, where $X_1^{\bullet}, X_2^{\bullet}, Y^{\bullet}$ are all positive. Then, the first linearization of the original system using the following substitutions:

$$X_1 = X_1^{\bullet} + u, \quad X_2 = X_2^{\bullet} + v, \quad Y = Y^{\bullet} + w. \qquad \textit{Eq. 9}$$

Given that u, v and w are small perturbations to the equilibrium points. By using Taylor's Theorem, all the terms of equilibrium point are developed, ignoring the higher order terms of u, v and w.
The characteristic polynomial has the form of

$$P_3(\lambda) = \lambda^3 + d_1\lambda^2 + d_2\lambda + d_3 = 0\ , \qquad \textit{Eq. 10}$$

where:

$$d_1 = -(j_{11} + j_{22} + j_{33}),$$
$$d_2 = j_{22}j_{11} - j_{12}j_{21} + j_{33}(j_{11} + j_{22}) - j_{31}j_{13} - j_{32}j_{23}, \qquad \textit{Eq. 11}$$
$$d_3 = -j_{11}j_{22}j_{33} + j_{21}j_{12}j_{33} - j_{12}j_{31}j_{23} - j_{11}j_{32}j_{23} - j_{13}j_{21}j_{32} + j_{13}j_{31}j_{22}\ .$$

For stability, getting negative real parts is necessary for the eigenvalues, λ in equation *Eq. 10.* The Routh-Hurwitz criteria satisfy these conditions, which indicate that there is a stable equilibrium occurs where eigenvalues will have negative real parts, if and only if

$$d_1 > 0, \quad d_3 > 0, \quad d_1d_2 - d_3 > 0. \qquad \textit{Eq. 12}$$

2.4 Bifurcations Analysis

Besides the stability of the point, there are also qualitative changes in the stability. A bifurcation indicates the change in stability of equilibrium in a system [2]. There are different types of bifurcations that can arise within the given system that would effect some stability regions.

3. Result and Discussion

3.1 The proposed Lotka-Volterra models

Simple Lotka-Volterra model is used to examine the competing companies by Lotka-Volterra systems modelling dynamics of competition in the market. This chapter started with the discussion of mathematical modelling of Lotka-Volterra model that use the numerical techniques as a tool to estimate the model parameters. In order to analyze and clarify market dynamics, this project need to define a class of non-autonomous Lotka-Volterra systems with strong economic foundations [4].

A general Lotka-Volterra competition system of N – species in a developed niche is expressed by the following ordinary differential equation.

$$\frac{dx_i(t)}{dt} = x_i(t)\left[g_i(t) - \sum_{j=1}^{N} g_j(t)x_j(t)\right], \qquad i = 1,\ldots,N\ . \qquad \textit{Eq. 13}$$

Using historical data as parameter estimation, the growth rates for both telco companies, $g_1(t)$ and $g_2(t)$ are obtained where this model utilizing Maxis Berhad company as the prey and Axiata Group Berhad as the predator and the system obtained as shown in *Eq. 14*.

$$\begin{aligned}\frac{dx_1}{dt} &= x_1\left[5.6796 - 5.6796x_1 - 4.6408x_2\right]\\ \frac{dx_2}{dt} &= x_2\left[4.6408 - 5.6796x_1 - 4.6408x_2\right]\end{aligned} \qquad \textit{Eq. 14}$$

The two populations can be in equilibrium with each other and we can determine when this arises by solving the Lotka-Volterra equations. Since equilibrium indicates that populations do not change with respect to time, the system of differential equations can be written as

$$\begin{aligned}x_1\left[5.6796 - 5.6796x_1 - 4.6408x_2\right] &= 0,\\ x_2\left[4.6408 - 5.6796x_1 - 4.6408x_2\right] &= 0\ .\end{aligned} \qquad \textit{Eq. 15}$$

By solving the system, we get $x_1 = 0, x_2 = 0$. Thus, the first fixed point is $\left(x_1^*, x_2^*\right) = (0,0)$

When $x_1 = 0,$ $4.6408 - 5.6796x_1 - 4.6408x_2 = 0$, $x_2 = 1.$ The second fixed point is $\left(x_1^*, x_2^*\right) = (0,1)$. When $x_2 = 0,$ $5.6796 - 5.6796x_1 - 4.6408x_2 = 0$, $x_1 = 1$. The third fixed point is $\left(x_1^*, x_2^*\right) = (1,0)$.

The system of equations obtained three fixed points which are $(0,0)$, $(0,1)$ and $(1,0)$.

3.2 Stability analysis

Analysis of the behaviour of the solutions for equilibrium points using linearization is one of the most valuable techniques for nonlinear systems. By computing the Jacobian matrix corresponding to each equilibrium points or fixed points, the stability analysis of the system can be carried out.

$$J\left(x_1, x_2\right) = \begin{bmatrix} 5.6796 - 11.3592x_1 - 4.6408x_2 & -4.6408x_1 \\ -5.6796x_2 & 4.6408 - 5.6796x_1 - 9.2816x_2 \end{bmatrix}$$

For the first fixed point which is $(0,0)$, the solution implies that if both populations are extinct, they will continue to be extinct until outside factor can change this. The tendency of a population to become extinct can be observed in this model when looking at the fixed point $(0,0)$.

$$J(0,0) = \begin{bmatrix} 5.6796 & 0 \\ 0 & 4.6408 \end{bmatrix}$$

The corresponding eigenvalues of the Jacobian matrix are $\lambda_1 = 5.6796$ and $\lambda_2 = 4.6408$. Since both of the eigenvalues are real and always positive, the fixed point at the origin is a saddle point which means it is unstable. The impact of this is that extinction of the species of predator and prey may be considered very uncertain. If it were stable, it could attract non-zero species and even so, the dynamics of the system might lead to the extinction of both species in certain instances of initial population levels.

For the second fixed point, $(0,1)$,

$$J(0,1)=\begin{bmatrix} 1.0388 & 0 \\ -5.6796 & -4.6408 \end{bmatrix}.$$

The fixed point of $(0,1)$ have eigenvalues which are $\lambda_1 = 1.0388$ and $\lambda_2 = -4.6408$ The point produces a saddle point but it possess unstable saddle point since the eigenvalues are all mixed positive and negative real values.

$$J(1,0)=\begin{bmatrix} -5.6796 & -4.6408 \\ 0 & -1.0388 \end{bmatrix}$$

For the third fixed point of $(1,0)$ have eigenvalues of $\lambda_1 = -1.0388$ and $\lambda_2 = -5.6796$. Since both of the eigenvalues are negative and less than zero, the stability for the fixed point means it is a stable node. This present the situation where only the prey species survives meanwhile the predator species will become extinct. In order to profitable to the company, the stock market parameter need to be within the stability interval.

Since the parameter values of the system have only real number of eigenvalues, there will be no bifurcation occurs as a Hopf Bifurcation only arises when there are two eigenvalues intersect the imaginary axis. Based on Figure 1, it is shows that the graph for the system of differential equations over time with the initial condition at 1.

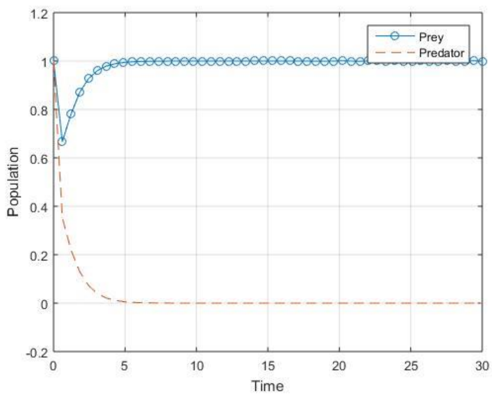

Figure 1

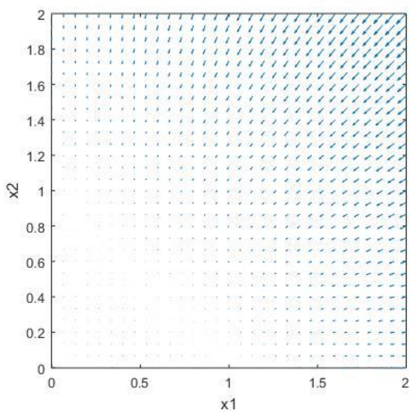

Figure 2

From Figure 2, the vector field drawn indicates both the direction and length of the system. The fixed point of the system equations are those point at which the vector field vanishes, where $F(x_1, x_2) = G(x_1, x_2) = 0$. Thus, the vectors are very short near the critical point.

Based on Figure 1, it is shows that the graph for the system of differential equations with the initial condition at 1 over time. In addition, the local bifurcation will exist when there is a pair of complex conjugate eigenvalues around the fixed point in the system by linearization which the dynamical system will lose the stability. Since the parameter values of the system have only real number of eigenvalues, there will be no bifurcation occurs as a bifurcation only arises when there are two eigenvalues intersect the imaginary axis.

This model provides the opportunity to identify insightful instances of change in company growth. As from the evaluation of the Jacobian matrix of the system, it will determine the type of solution behaviour around each fixed point. From the stability analysis that we have done, the third fixed point reflects that the type of solution defined that the behaviour of the system is a stable node as the value of eigenvalues are both negative. This implies that the relative market share of both prey company and predator company can survive or coexist with the parameter of the model. It is important to be in stability region to remains profitable to both companies. The companies are advised to carefully monitoring the changes in estimating the parameter of the model that will eventually give impact to the stability of the market share.

4. Conclusion

In this paper, we have numerically studied the parameter estimation for the net growth rates, $g_i(t)$ by utilizing yearly stock data between two companies. In a competitive market, the model can analyse the competitiveness among N firms. The competitive roles between the market shares are defined by the representations of the $g_i(t)$ functions. The parameter estimation for the model will then discover the stability of the predator-prey system. The Jacobian matrix are used to analyse the solutions for equilibrium points which is the techniques to determine the stability of the system. The stock market also has the equilibrium which can be determined by the stability analysis that discover the type of equilibrium point of the system. If the system exhibits the oscillatory behaviour, it possible to happen for a company to suffer business losses.

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References

[1] Addison, L., Bhatt, B., & Owen, D. (2017). A Financial Prey-predator Model with Infection in the Predator. *Journal of Advances in Mathematics and Computer Science*, *25*(6), 1–16. https://doi.org/10.9734/jamcs/2017/38379.

[2] Addison, L. M., Bhatt, B. S., & Owen, D. R. (2016). A Prey-Predator Model For Investing In Stocks. *International Journal of Pure and Apllied Mathematics*, *107*(2), 487–504. https://doi.org/10.12732/ijpam.v107i2.17.

[3] Editor, C., & Cheng, L. (2014). *International Journal of Chief Editor Law , Language &*. *4*(March), 0–47.

[4] Marasco, A., Picucci, A., & Romano, A. (2016). Market share dynamics using Lotka-Volterra models. *Technological Forecasting and Social Change*, *105*, 49–62. https://doi.org/10.1016/j.techfore.2016.01.017.