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Predicting Transmission of Tuberculosis Mathematical Model Using Adams-Moulton Three-Step Implicit Method

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Abstract

Tuberculosis is an infectious disease that primarily affect the lungs and sometimes other organ. It remains a serious public health issue in Malaysia. SIR model is applied in this study for modelling infectious disease dynamics to predict the trends of tuberculosis transmission. Adams-Moulton three-step implicit method is implemented to solve SIR models. The main purposes of this study are to examine SIR model without demography and with demography as the tuberculosis transmission mathematical model and predict the tuberculosis transmission trend in Malaysia for 2024 by changing the transmission rate of tuberculosis, β and recovery rate of tuberculosis, γ. MATLAB is employed to simplify the calculation works. The results show that SIR model with demography is a better model in predicting the trend of tuberculosis transmission. In conclusion, transmission rate, β and recovery rate, γ significantly impact tuberculosis transmission trend.

1. Introduction

Tuberculosis is a disease that commonly caused by Mycobacterium tuberculosis. Tuberculosis can have a prolonged incubation phase and leads to a persistent infection that can reactivate over time. Besides, it can frequently result in death without proper treatment [1]. Tuberculosis has been proved to be a fatal disease throughout human history. The Bacillus Calmette–Guérin (BCG) vaccine was widely used after World War I. The discovery of streptomycin in 1944 and isoniazid in 1952 marked the beginning of the modern era in tuberculosis treatment and control [2].

After COVID-19, tuberculosis was the second most prevalent infectious illness killer globally in 2022. It was also the main cause of death for HIV-positive individuals and a significant contributor to deaths from antibiotic resistance. Approximately 10.6 million cases of tuberculosis are indicated globally in 2022, affecting 5.8 million men, 3.5 million women, and 1.3 million children [3]. Since tuberculosis is endemic in Malaysia, it remains a serious public health problem. To cope with this problem, Susceptible-Infected-Recovered (SIR) model is used to predict the transmission of tuberculosis. The basic SIR model is displayed by following equations [4]:

$$\frac{dS}{dt} = -\beta IS$$
$$\frac{dI}{dt} = \beta IS - \gamma I \quad (1)$$
$$\frac{dR}{dt} = \gamma I$$

where β is the transmission rate and γ is the recovery rate. SIR model is a mathematical model that was introduced by Kermark & McKendrick in 1991 to predict epidermic dynamics [5]. In SIR model, "S" stands for Susceptible, "I" stands for Infectious, and "R" stands for Recovered. The model assumes that individuals transition from being susceptible to infectious to recovered lastly.

The model that used equation (1) cannot be solved explicitly due to its extreme simplicity. This is because an exact analytical expression for the dynamics of S, I and R cannot be obtained over time. Thus, the model needed to use numerical methods to solve it. Hence, Adams-Moulton method is employed to solve equation (1) due to its implicit nature. Adams-Moulton's method has better performance in approximating the solution of first-order ordinary differential equations (ODEs) compared to other numerical solutions [6], [7], [8]. In this study, SIR model consists of nonlinear ODEs which are suitable to solve by Adams-Moulton method [8]. MATLAB software will be used to simplify the calculation work.

This research consists of three objectives. First, to examine SIR model without demography and with demography as the transmission of tuberculosis mathematical model. Second, to solve tuberculosis transmission mathematical model using Adams-Moulton three-step implicit method. Third, to predict the future trend of transmission of tuberculosis in Malaysia in the year 2024 by changing the value of parameters which are the transmission rate of tuberculosis, β and recovery rate of tuberculosis, γ.

2. Methodology

2.1 SIR Model Without Demography

In SIR model without demography, the assumption will be made for population remained the same during the duration for investigation. The birth rate, the mortality rate and migration in this period are not considered as a parameter to measure this model. To build SIR model without demography, let's consider the modelling differential equation for SIR model without demography as follows [9]:

$$\begin{aligned} f_S(t,I,S) &= \frac{dS}{dt} = -\frac{\beta IS}{N} \\ f_I(t,I,S) &= \frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I \\ f_R(t,I) &= \frac{dR}{dt} = \gamma I \end{aligned} \tag{2}$$

with total population, $N = S + I + R$ and initial condition $S(0) = S_0, I(0) = I_0$ and $R(0) = R_0$ where S is the number of susceptible individuals, I is the number of infected individuals, R is the number of recovered individuals, β is the transmission rate of tuberculosis and γ is the recovery rate of tuberculosis. The formula of β and γ are shown as follows:

$$\text{Transmission rate}, \beta = \frac{1}{\text{the number of people infected by a tuberculosis patient}} \tag{3}$$

$$\text{Recovery rate}, \gamma = \frac{1}{\text{the average time needed for recovering tuberculosis}} \tag{4}$$

In this study, the model (2) is solved using Adams-Moulton three-step implicit method. Adams-Moulton method requires starting values. The starting values can be computed by any one step methods such as Euler's and Runge-Kutta methods. After obtaining the starting values, the predictor is required to find approximation of predict values. Hence, in this study, Runge-Kutta fourth-order method will be considered as initiator, Adams-Bashforth four-step explicit method as predictor and Adams-Moulton three-step implicit method as corrector. To apply this method, several steps need to be implemented [10].

Step 1: Runge-Kutta fourth-order method (RK4)

The multistep method needs the starting values, thus RK4 is used to obtain the starting values. Therefore, for $i = 1, 2, 3$ iterations, the formulation of SIR model without demography using RK4 is given as follows:

$$\begin{aligned} S_{i+1} &= S_i + \frac{1}{6}h(p_1 + 2p_2 + 2p_3 + p_4) \\ I_{i+1} &= I_i + \frac{1}{6}h(q_1 + 2q_2 + 2q_3 + q_4) \end{aligned} \tag{5}$$

$$R_{i+1} = R_i + \frac{1}{6}h(r_1 + 2r_2 + 2r_3 + r_4)$$

The coefficients of p_j, q_j and r_j will be found for $j \in \{1,2,3,4\}$ in every iteration i. In every iteration i, the coefficient of p_j, q_j and r_j is illustrated as follows:

$$\begin{gathered} p_1 = f_s(t_i, I_i, S_i) = -\frac{\beta I_i S_i}{N} \\ q_1 = f_I(t_i, I_i, S_i) = \frac{\beta I_i S_i}{N} - \gamma I_i \\ r_1 = f_R(t_i, I_i) = \gamma I_i \end{gathered} \tag{6}$$

$$\begin{gathered} p_2 = f_s\left(t_i + \frac{h}{2}, I_i + \frac{hq_1}{2}, S_i + \frac{hp_1}{2}\right) = -\frac{\beta\left(I_i + \frac{hq_1}{2}\right)\left(S_i + \frac{hp_1}{2}\right)}{N} \\ q_2 = f_I\left(t_i + \frac{h}{2}, I_i + \frac{hq_1}{2}, S_i + \frac{hp_1}{2}\right) = \frac{\beta\left(I_i + \frac{hq_1}{2}\right)\left(S_i + \frac{hp_1}{2}\right)}{N} - \gamma\left(I_i + \frac{hq_1}{2}\right) \\ r_2 = f_R\left(t_i + \frac{h}{2}, I_i + \frac{hq_1}{2}\right) = \gamma\left(I_i + \frac{hq_1}{2}\right) \end{gathered} \tag{7}$$

$$\begin{gathered} p_3 = f_s\left(t_i + \frac{h}{2}, I_i + \frac{hq_2}{2}, S_i + \frac{hp_2}{2}\right) = -\frac{\beta\left(I_i + \frac{hq_2}{2}\right)\left(S_i + \frac{hp_2}{2}\right)}{N} \\ q_3 = f_I\left(t_i + \frac{h}{2}, I_i + \frac{hq_2}{2}, S_i + \frac{hp_2}{2}\right) = \frac{\beta\left(I_i + \frac{hq_2}{2}\right)\left(S_i + \frac{hp_2}{2}\right)}{N} - \gamma\left(I_i + \frac{hq_2}{2}\right) \\ r_3 = f_R\left(t_i + \frac{h}{2}, I_i + \frac{hq_2}{2}\right) = \gamma\left(I_i + \frac{hq_2}{2}\right) \end{gathered} \tag{8}$$

$$\begin{gathered} p_4 = f_s(t_i + h, I_i + hq_3, S_i + hp_3) = -\frac{\beta(I_i + hq_3)(S_i + hp_3)}{N} \\ q_4 = f_I(t_i + h, I_i + hq_3, S_i + hp_3) = \frac{\beta(I_i + hq_3)(S_i + hp_3)}{N} - \gamma(I_i + hq_3) \\ r_4 = f_R(t_i + h, I_i + hq_3) = \gamma(I_i + hq_3) \end{gathered} \tag{9}$$

Step 2: Adams-Bashforth four-step explicit method

After obtaining the starting values from RK4, the (2) is solved by Adams-Bashforth four-step explicit method to find the predicted value. For $i = 4, 5, 6, \ldots, N$ iterations, the calculation will be continued as follows:

$$\begin{gathered} S_{i+1}^p = S_i + \frac{h}{24}[55f_s(t_i, I_i, S_i) - 59f_s(t_{i-1}, I_{i-1}, S_{i-1}) + 37f_s(t_{i-2}, I_{i-2}, S_{i-2}) - 9f_s(t_{i-3}, I_{i-3}, S_{i-3})] \\ I_{i+1}^p = I_i + \frac{h}{24}[55f_I(t_i, I_i, S_i) - 59f_I(t_{i-1}, I_{i-1}, S_{i-1}) + 37f_I(t_{i-2}, I_{i-2}, S_{i-2}) - 9f_I(t_{i-3}, I_{i-3}, S_{i-3})] \\ R_{i+1}^p = R_i + \frac{h}{24}[55f_R(t_i, I_i) - 59f_R(t_{i-1}, I_{i-1}) + 37f_R(t_{i-2}, I_{i-2}) - 9f_R(t_{i-3}, I_{i-3})] \end{gathered} \tag{10}$$

Step 3: Adams-Moulton three-step implicit method

The predicted values from Adams-Bashforth four-step explicit method are then applied to Adams-Moulton three-step explicit method to find the approximate values. For $i = 4, 5, 6, \ldots, N$ iterations, the calculation will be continued as follows:

$$\begin{gathered} S_{i+1} = S_i + \frac{h}{24}\left[9f_s\left(t_{i+1}, I_{i+1}^p, S_{i+1}^p\right) + 19f_s(t_i, I_i, S_i) - 5f_s(t_{i-1}, I_{i-1}, S_{i-1}) + f_s(t_{i-2}, I_{i-2}, S_{i-2})\right] \\ I_{i+1} = I_i + \frac{h}{24}\left[9f_I\left(t_{i+1}, I_{i+1}^p, S_{i+1}^p\right) + 19f_I(t_i, I_i, S_i) - 5f_I(t_{i-1}, I_{i-1}, S_{i-1}) + f_I(t_{i-2}, I_{i-2}, S_{i-2})\right] \\ R_{i+1} = R_i + \frac{h}{24}\left[9f_R\left(t_{i+1}, I_{i+1}^p\right) + 19f_R(t_i, I_i) - 5f_R(t_{i-1}, I_{i-1}) + f_R(t_{i-2}, I_{i-2})\right] \end{gathered} \tag{11}$$

Step 1 is applied to find i=1, 2, 3 iterations while steps 2 and 3 are applied to find i = 4, ..., 365 iterations.

2.2 SIR Model With Demography

For SIR model with demography, the birth rate and the mortality rate are considered during the duration for investigation as parameters to measure this model. For the population size to be considered constant throughout the period, the birth rate and the mortality rate must be equal. To build SIR model with demography, let's consider the modelling differential equation for SIR model without demography as follow [9].

$$\begin{aligned} f_s(t,I,S) &= \frac{dS}{dt} = \mu - \frac{\beta IS}{N} - \mu S \\ f_I(t,I,S) &= \frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I - \mu I \\ f_R(t,I,R) &= \frac{dR}{dt} = \gamma I - \mu R \end{aligned} \tag{12}$$

with total population, $N = S + I + R$ and initial conditions $S(0) = S_0, I(0) = I_0$ and $R(0) = R_0$ where S is the number of susceptible individuals, I is the number of infected individuals, R is the number of recovered individuals, β is the transmission rate of tuberculosis, γ is the recovery rate of tuberculosis and μ is the tuberculosis mortality rate.

Step 1: Runge-Kutta fourth-order method (RK4)
The multistep method needs the starting values, thus RK4 is used to obtain the starting values. Therefore, for $i = 1, 2, 3$ iterations, the formulation of SIR model without demography using RK4 is shown as follows.

$$\begin{aligned} S_{i+1} &= S_i + \frac{1}{6}h(p_1 + 2p_2 + 2p_3 + p_4) \\ I_{i+1} &= I_i + \frac{1}{6}h(q_1 + 2q_2 + 2q_3 + q_4) \\ R_{i+1} &= R_i + \frac{1}{6}h(r_1 + 2r_2 + 2r_3 + r_4) \end{aligned} \tag{13}$$

The coefficients of p_j, q_j and r_j will be found for $j \in \{1,2,3,4\}$ in every iteration i. In every iteration i, the coefficients of p_j, q_j and r_j is illustrated as follows:

$$\begin{aligned} p_1 &= f_s(t_i, I_i, S_i) = \mu - \frac{\beta I_i S_i}{N} - \mu S_i \\ q_1 &= f_I(t_i, I_i, S_i) = \frac{\beta I_i S_i}{N} - \gamma I_i - \mu I_i \\ r_1 &= f_R(t_i, I_i, R_i) = \gamma I_i - \mu R_i \end{aligned} \tag{14}$$

$$\begin{aligned} p_2 &= f_s\left(t_i + \frac{h}{2}, I_i + \frac{hq_1}{2}, S_i + \frac{hp_1}{2}\right) = \mu - \frac{\beta\left(I_i + \frac{hq_1}{2}\right)\left(S_i + \frac{hp_1}{2}\right)}{N} - \mu\left(S_i + \frac{hp_1}{2}\right) \\ q_2 &= f_I\left(t_i + \frac{h}{2}, I_i + \frac{hq_1}{2}, S_i + \frac{hp_1}{2}\right) = \frac{\beta\left(I_i + \frac{hq_1}{2}\right)\left(S_i + \frac{hp_1}{2}\right)}{N} - \gamma\left(I_i + \frac{hq_1}{2}\right) - \mu\left(I_i + \frac{hq_1}{2}\right) \\ r_2 &= f_R\left(t_i + \frac{h}{2}, I_i + \frac{hq_1}{2}, R_i + \frac{hr_1}{2}\right) = \gamma\left(I_i + \frac{hq_1}{2}\right) - \mu\left(R_i + \frac{hr_1}{2}\right) \end{aligned} \tag{15}$$

$$\begin{aligned} p_3 &= f_s\left(t_i + \frac{h}{2}, I_i + \frac{hq_2}{2}, S_i + \frac{hp_2}{2}\right) = \mu - \frac{\beta\left(I_i + \frac{hq_2}{2}\right)\left(S_i + \frac{hp_2}{2}\right)}{N} - \mu\left(S_i + \frac{hp_2}{2}\right) \\ q_3 &= f_I\left(t_i + \frac{h}{2}, I_i + \frac{hq_2}{2}, S_i + \frac{hp_2}{2}\right) = \frac{\beta\left(I_i + \frac{hq_2}{2}\right)\left(S_i + \frac{hp_2}{2}\right)}{N} - \gamma\left(I_i + \frac{hq_2}{2}\right) - \mu\left(I_i + \frac{hq_2}{2}\right) \\ r_3 &= f_R\left(t_i + \frac{h}{2}, I_i + \frac{hq_2}{2}, R_i + \frac{hr_2}{2}\right) = \gamma\left(I_i + \frac{hq_2}{2}\right) - \mu\left(R_i + \frac{hr_2}{2}\right) \end{aligned} \tag{16}$$

$$\begin{aligned} p_4 &= f_s(t_i + h, I_i + hq_3, S_i + hp_3) = \mu - \frac{\beta(I_i + hq_3)(S_i + hp_3)}{N} + \mu(S_i + hp_3) \\ q_4 &= f_I(t_i + h, I_i + hq_3, S_i + hp_3) = \frac{\beta(I_i + hq_3)(S_i + hp_3)}{N} - \gamma(I_i + hq_3) - \mu(I_i + hq_3) \\ r_4 &= f_R(t_i + h, I_i + hq_3, R_i + hr_3) = \gamma(I_i + hq_3) - \mu(R_i + hr_3) \end{aligned} \tag{17}$$

Step 2: Adams-Bashforth four-step explicit method
After obtaining the starting values from RK4, the (12) is solved to Adams-Bashforth four-step explicit method to find the predicted values. For $i = 4, 5, 6, \ldots, N$ iterations, the calculation will be continued as follows:

$$\begin{aligned} S_{i+1}^{p} &= S_i + \frac{h}{24}[55f_s(t_i, I_i, S_i) - 59f_s(t_{i-1}, I_{i-1}, S_{i-1}) + 37f_s(t_{i-2}, I_{i-2}, S_{i-2}) - 9f_s(t_{i-3}, I_{i-3}, S_{i-3})] \\ I_{i+1}^{p} &= I_i + \frac{h}{24}[55f_I(t_i, I_i, S_i) - 59f_I(t_{i-1}, I_{i-1}, S_{i-1}) + 37f_I(t_{i-2}, I_{i-2}, S_{i-2}) - 9f_I(t_{i-3}, I_{i-3}, S_{i-3})] \\ R_{i+1}^{p} &= R_i + \frac{h}{24}[55f_R(t_i, I_i, R_i) - 59f_R(t_{i-1}, I_{i-1}, R_{i-1}) + 37f_R(t_{i-2}, I_{i-2}, R_{i-2}) - 9f_R(t_{i-3}, I_{i-3}, R_{i-3})] \end{aligned} \tag{18}$$

Step 3: Adams-Moulton three-step implicit method
After obtaining the predicted values from Adams-Bashforth four-step explicit method are then applied to Adams-Moulton three-step explicit method to find the approximate values. For $i = 4, 5, 6, \ldots, N$ iterations, the calculation will be continued as follows:

$$\begin{aligned} S_{i+1} &= S_i + \frac{h}{24}\left[9f_s\left(t_{i+1}, I_{i+1}^{p}, S_{i+1}^{p}\right) + 19f_s(t_i, I_i, S_i) - 5f_s(t_{i-1}, I_{i-1}, S_{i-1}) + f_s(t_{i-2}, I_{i-2}, S_{i-2})\right] \\ I_{i+1} &= I_i + \frac{h}{24}\left[9f_I\left(t_{i+1}, I_{i+1}^{p}, S_{i+1}^{p}\right) + 19f_I(t_i, I_i, S_i) - 5f_I(t_{i-1}, I_{i-1}, S_{i-1}) + f_I(t_{i-2}, I_{i-2}, S_{i-2})\right] \\ R_{i+1} &= R_i + \frac{h}{24}\left[9f_R\left(t_{i+1}, I_{i+1}^{p}, R_{i+1}^{p}\right) + 19f_R(t_i, I_i, R_i) - 5f_R(t_{i-1}, I_{i-1}, R_{i-1}) + f_R(t_{i-2}, I_{i-2}, R_{i-2})\right] \end{aligned} \tag{19}$$

Step 1 is applied to find i=1, 2, 3 iterations while steps 2 and 3 are applied to find i = 4, ..., 365 iterations.

2.3 Incidence Rate and Success Rate

The number of newly diagnosed cases of tuberculosis (TB) per 100,000 people is known as the incidence rate of tuberculosis. Besides, success rate is a metric to evaluate the effectiveness of tuberculosis treatment. Formula for these parameters are shown as follows.

$$\text{Incidence rate, IR} = \frac{\text{Number of the newly tuberculosis diagnosed in the period}}{\text{Total population at risk in period}} \times 100000 \tag{20}$$

$$\text{Success rate, SR} = \frac{\text{Number of the recovered people in the period}}{\text{Total population of infected people in the period}} \times 100\% \tag{21}$$

3. Results and Discussions

3.1 Accuracy of SIR Models

To build the SIR models without demography and with demography, the initial values which are the historical data from 2019 is needed. The number of susceptible and infected individuals in 2019 can be obtained from National Strategic Plan to End TB (2021-2030) [11]. The number of susceptible individuals is 177121 and the number of recovered individuals is 26352. The number of recovered individuals can be estimated using the success treatment rate. The success treatment of tuberculosis in Malaysia in 2019 is approximately 80% [12]. By applying (21), the number of recovered individuals can be estimated. The number of recovered individuals in Malaysia in 2019 is around 21082.

An assumption about transmission rate, β and recovery rate, γ have been made. A tuberculosis patient can infect about 20.6 people and tuberculosis requires 140 days to recover from the disease with treatment [13]. In 2019, there are 5.8 tuberculosis death per 100000 people [14]. Transmission rate, β and recovery rate, γ of tuberculosis can be calculated using (3) and (4) respectively. Hence, the physical parameters $\beta = 0.04854$, $\gamma = 0.007142$ and $\mu = 0.000058$ for 2019 are obtained after the calculation. With these parameters, the prediction of transmission of tuberculosis in the year 2020 can be made using SIR model without demography (2) and SIR model with demography (12).

3.2 SIR Model Without Demography

The initial value of susceptible individuals, $S(0)$ is 177121, infected individuals, $I(0)$ is 26352 and recovered individuals, $R(0)$ is 21082. While the transmission rate, β is 0.04854 and the recovery rate, γ is 0.007143. The initial conditions and value for each parameter are used in SIR model without demography.

By applying (2) to (11), the SIR data of tuberculosis for the year 2020 is obtained as Table 1. The step size, h is 1 for the model. The initial values $S(0), I(0)$ and $R(0)$, are employed as day 0 in the model.

Table 1 SIR data for tuberculosis in Malaysia in the year 2020

Day	S	I	R
0	177121	26352	21082
1	176099	27183	21273
⋮	⋮	⋮	⋮
364	798	24150	199607
365	794	23982	199779

Table 1 shows the SIR data without demography of tuberculosis in Malaysia for year 2020. The data is applied to plot the graph to view the trend of SIR in year 2020.

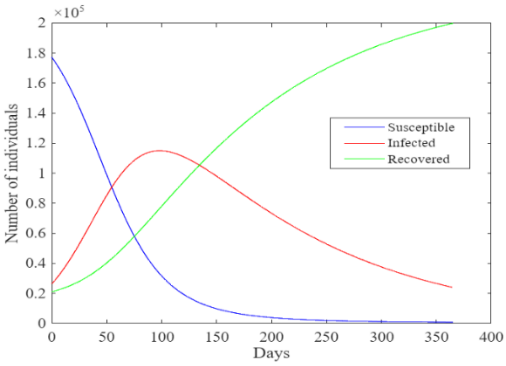


Fig. 1 Graph of SIR model without demography for year 2020

Fig. 1 displays the graph of SIR model without demography of tuberculosis in Malaysia for year 2020. Since the graph is a time series graph, the trend of SIR model can be analysed. From Fig. 1, the peak of infected individuals is notified on day 98 which is 114947 people. After day 98, the case of infected individuals starts to decrease over time. Next, the number of suspected individuals scores the highest count at the first day, then it started to decrease gradually until the end of the year 2020. Besides, the number of recovered individuals rose steadily and it reaches the highest peak at the end of the year 2020. The number of infected individuals is 23982 people at the last day of 2020.

3.2.1 SIR Model With Demography

The initial value of susceptible individuals, $S(0)$ is 177121, infected individuals, $I(0)$ is 26352 and recovered individuals, $R(0)$ is 21082. While the transmission rate, β is 0.04854, the recovery rate, γ is 0.007143 and mortality rate or birth rate, μ is 0.000058. The initial conditions and value for each parameter are used in SIR model without demography.

By applying (12) to (19), the SIR data of tuberculosis for the year 2020 is obtained as Table 2. The step size, h is 1 for the model. The initial values $S(0) = S_0, I(0) = I_0$ and $R(0) = R_0$ are employed as day 0 in the model.

Table 2 SIR data for tuberculosis in Malaysia in the year 2020

Day	S	I	R
0	177121	26352	21082
1	176089	27181	21272
⋮	⋮	⋮	⋮
364	822	23683	195358

365	818	23518	195516

Table 2 shows the SIR data with demography of tuberculosis in Malaysia for year 2020. The data is applied to plot the graph to view the trend of SIR in year 2020.

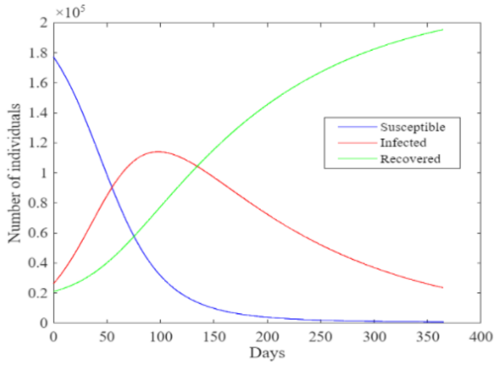


Fig. 2 Graph of SIR model with demography for year 2020

Fig. 2 displays the graph of SIR model with demography of tuberculosis in Malaysia for year 2020. From Fig. 2, the peak of infected individuals is notified on day 98 which is 114104 people. After day 98, the case of infected individuals starts to decrease over time. Next, the number of suspected individuals score the highest count at the first day, then it starts to decrease gradually until the end of the year 2020. Besides, the number of recovered individuals rose steadily and it reaches the highest peak at the end of the year 2020. The number of infected individuals is 23518 people at the last day of 2020.

3.2.2 Comparison With Actual Data

To examine the accuracy of model (2) and (12), the error of both models is calculated. The number of infected individuals in the last day of the year 2020 will be counted as the number of infected individuals for the year 2020. To simplify the number of infected individuals, incidence rate is employed to measure the incidence of infected individuals of tuberculosis per 100,000 people. The equation (20) is used to calculate the incidence rate. The total population for year 2020 is approximately 33.20 million people [15]. The actual number of infected individuals for 2020 is 23644, it indicates the incidence rate is 71.22%.

Table 3 The actual and approximated incidence rate of TB for year 2020

Type of SIR Models	The Approximated Incidence Rate	The Actual Incidence Rate	Error (%)
Without demography	72.23	71.22	1.42
With demography	70.84	71.22	0.53

Table 3 shows the actual and approximated incidence rate of tuberculosis of the two models and the error between the actual and approximated incidence rate of tuberculosis in the year 2020. For SIR model without demography and SIR model with demography, the incidence rate of both models illustrated small error value which were 1.42% and 0.53% respectively. This exhibited that the SIR models using Adams-Moulton three step implicit method is suitable method in predicting the transmission of tuberculosis. By comparing the error of both models, it shows that SIR model with demography has smaller values in error compared to SIR model without demography. This proves that SIR model with demography is a better model to predict the trend of tuberculosis. Thus, SIR model with demography is selected to predict the transmission of tuberculosis in Malaysia for year 2024.

3.3 Prediction of Tuberculosis in 2024

In this section, SIR model with demography using Adams-Moulton three-step implicit method is applied to predict the transmission of tuberculosis in Malaysia in the year 2024. The estimated number of susceptible of tuberculosis for the year 2023 is 190000 people. The number of infected individuals was 26781. The success treatment rate of tuberculosis in Malaysia for the year 2022 was 81% [12]. With the given success treatment rate, the estimated number of recovered individuals was 21693.

Based on the tuberculosis profile: Malaysia, the estimated number of deaths caused by tuberculosis in 2023 are 3140 [16]. The total population in Malaysia for the year 2023 is 34.31 million approximately. Thus, the estimated mortality rate or birth rate, μ is 0.000092.

3.3.1 Effect to SIR model When Parameter β is Changing

In this section, the transmission rate, β will be changing from 0.1, 0.05, 0.04854 and 0.03333 while the other parameters remain the same. The values of transmission rate, β equals to 0.1, 0.05, 0.04854 and 0.03333 which present a tuberculosis patient can infect to 10 people, 20 people, 20.6 people and 30 people respectively. The different values of β were selected to view its effect in SIR model.

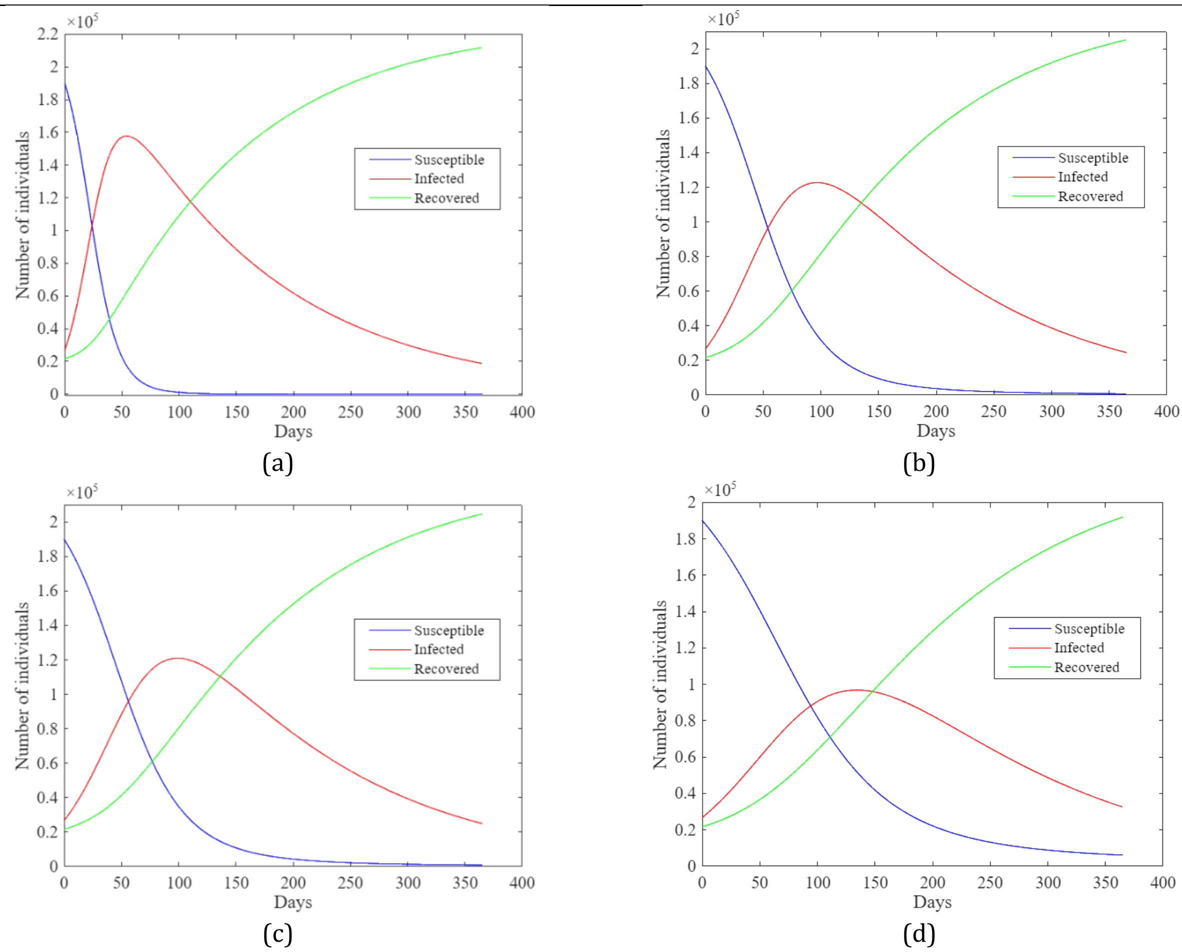


Fig. 3 Graph of SIR model with different $\boldsymbol{\beta}$ (a) $\boldsymbol{\beta = 0.1}$; (b) $\boldsymbol{\beta = 0.05}$; (c) $\boldsymbol{\beta = 0.04854}$; (d) $\boldsymbol{\beta = 0.03333}$

Table 4 The number of infected people at the peak day for different $\boldsymbol{\beta}$

β	Number of Infected People	Peak Day
0.1	157746	54
0.05	122741	97
0.04854	120973	99
0.03333	96858	134

Fig. 3 illustrates the trend of SIR models when $\beta = 0.1$, $\beta = 0.05$, $\beta = 0.04854$ and $\beta = 0.03333$. Table 4 displays the number of infected people at the peak day for different β. By observing Table 4, when the greater the value of transmission rate, the earlier the peak time for number of infected people is recorded. When β is 0.1, it has the highest number of infected people at its peak time and the earliest peak time for infection. When β is 0.03333, it has the lowest number of infected people at its peak time and the slowest peak time for infection of tuberculosis. This proves the spread of the tuberculosis will be faster when the transmission rate was higher.

Table 5 The number of infected people and number of recovered people at the last day of 2024 for different $\boldsymbol{\beta}$

β	Number of Infected People	Number of Recovered people
0.1	18709	211888

0.05	24516	205345
0.04854	24964	204753
0.03333	32534	191965

Table 5 presents the number of infected people and number of recovered people at the last day of 2024 for different β. A higher value of transmission rate will gain lower number of infected individuals for the end of the year 2024 but a higher number of recovered individuals. This pattern develops because a higher transmission rate suggests a faster pace of disease spread, which causes the graph to peak earlier and increases the number of recovered people. Conversely, a lower rate of transmission causes the disease to spread more slowly and leaving higher amount of infected people at year-end. Assume the total population in Malaysia for 2024 is approximately 34.69 million people, the incidence rate of tuberculosis per 100000 people can be calculated using (18).

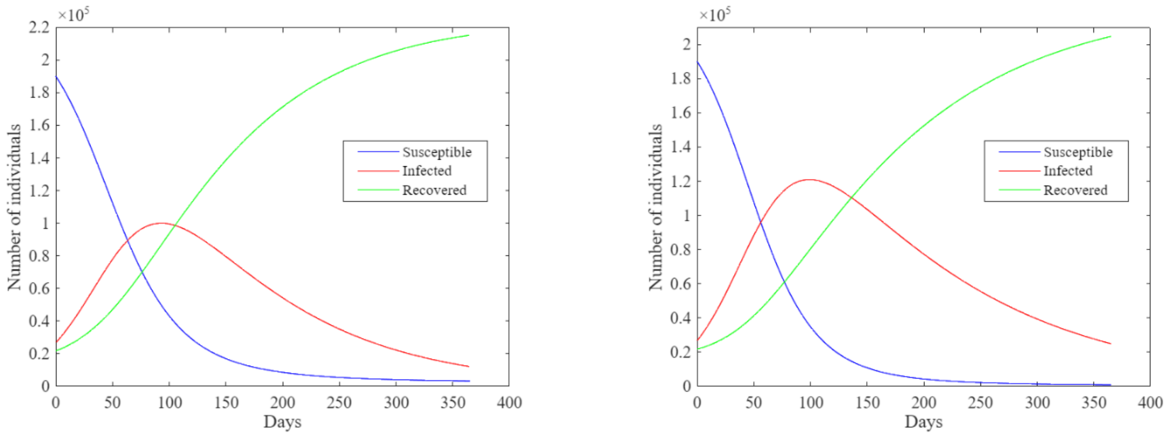
Table 6 The estimated incidence rate of tuberculosis per 100000 people in 2024 for different $\boldsymbol{\beta}$

β	Number of Infected People	Incidence Rate of TB Per 100000 People
0.1	18709	53.94
0.05	24516	70.67
0.04854	24964	71.96
0.03333	32534	93.78

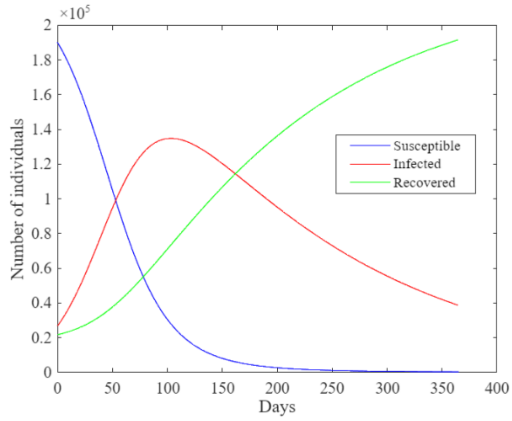
Table 6 displays the estimated incidence rate of tuberculosis per 100000 people in 2024 for different β. Based on Table 6, when the $\beta = 0.1$, it has the lowest incidence rate which is 53.96 while when the $\beta = 0.03333$, it has the highest incidence rate which is 93.78 per 100000 people. This concludes that incidence rate of tuberculosis increases when the transmission rate decreases.

3.3.2 Effect to SIR Model when Parameter γ is Changing

In this section, the recovery rate, γ will be changing from 0.01, 0.007143 and 0.005566 while the other parameters remain the same. The values of recovery rate, γ equals to 0.01, 0.007143 and 0.005566 which represents a tuberculosis patient take 100 days, 140 days and 180 days to recover from the disease.



(a) (b)



(c)

Fig. 4 Graph of SIR model with different γ (a) $\gamma = 0.01$; (b) $\gamma = 0.007143$; (c) $\gamma = 0.005566$

Table 7 The number of infected people at the peak day for different γ

γ	Number of Infected People	Peak Day
0.01	99996	93
0.007143	120973	99
0.005566	134823	103

Fig. 4 illustrates the trend of SIR models when $\gamma = 0.01$, $\gamma = 0.007143$ and $\gamma = 0.005566$. Table 7 illustrates the number of infected people at the peak day for different recovery rate, γ. A higher recovery rate of tuberculosis will have smaller number of infected individuals for the peak time and earlier peak day for infected people.

Table 8 The number of infected people and number of recovered people at the end of 2024 for different γ

γ	Number of infected people	Number of recovered people
0.01	12103	215251
0.007143	24964	204753
0.005566	38657	191623

Table 8 displays the number of infected people and number of recovered people at the end of 2024 for different recovery rate, γ. Based on Table 8, higher recovery rate has a smaller number of infected people and a higher number of recovered individuals at the end of the year 2024. This showed that the number of infected people will be reduced by the high recovery rate. When the recovery rate increases, it will increase the number of recovered people and reduce the number of infected people. Let consider the total population in Malaysia for 2024 is around 34.69 million people, the incidence rate of tuberculosis per 100000 people can be calculated using (18).

Table 9 The estimated incidence rate of tuberculosis in 2024 for different γ

γ	Number of Infected People	Incidence Rate of TB Per 100000 People
0.01	12103	34.88
0.007143	24964	71.96
0.005566	38657	111.44

Table 9 shows the estimated incidence rate of tuberculosis per 100000 people in 2024 for different value of recovery rate, γ. Based on the given table, incidence rates range from 34.88 (for $\gamma = 0.01$) to 111.44 (for $\gamma = 0.005566$). These results demonstrate that improving recovery rates significantly reduces tuberculosis incidence rate.

4. Conclusion

This study analysed the dynamic of tuberculosis transmission in Malaysia using SIR model with demography and SIR model without demography. Adams-Moulton three-step implicit method is used to solve both models. Both SIR models illustrated small errors when compared to actual data. The SIR model with demography has a lower error (0.53%) compared to the SIR model without demography (1.42%). This proves that including demographic factors such as birth rate and mortality rate enhances predictive accuracy of tuberculosis transmission dynamic. Consequently, SIR model with demography is used in predicting the trend of SIR model with demography in Malaysia for the year 2024.

The prediction for 2024 revealed that the transmission rate, β and recovery rate, γ significantly affect the dynamic of tuberculosis transmission. A higher transmission rate, β accelerates the spread of tuberculosis and the peak time for infection while reducing the numbers of infected individuals at the end of 2024. Similarly, a higher recovery rate, γ reduces the peak time for infections and the number of infected individuals for 2024. In conclusion, these findings emphasise that the transmission and recovery rate of tuberculosis play vital roles in shaping the transmission dynamics. The different values of parameters which are the transmission and recovery rate of tuberculosis provide valuable guidance for public health strategies against tuberculosis.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Tan Kim Yeong, Syahirbanun Isa;* ***data collection:*** *Tan Kim Yeong;* ***analysis and interpretation of results:*** *Tan Kim Yeong;* ***draft manuscript preparation:*** *Tan Kim Yeong, Syahirbanun Isa. All authors reviewed the results and approved the final version of the manuscript.*

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