

Solving Magnetohydrodynamic Stagnation-Point Flow Towards a Stretching/Shrinking Sheet using bvp4c Solver

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Abstract

Magnetohydrodynamic (MHD) flows involve the interaction between magnetic fields and electrically conductive fluids, frequently observed in various natural and industrial processes. The governing equations for MHD stagnation-point flow towards a stretching/shrinking sheet are transformed into ordinary differential equations via similarity transformation and solved numerically using the bvp4c solver in MATLAB. Results reveal that an increase in thermal slip and velocity slip parameters leads to a reduction in heat transfer rates, while stronger magnetic field strength decreases the fluid velocity near the sheet but enhances the boundary layer thickness. These findings provide valuable insights for engineering applications, including heat transfer systems and aerodynamic flow control.

1. Introduction

Magnetohydrodynamic (MHD) flows arise from interactions between magnetic fields and conductive fluids, playing a crucial role in engineering and astrophysical applications. Stagnation-point flows, where fluid velocity becomes zero at a surface, introduce complex dynamics. Industrial applications, such as polymer manufacturing and sheet metal processing, benefit significantly from a deeper understanding of MHD flows to optimize heat and mass transfer processes.

Previous studies have focused on dual solutions and the stability of MHD stagnation-point flows, with particular attention to slip conditions and their influence on heat and mass transfer properties. [1] successfully solved MHD stagnation-point flow equations using similarity transformations and shooting methods in Maple, demonstrating the existence of dual solutions under certain parametric conditions. However, their approach faced limitations in handling complex boundary conditions, highlighting the need for alternative numerical solvers for higher reliability and accuracy.

MATLAB's bvp4c solver has since been utilized to overcome these challenges. [2] provided a detailed formulation of this robust boundary value problem solver, showcasing its capability to handle nonlinear differential equations efficiently. [5] further validated the solver's potential for solving MHD flow problems involving intricate boundary conditions, such as velocity and thermal slip effects.

Research on MHD stagnation-point flows has also highlighted the significance of boundary layer behaviour in the presence of stretching and shrinking sheets. [3] and [4] demonstrated how varying sheet stretching/shrinking parameters significantly influence flow separation and heat transfer rates. However, limited attention has been given to the interplay of velocity slip, thermal slip, and magnetic field strength on dual solutions and their stability, which are critical for optimizing engineering processes.

In addressing these gaps, the present study extends the work of [1] and others by leveraging MATLAB’s `bvp4c` solver for more efficient and accurate analysis of MHD stagnation-point flow. Additionally, recent advancements in numerical techniques, as discussed in [6] and [7], provide a strong foundation for exploring dual solutions and stability analysis. These references emphasize the need for parameter-specific investigations to better understand the effects of slip conditions and magnetic field strength on boundary layer behaviour.

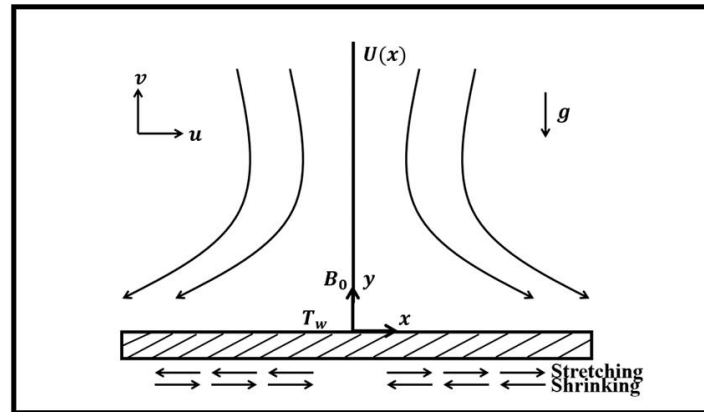


Fig. 1 Physical model of a two-dimensional stagnation-point flow towards a linearly stretching/shrinking sheet [1]

Fig. 1 illustrates the physical model of two-dimensional MHD stagnation-point flow toward a stretching or shrinking sheet. The model highlights the interaction between the magnetic field and the fluid flow, where the sheet’s stretching or shrinking modifies the boundary layer behaviour. The magnetic field exerts a Lorentz force, influencing velocity profiles and the overall flow structure. This setup is crucial for analysing heat and mass transfer in engineering processes like cooling systems and material processing.

2. Problem Formulation

The governing equations for the two-dimensional MHD stagnation-point flow over a stretching/shrinking sheet are as follows [7]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + v \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho} (U - u) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

The boundary conditions for this problem are specified as follows (Bhattacharyya et al. [7]):

$$u = cx + L \left(\frac{\partial y}{\partial x} \right), v = 0, T = T_w + S \frac{\partial T}{\partial y} \text{ at } y = 0$$

$$u \rightarrow U(x), T \rightarrow T_\infty \text{ as } y \rightarrow \infty \tag{4}$$

To reduce the governing equations to a system of ordinary differential equations, we apply the following similarity transformation [1]:

$$\eta = \left(\frac{U}{vx} \right)^{1/2} y, \psi = (vxU)^{1/2} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

where η represents the independent similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ denotes the dimensionless temperature, and ψ is the stream function. The stream function is defined such that $u = \frac{\partial \psi}{\partial y}$

and $v = - \frac{\partial \psi}{\partial x}$, which inherently satisfies Eq. (1). By employing Eq. (6), we derive the following

$$u = \alpha x f'(\eta) \text{ and } v = -(\nu \alpha)^{1/2} f(\eta) \quad (6)$$

primes indicate differentiation with respect to η . By substituting Eq. (6) and (7) into Eq. (2) and (3), the resulting nonlinear ordinary differential equations are obtained as follows:

$$\rightarrow f''' + f f'' + 1 - f'^2 + M(1 - f') = 0 \quad (7)$$

$$\rightarrow \theta'' + Pr f \theta' = 0 \quad (8)$$

Subject to boundary conditions,

$$f(0) = 0, f'(0) = \varepsilon + \delta f'(0), \theta(0) = 1 + \gamma \theta'(0) \quad (9)$$

$$f'(\eta) \rightarrow 1, \theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty$$

where the surface shear stress τ_w and the surface heat flux q_w are given by

$$q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad \tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad (10)$$

where μ and k represent the dynamic viscosity and the thermal conductivity, respectively. Using the similarity variables eq. (6), we derive

$$\frac{1}{2} C_f Re_x^{1/2} x = f''(0) \quad \frac{Nu_x}{Re_x^{1/2}} = -\theta'(0) \quad (11)$$

where $Re_x = U_x/\nu$ is the local Reynolds number

3. Numerical Solution with `bvp4c`

The MATLAB `bvp4c` solver uses a collocation method based on a three-point Lobatto formula to solve boundary value problems (BVPs). This method adapts the computational mesh dynamically to achieve accurate solutions while minimizing computational effort. The system of transformed ordinary differential equations (ODEs) was reformulated as a set of first-order ODEs, which BVP4C solved iteratively.

Proper initial guesses for the solution profiles and boundary conditions were provided to enhance solver convergence and ensure stability. The solver's adaptive mesh refinement capability was instrumental in accurately resolving steep gradients in the velocity and temperature profiles.

4. Results and Discussion

To validate the numerical solutions' accuracy, the values of the skin friction coefficient are compared with previous studies under no-slip conditions ($M = 0$ and $\delta = 0$). Table 1 shows the results align closely with [1], [6] and [7]. For example, at $\varepsilon = -0.25$, the computed skin friction coefficient ($f''(0) = 1.40224$) matches values from these studies, confirming the robustness of the numerical method.

4.1 Skin Friction Coefficient Comparison

Table 1 Comparison with previously published data for the values of $f''(0)$, when $M = 0$ and $\delta = 0$ (no slip) for shrinking case

ε	Present Results	[1]	[6]	[7]
-0.25	1.40224	1.4022	1.40224	1.40224051
-0.3	1.42764	1.4276		
-0.4	1.46870	1.4686		
-0.5	1.49617	1.4957	1.49567	1.49566948
-0.615	1.50936	1.5072		1.50724089
-0.75	1.49709	1.4893	1.48930	1.48929834

Table 1 shows that as the parameter ε becomes more negative, the skin friction coefficient increases. This means that the fluid sticks more strongly to the surface when it is shrinking. The pattern makes sense because shrinking surfaces create higher shear forces on the fluid. The close match between the current results and previous studies proves that the numerical method used is accurate and reliable for solving problems involving magnetohydrodynamic (MHD) flow near a stretching or shrinking sheet.

4.2 Parameter Effects on Thermal Profiles

Fig. 2 shows the effect of the thermal slip parameter (γ) on the boundary layer thickness. Increasing thermal slip parameter ($\gamma = 0.1, 0.5, 1.0, 1.5$) reduces the thermal boundary layer thickness, indicating diminished heat transfer efficiency. These findings highlight the role of slip conditions in controlling heat transfer and validate the numerical approach through consistent skin friction comparisons with previous studies.

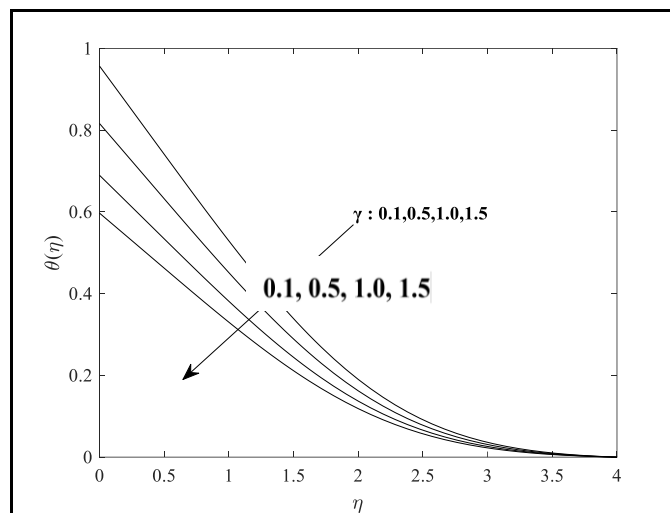


Fig. 2 Thermal slip parameter γ , when $Pr = 0.7, M = 0, \varepsilon = -0.25, \delta = 0$

Fig. 3 illustrates the effect of the magnetic parameter (M) on temperature $\theta(\eta)$ for $M = 0.1, 0.5, 1.0, 1.5$. As M increases, both temperature and thermal boundary layer thickness decrease due to the enhanced Lorentz force, which suppresses fluid motion and heat transfer, reducing thermal conductivity in MHD flows.

Fig. 4 illustrates the effect of the stretching/shrinking parameter ε on temperature $\theta(\eta)$ for various ε values ($\varepsilon = 0.1, 0.5, 1.0, 1.5$). As ε increases, the temperature and thermal boundary layer thickness decrease, indicating enhanced heat dissipation and reduced thermal resistance. This demonstrates the significant role of ε in thermal boundary layer dynamics.

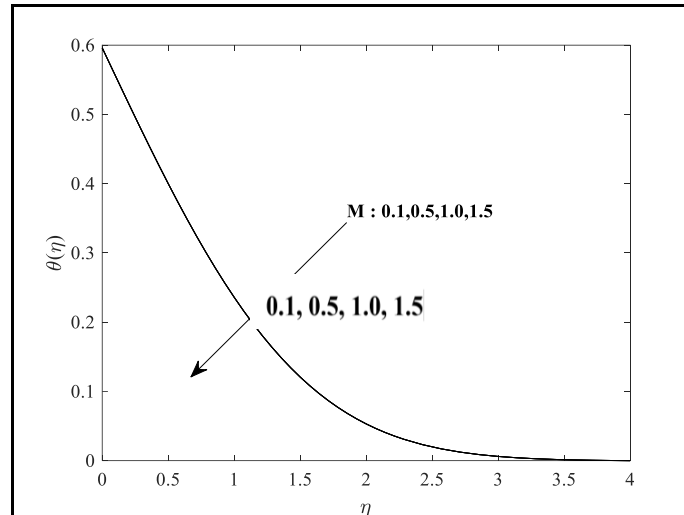


Fig. 3 Several magnetic parameter M , when $Pr = 0.7, \varepsilon = 0.1, \gamma = 1, \delta = 1$

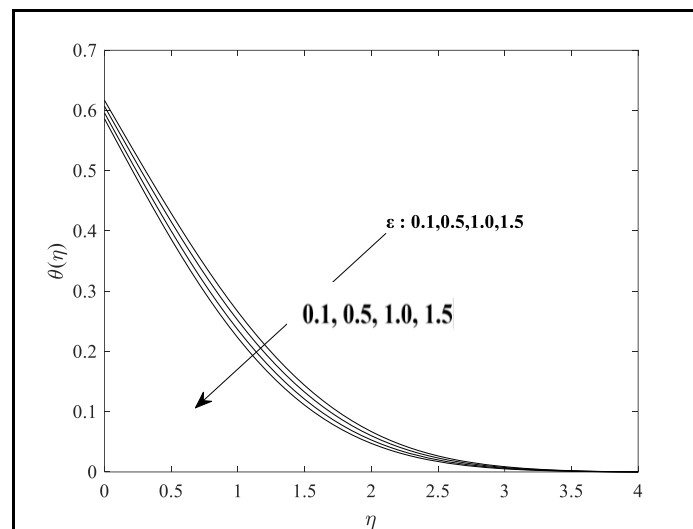


Fig. 4 Several stretching/shrinking parameter ε , when $Pr = 0.7, M = 0.1, \gamma = 1, \delta = 1$

5. Conclusion

This study investigates the magnetohydrodynamic (MHD) stagnation-point flow toward a stretching/shrinking sheet in a viscous and incompressible fluid, considering the effects of thermal slip and other key parameters. The numerical results, validated through comparison with previous studies by [1], [6] and [7], demonstrated excellent agreement, confirming the accuracy of the computational approach. The findings indicate that increasing the thermal slip parameter reduces the boundary layer thickness and decreases the heat transfer rate at the surface, making the thermal boundary layer thinner and less efficient in transferring heat. Additionally, thermal slip significantly impacts both the skin friction coefficient and the surface heat transfer rate. The study successfully employed the shooting technique in combination with MATLAB's bvp4c solver, which proved effective in solving complex boundary layer problems involving nonlinear differential equations. These insights contribute to a deeper understanding of MHD stagnation-point flow behaviour and provide valuable guidance for future studies in this field.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: **study conception and design:** Muhamad Salman Mohd Nazri, Fazlina Aman; **analysis and interpretation of results:** Muhamad Salman Mohd Nazri, Fazlina Aman; **draft manuscript preparation:** Muhamad Salman Mohd Nazri, Fazlina Aman. All authors reviewed the results and approved the final version of the manuscript.

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