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Solving SIR Model of Yellow Fever Using Dormand-Prince and Euler Method

Muhammad Irham Hisham[1], Norzuria Ibrahim[1]*

[1] *Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology, UTHM Kampus Cawangan Pagoh, Hab Pendidikan Tinggi Pagoh, KM 1, Jalan Panchor, 84600, Pagoh, Muar, Johor, MALAYSIA.*

*Corresponding Author: norzuria@uthm.edu.my
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Abstract

This study investigated numerical methods for solving the SIR model, which represents susceptible (S), infectious (I), and recovery (R) populations, in the context of yellow fever dynamics. The aim of this study is to evaluate the effectiveness of two numerical methods, the Dormand-Prince method and the Euler method, in solving the SIR model of yellow fever. Through numerical simulations using MATLAB R2024b software, the study examines the dynamics of susceptible, infected, and recovered populations over time to gain insights into the spread and control of yellow fever. The research includes parameters in SIR for validation, and comparison to ensure better approximation of the numerical solutions. Comparisons were made between the Dormand-Prince and Euler methods at step sizes of h = 0.01 and h = 0.05, with results indicating that the Dormand-Prince method, especially at h = 0.01, provided better approximations of yellow fever dynamics compared to the Euler method. Smaller step sizes improved performance and reduced errors, emphasizing the critical role of method selection and step size in studying disease spread. By addressing these aspects, the study provides valuable insight into yellow fever dynamics, supports public health strategies, and contributes to yellow fever epidemiology through the application of mathematical modelling and numerical methods.

1. Introduction

Yellow fever is a serious viral disease spread by the Aedes aegypti mosquito, which is common in tropical regions of Africa and South America. The disease can cause severe symptoms, including fever, organ damage, and in many cases, death if not treated properly [1]. Controlling yellow fever is especially important in crowded cities where mosquitoes thrive and can spread the disease quickly. Vaccination is one of the best ways to prevent outbreaks. However, environmental changes like climate change and deforestation are making the problem worse by creating better habitats for mosquitoes and increasing the chances of humans getting infected [2].

To predict and control yellow fever outbreaks, scientists use a mathematical tool called the Susceptible-Infected-Recovered (SIR) model. This model helps us understand how the disease spreads by dividing people into three groups: those who can catch the disease (susceptible), those who are currently sick (infected), and those who have recovered. By studying how these numbers change over time, the SIR model can help predict outbreaks and test ways to stop them [3]. For example, studies using the SIR model have shown that targeted vaccination campaigns can reduce the number of cases in high-risk areas [4].

To solve the equations in the SIR model, scientists often use special calculation methods. One of the highly advanced methods is called the Dormand-Prince method [5]. This method provides better approximation because it adjusts the calculation steps as it goes, balancing precision and speed [6]. It is commonly used in simulations of yellow fever to test different vaccination strategies or predict how the disease will spread. This method helps public health officials make the best use of resources to control outbreaks effectively [7].

Another method, called the Euler method, is the basic and simplest. The Euler method uses fixed steps to calculate how the disease spreads. It is easier to use and helpful for quick tests or when learning how the SIR model works. The Euler method is useful for understanding basic ideas about disease spread. Researchers have used it to model the early stages of yellow fever outbreaks and as a starting point for more detailed studies [8].

Both the Dormand-Prince and Euler methods are important tools for understanding yellow fever. The Dormand-Prince method is used for detailed, better approximations, while the Euler method is helpful for simple, quick calculations. Together, they help scientists and public health officials plan better ways to fight the disease.

By focusing on vaccination, protecting the environment, and using tools like the SIR model, we can reduce the spread of yellow fever. These efforts show how science and teamwork can make a big difference in keeping people safe from this deadly disease.

The objectives of this study are to examine a mathematical formulation for the SIR model of the yellow fever problem that is suitable for numerical analysis. The model is solved using both the Dormand-Prince method and Euler's method to explore their effectiveness. A comparison of the solutions obtained from these methods is conducted using MATLAB as the computational tool, highlighting their performance and better approximation in solving the SIR model.

2. Methodology

This section provides a detailed explanation of the SIR model of yellow fever and the two numerical methods used, namely the Dormand-Prince method and Euler method. The explanation includes the formulas, initial conditions, and vector notation. The selected step sizes, h = 0.01 and h = 0.05, are used to evaluate the performance of the SIR model for yellow fever by employing the Dormand-Prince method and the Euler method. All calculations are performed using MATLAB R2024b software.

2.1 SIR Model of Yellow Fever

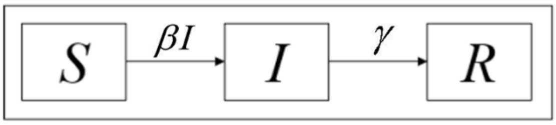


Fig. 1 *The SIR Model compartment*

Fig. 1 shows the SIR model, which is used to study how diseases spread in a population. It has three groups: Susceptible (S) for people who can catch the disease, Infected (I) for people who have the disease and can spread it, and Recovered (R) for people who have recovered and cannot get the disease again. The disease spreads based on the infection rate (β), and infected people recover at a recovery rate (γ). This model helps to understand and control disease outbreaks [9].

$$N(t) = S(t)+I(t)+R(t) \tag{1}$$

where;

- $N(t)$ is the total population at any time, t
- $S(t)$ is the susceptible population at any time, t
- $I(t)$ is the infectious population at any time, t
- $R(t)$ is the recovered population at any time, t.

In the SIR model, the parameters β (infection rate) and γ (recovery rate) play a crucial role in determining the dynamics of disease transmission. In this study, the value of β is 0.3, representing the rate at which susceptible individuals become infected when exposed to infected individuals. The value of γ is 0.1, representing the rate at which infected individuals recover or gain immunity. These parameter values are commonly used in literature as estimates for modelling the dynamics of infectious diseases, including yellow fever. The values were sourced from [10].

The following is a set of mathematical equations that may be derived from the model that was utilized in this investigation. These equations were obtained from [11]:

$$\frac{dS}{dt} = -\frac{\beta SI}{N}, \tag{2}$$

$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I, \tag{3}$$

$$\frac{dR}{dt} = \gamma I. \tag{4}$$

where;

- $\frac{dS}{dt}$ is the rate of susceptible population, S with respect to any time, t
- $\frac{dI}{dt}$ is the rate of infected population, I with respect to any time, t
- $\frac{dR}{dt}$ is the rate of recovered population, R with respect to any time, t
- β is constant rate of disease transmission
- γ is recovery rate.

2.2 Dormand-Prince Method

The Dormand-Prince method, known for its enhanced precision and adaptability, has replaced traditional methods such as the Runge-Kutta technique. Renowned for its accuracy, it is categorized under adaptive step methods. The specific iterative equations defining $S(t)$, $I(t)$ and $R(t)$ within the framework of the SIR model are outlined as follows [12]:

$$k_1 = hf(t_k, y_k) \tag{5}$$

$$k_2 = hf(t_k + \frac{1}{5}h, y_k + \frac{1}{5}k_1) \tag{6}$$

$$k_3 = hf(t_k + \frac{3}{10}h, y_k + \frac{3}{40}k_1 + \frac{9}{40}k_2) \tag{7}$$

$$k_4 = hf(t_k + \frac{4}{5}h, y_k + \frac{44}{45}k_1 - \frac{56}{15}k_2 + \frac{32}{9}k_3) \tag{8}$$

$$k_5 = hf(t_k + \frac{8}{9}h, y_k + \frac{19372}{6561}k_1 - \frac{25360}{2187}k_2 + \frac{64448}{6561}k_3 - \frac{212}{729}k_4) \tag{9}$$

$$k_6 = hf(t_k + h, y_k + \frac{9017}{3168}k_1 - \frac{355}{33}k_2 - \frac{46732}{5247}k_3 + \frac{49}{176}k_4 - \frac{5103}{18656}k_5) \tag{10}$$

$$k_7 = hf(t_k + h, y_k + \frac{35}{384}k_1 + \frac{500}{1113}k_3 + \frac{125}{192}k_4 - \frac{2187}{6784}k_5 + \frac{11}{84}k_6) \tag{11}$$

Compute the next step value z_{k+1} using the Runge-Kutta method of fifth order, as follows:

$$z_{k+1} = y_k + \frac{5179}{57600}k_1 + \frac{7571}{16695}k_3 + \frac{393}{640}k_4 - \frac{92097}{339200}k_5 + \frac{187}{2100}k_6 + \frac{1}{40}k_7) \tag{12}$$

2.3 Euler Method

In this section, this study employed a different numerical solution method which is Euler method. At this stage, we formulated and converted the SIR model equations (13-15) into Euler formula. Then, the Euler formulas from [9] became:

$$S_{i+1} = S_i + h(-\beta S_i I_i), \tag{13}$$

$$I_{i+1} = I_i + h(\beta S_i I_i - \gamma I_i), \tag{14}$$

$$R_{i+1} = R_i + h(\gamma I_i). \tag{15}$$

where,

- S_i, I_i, R_i = susceptible, infected, and recovered population.
- $S_{i+1}, I_{i+1}, R_{i+1}$ = sequence of susceptible, infected, and recovered population.
- h = step size in the time interval.

3. Results and Discussion

This section discussed the solution of the SIR model of yellow fever with initial conditions and proposed solving the SIR model of yellow fever using numerical analysis methods, specifically the Dormand-Prince method and Euler method, implemented in MATLAB R2024b with the ode45 solver. This section explored different step sizes, h and their impact on these methods when solving the SIR model. It sought to determine which step size provided a better approximation of the behaviour of the results.

3.1 Simulation of SIR Model of Yellow Fever in African Region

The analysis of yellow fever in the Africa region was conducted using the SIR model simulated with the ode45 solver in MATLAB R2024b. Initial values, S=1513208983, I=112093, R=44715 were taken from [1], while β=0.3 and γ=0.1 were obtained from [7]. The model equation (2), (3) and (4) over $0 \leq t \leq 90$, predicted the disease's progression in the region.

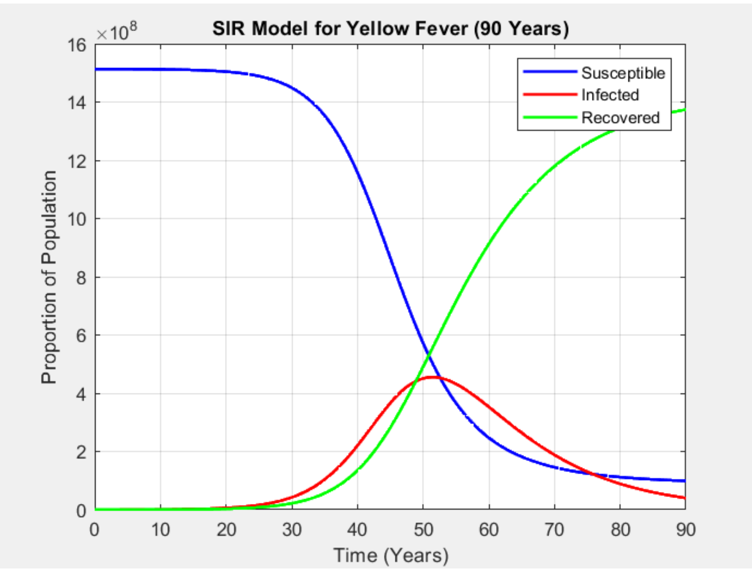


Fig. 2 *The SIR Model compartment Simulation of the SIR Model of Yellow Fever in African Region*

Fig. 2 illustrates the spread of yellow fever over 90 years using the SIR model. The blue line represents the susceptible population, $S(t)$, which starts high but decreases rapidly, especially in the first 30–40 years, showing how fast yellow fever spreads less vaccinations or mosquito control [13].

Yellow fever is a severe disease caused by the Aedes aegypti mosquito. According to [14], outbreaks are worsened by urbanization, deforestation, and climate change. Poor vaccination coverage and mosquito control further contribute to the spread of the disease.

The red line represents the infected population, $I(t)$, which peaks around 40–50 years, showing the rapid spread of the disease among unvaccinated individuals. This highlights the need for vaccinations and health measures to prevent yellow fever. The green line shows the recovered population, $R(t)$, steadily increasing due to natural recovery and more vaccination efforts. Vaccination remains the best way to prevent yellow fever. As noted by [1], mass vaccination programs play a crucial role in saving lives.

3.2 Approximations Method

Refer to the SIR model of yellow fever in equation (2), (3) and (4) with the initial conditions from [1], time interval, $0 \le t \le 90$. Let the parameters are $\beta = 0.3$ and $\gamma = 0.1$, this will give [10],

$$\frac{dS}{dt} = -\frac{(0.3)SI}{N}, \tag{16}$$

$$\frac{dI}{dt} = \frac{(0.3)SI}{N} - (0.1)I, \tag{17}$$

$$\frac{dR}{dt} = (0.1)I. \tag{18}$$

Table 1 Numerical results for susceptible of SIR model of yellow fever using Dormand-Prince Method and Euler Method with step size, h = 0.01 and h = 0.05

Time (Year)	Susceptible of SIR	Dormand Prince Method h=0.01	Dormand Prince Method h=0.05	Euler Method h=0.01	Euler Method h=0.05	Error Dormand Prince Method h=0.01	Error Dormand Prince Method h=0.05	Error Euler Method h=0.01	Error Euler Method h=0.05
1	1.5E+09	1.5E+09	1.5E+09	1.5E+09	1.5E+09	1326	1684	1657	3319
3	1.5E+09	1.5E+09	1.5E+09	1.5E+09	1.5E+09	5383	8890	8629.4	24672
6	1.5E+09	1.5E+09	1.5E+09	1.5E+09	1.5E+09	11048	26063	24949	92638
9	1.5E+09	1.5E+09	1.5E+09	1.5E+09	1.5E+09	17771	70143	66267	3E+05
12	1.5E+09	1.5E+09	1.5E+09	1.5E+09	1.5E+09	67212	2E+05	220460	9E+05
15	1.5E+09	1.5E+09	1.5E+09	1.5E+09	1.5E+09	2E+05	7E+05	692545	3E+06
18	1.5E+09	1.5E+09	1.5E+09	1.5E+09	1.5E+09	7E+05	2E+06	2E+06	8E+06
21	1.4E+09	1.4E+09	1.4E+09	1.4E+09	1.4E+09	2E+06	5E+06	5E+06	2E+07
24	1.3E+09	1.3E+09	1.3E+09	1.3E+09	1.3E+09	2E+06	1E+07	1E+07	4E+07
27	1.1E+09	1.1E+09	1.1E+09	1.1E+09	1.1E+09	5E+06	2E+07	2E+07	8E+07
30	7.8E+08	7.9E+08	8E+08	8E+08	8.7E+08	4E+06	2E+07	2E+07	9E+07
33	5.1E+08	5.1E+08	5.2E+08	5.2E+08	5.8E+08	1E+06	1E+07	1E+07	7E+07
36	3.1E+08	3.2E+08	3.2E+08	3.2E+08	3.6E+08	1E+06	8E+06	7E+06	4E+07
39	2E+08	2E+08	2E+08	2E+08	2.2E+08	3E+05	4E+06	3E+06	2E+07
42	1.3E+08	1.3E+08	1.3E+08	1.3E+08	1.4E+08	3E+05	2E+06	2E+06	1E+07
45	9.5E+07	9.6E+07	9.6E+07	9.6E+07	1E+08	3E+05	8E+05	825012	4E+06
48	7.3E+07	7.3E+07	7.4E+07	7.4E+07	7.5E+07	3E+05	5E+05	458736	1E+06
51	6E+07	6E+07	6E+07	6E+07	5.9E+07	2E+05	85887	101403	16954
54	5.1E+07	5.1E+07	5.1E+07	5.1E+07	5E+07	1E+05	31599	10429	7E+05
57	4.5E+07	4.5E+07	4.5E+07	4.5E+07	4.4E+07	1E+05	1E+05	103203	1E+06
60	4.1E+07	4.1E+07	4E+07	4E+07	3.9E+07	1E+05	1E+05	93564	1E+06

63	3.8E+07	3.8E+07	3.8E+07	3.8E+07	3.6E+07	77582	2E+05	163545	1E+06
66	3.6E+07	3.6E+07	3.5E+07	3.6E+07	3.4E+07	71065	2E+05	176154	1E+06
69	3.4E+07	3.4E+07	3.4E+07	3.4E+07	3.3E+07	82951	2E+05	166926	1E+06
72	3.3E+07	3.3E+07	3.3E+07	3.3E+07	3.2E+07	64511	2E+05	186184	1E+06
75	3.2E+07	3.2E+07	3.2E+07	3.2E+07	3.1E+07	64046	2E+05	186507	1E+06
78	3.2E+07	3.2E+07	3.1E+07	3.2E+07	3E+07	67622	2E+05	182315	1E+06
81	3.1E+07	3.1E+07	3.1E+07	3.1E+07	3E+07	61880	2E+05	187241	1E+06
84	3.1E+07	3.1E+07	3.1E+07	3.1E+07	2.9E+07	64222	2E+05	184033	1E+06
87	3.1E+07	3.1E+07	3E+07	3E+07	2.9E+07	64192	2E+05	183230	1E+06
90	3.1E+07	3.1E+07	3E+07	3E+07	2.9E+07	64582	2E+05	182322	1E+06

Table 2 Numerical results for infection of SIR model of yellow fever using Dormand-Prince Method and Euler Method with step size, h = 0.01 and h = 0.05

Time (Year)	Infection of SIR	Dormand Prince Method h=0.01	Dormand Prince Method h=0.05	Euler Method h=0.01	Euler Method h=0.05	Error Dormand Prince Method h=0.01	Error Dormand Prince Method h=0.05	Error Euler Method h=0.01	Error Euler Method h=0.05
1	152229	151235	150966	150986	149739	994.034	1262.66	1242.62	2489.31
3	375414	371378	368748	368944	356913	4036.07	6666.42	6470.75	18501.2
6	919913	911630	900372	901208	850456	8282.27	19540.3	18705	69456.5
9	2249059	2235740	2196491	2199395	2024951	13319.5	52568.6	49663.9	224108
12	5521002	5470723	5346850	5355992	4812828	50279	174152	165010	708173
15	1.3E+07	1.3E+07	1.3E+07	1.3E+07	1.1E+07	177922	543487	516583	2100578
18	3.2E+07	3.2E+07	3.1E+07	3.1E+07	2.7E+07	500563	1511241	1437086	5782158
21	7.6E+07	7.4E+07	7.2E+07	7.2E+07	6.1E+07	1126738	3671055	3484977	1.4E+07
24	1.6E+08	1.6E+08	1.6E+08	1.6E+08	1.3E+08	1694877	7098601	6704781	3.1E+07
27	3.1E+08	3.1E+08	3E+08	3E+08	2.6E+08	2981161	1.1E+07	1.1E+07	5E+07
30	4.8E+08	4.8E+08	4.7E+08	4.7E+08	4.3E+08	1326371	8872252	8333800	4.8E+07
33	5.9E+08	5.9E+08	5.9E+08	5.9E+08	5.7E+08	1839579	469334	323051	1.7E+07
36	6E+08	6.1E+08	6.1E+08	6.1E+08	6.2E+08	1252084	3865449	3651130	1.3E+07
39	5.5E+08	5.5E+08	5.5E+08	5.5E+08	5.7E+08	1431351	6186495	5823272	2.8E+07
42	4.6E+08	4.6E+08	4.7E+08	4.7E+08	4.9E+08	923845	5822334	5457863	3E+07
45	3.7E+08	3.7E+08	3.8E+08	3.8E+08	4E+08	567345	4776024	4469012	2.6E+07
48	3E+08	3E+08	3E+08	3E+08	3.2E+08	537324	3868034	3629506	2.1E+07
51	2.3E+08	2.3E+08	2.3E+08	2.3E+08	2.5E+08	95112.7	2618490	2441153	1.6E+07
54	1.8E+08	1.8E+08	1.8E+08	1.8E+08	1.9E+08	17833.7	1880117	1751870	1.1E+07
57	1.4E+08	1.4E+08	1.4E+08	1.4E+08	1.5E+08	10685.3	1339582	1248677	8318907
60	1.1E+08	1.1E+08	1.1E+08	1.1E+08	1.1E+08	178992	1144938	1081576	6167311
63	8.1E+07	8.1E+07	8.1E+07	8.1E+07	8.5E+07	117035	566116	522655	4141973
66	6.1E+07	6.1E+07	6.2E+07	6.2E+07	6.4E+07	47567.8	430400	401094	2952125
69	4.7E+07	4.7E+07	4.7E+07	4.7E+07	4.9E+07	77597.1	408321	388952	2170055
72	3.6E+07	3.6E+07	3.6E+07	3.6E+07	3.7E+07	60705.6	165353	152871	1384020
75	2.7E+07	2.7E+07	2.7E+07	2.7E+07	2.8E+07	32578	119747	111973	953470

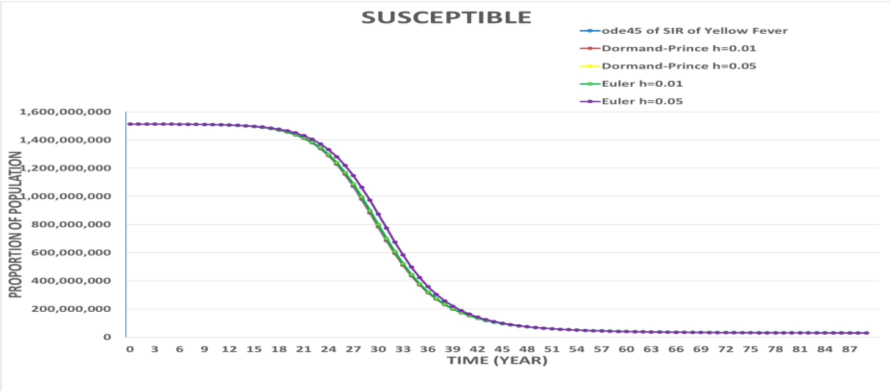
78	2.1E+07	2.1E+07	2.1E+07	2.1E+07	2.1E+07	16046.3	116901	112297	679908
81	1.6E+07	1.6E+07	1.6E+07	1.6E+07	1.6E+07	32567.3	32710.4	30203.2	406896
84	1.2E+07	1.2E+07	1.2E+07	1.2E+07	1.2E+07	2659.12	43622.1	42468.5	287291
87	8989244	8997619	9022186	9021878	9176572	8374.22	32941	32633.7	187327
90	7470359	7486346	7503168	7503227	7614721	15987.4	32808.7	32867.8	144362

Table 3 Numerical results for recovery of SIR model of yellow fever using Dormand-Prince Method and Euler Method with step size, h = 0.01 and h = 0.05

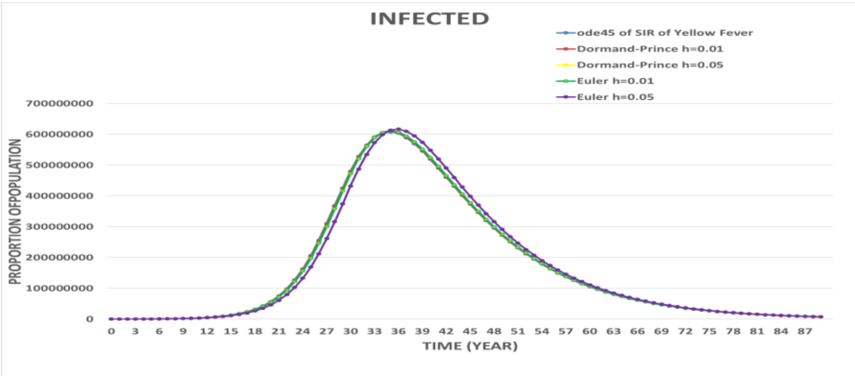
Time (Year)	Recovery of SIR	Dormand Prince Method h=0.01	Dormand Prince Method h=0.05	Euler Method h=0.01	Euler Method h=0.05	Error Dormand Prince Method h=0.01	Error Dormand Prince Method h=0.05	Error Euler Method h=0.01	Error Euler Method h=0.05
1	58095.8	57764.3	57674.7	57681.4	57265.7	331.472	421.041	414.36	830.036
3	132515	131168	130291	130356	126344	1346.61	2223.92	2158.65	6170.99
6	314155	311389	307632	307911	290973	2765.46	6523.09	6244.24	23181.6
9	758030	753578	740456	741427	683119	4451.84	17574.7	16603.2	74911.4
12	1853746	1836813	1795225	1798296	1616067	16932.8	58521.5	55450.5	237679
15	4541306	4480176	4356212	4365344	3829194	61130	185093	175962	712112
18	1.1E+07	1.1E+07	1.1E+07	1.1E+07	9043325	177897	528923	503124	2005126
21	2.6E+07	2.6E+07	2.5E+07	2.5E+07	2.1E+07	430472	1365750	1297136	5277592
24	6.1E+07	6E+07	5.8E+07	5.8E+07	4.8E+07	760329	3023788	2857861	1.3E+07
27	1.3E+08	1.3E+08	1.2E+08	1.2E+08	1E+08	2076637	6729693	6388347	2.7E+07
30	2.5E+08	2.5E+08	2.4E+08	2.4E+08	2.1E+08	2387491	9907712	9355442	4.4E+07
33	4.1E+08	4.1E+08	4E+08	4E+08	3.6E+08	2957585	1.2E+07	1.2E+07	5.6E+07
36	5.9E+08	5.9E+08	5.8E+08	5.8E+08	5.4E+08	2360328	1.2E+07	1.1E+07	5.7E+07
39	7.7E+08	7.7E+08	7.6E+08	7.6E+08	7.2E+08	1774832	9722290	9149500	5E+07
42	9.2E+08	9.2E+08	9.1E+08	9.1E+08	8.8E+08	1192883	7487778	7040627	4E+07
45	1E+09	1E+09	1E+09	1E+09	1E+09	883375	5624737	5294024	3E+07
48	1.1E+09	1.1E+09	1.1E+09	1.1E+09	1.1E+09	871559	4323577	4088242	2.2E+07
51	1.2E+09	1.2E+09	1.2E+09	1.2E+09	1.2E+09	259311	2704377	2542556	1.5E+07
54	1.3E+09	1.3E+09	1.3E+09	1.3E+09	1.3E+09	163783	1848519	1741441	1.1E+07
57	1.3E+09	1.3E+09	1.3E+09	1.3E+09	1.3E+09	90208.2	1212670	1145475	7257425
60	1.4E+09	1.4E+09	1.4E+09	1.4E+09	1.4E+09	314064	1026611	988012	4967181
63	1.4E+09	1.4E+09	1.4E+09	1.4E+09	1.4E+09	39453.8	377466	359109	2794418
66	1.4E+09	1.4E+09	1.4E+09	1.4E+09	1.4E+09	23497.5	229137	224940	1552297
69	1.4E+09	1.4E+09	1.4E+09	1.4E+09	1.4E+09	160548	216434	222026	760290
72	1.4E+09	1.4E+09	1.4E+09	1.4E+09	1.4E+09	3805.53	45585.5	33313.8	52982.1
75	1.5E+09	1.5E+09	1.5E+09	1.5E+09	1.5E+09	31467.8	91295.7	74533.5	485782
78	1.5E+09	1.5E+09	1.5E+09	1.5E+09	1.5E+09	83667.8	89743.5	70017.7	753915
81	1.5E+09	1.5E+09	1.5E+09	1.5E+09	1.5E+09	29312.5	178673	157037	1029084
84	1.5E+09	1.5E+09	1.5E+09	1.5E+09	1.5E+09	66881.3	164391	141565	1142103
87	1.5E+09	1.5E+09	1.5E+09	1.5E+09	1.5E+09	72565.9	174129	150596	1237790
90	1.5E+09	1.5E+09	1.5E+09	1.5E+09	1.5E+09	80569.4	173273	149455	1277619

The table presents the susceptible, infected, and recovered populations over time using the Dormand-Prince method and Euler method with step sizes h=0.01 and h=0.05, compared to ode45 (MATLAB R2024b). The Dormand-Prince method at h=0.01 consistently shows the lowest errors, while the Euler method at h=0.05 has the highest, reaching over 100 million for susceptibility, 50 million for infections, and 1.2 million for recoveries. The results highlight that smaller step sizes and advanced numerical methods provide better approximation in disease modelling.

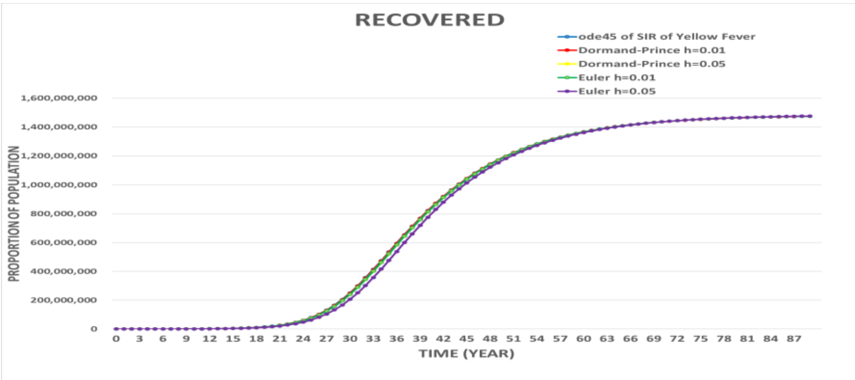
Fig. 3 presented results from solving the SIR model of Yellow Fever with ode45 (MATLAB R2024b) using Dormand-Prince method and Euler method. Calculations were performed in MATLAB R2024b, testing step sizes h = 0.01 and h = 0.05 to evaluate the behaviour of each SIR variable over time. Comparisons were made to determine a better approximation, with a focus on the performance of the smaller step size, h = 0.01.



(a)



(b)



(c)

Fig. 3 Graph depicted the numerical results of SIR model of Yellow Fever using Dormand-Prince Method and Euler Method with step size, h=0.01 and h=0.05 which (a) is susceptible, (b) is infected, (c) is recovered

For all three graphs showing the Susceptible (S), Infected (I), and Recovered (R) populations, the errors were calculated by comparing the results of the Dormand-Prince and Euler methods with the ode45 method.

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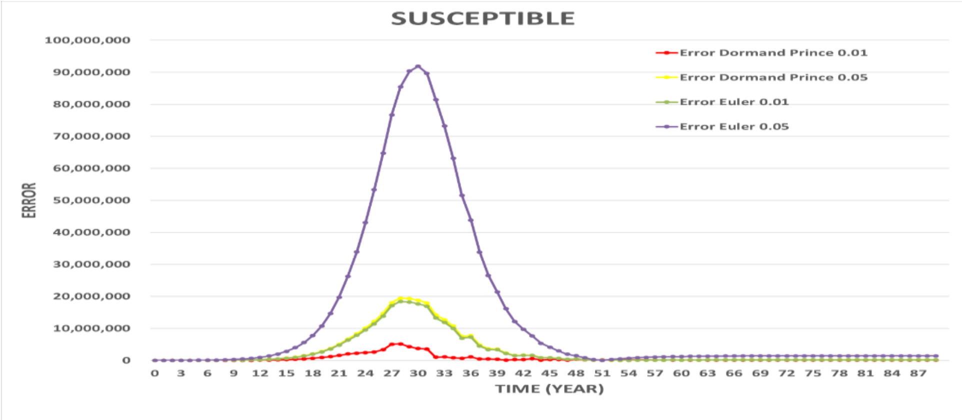
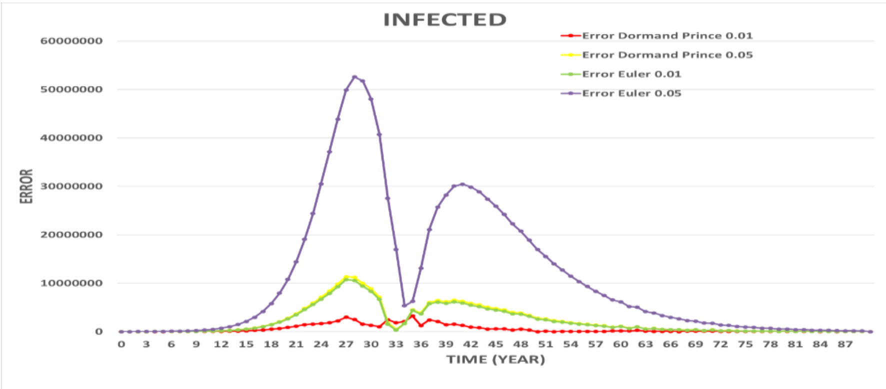
The Dormand-Prince method with a smaller step size which is h=0.01 consistently provides the lowest errors across all SIR model populations. In contrast, the Euler method with h=0.05 shows the largest errors. This highlights the Dormand-Prince method's better approximation and the importance of using smaller step sizes for improved results in solving the SIR model

3.3 Error Analysis

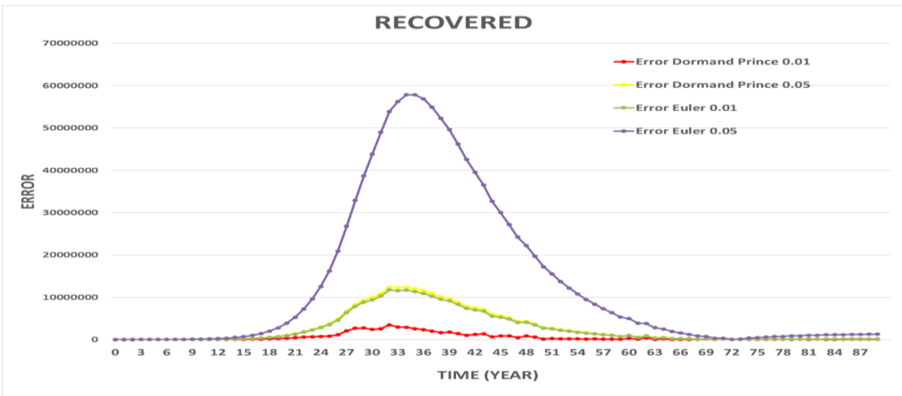
Fig. 4 visually displayed error analysis results of the SIR model of yellow fever with ode45 (MATLAB R2024b) solved via approximate methods, Dormand-Prince method and Euler method. It assessed computational disparities and the impact of step sizes, h=0.01 and h=0.01 on models through graphical analysis.



(a)



(b)



(c)

Fig. 4 Graphs depicting the errors for (a) is susceptible, (b) is infectious and (c) is recovery compartment for the SIR model of yellow fever with different step sizes

For all three graphs showing the error analysis for the Susceptible (S), Infected (I), and Recovered (R) populations in the SIR model. The errors were compared between the Dormand-Prince and Euler methods using the ode45 method as a reference. The Dormand-Prince method with a smaller step size, h = 0.01 had the smallest errors, showing it gives a better approximation. When the step size increased to h = 0.05, the errors for Dormand-Prince increased slightly but were still smaller than Euler's errors. The Euler method had the biggest errors, especially with h=0.05. This shows that the Dormand-Prince method is better, and smaller step sizes give better approximations for the SIR model.

4. Discussion

The study compared the Dormand-Prince method and Euler method with ode45 (MATLAB R2024b) in solving the SIR model of yellow fever using different step sizes, h=0.01 and h=0.05. Fig. 3 illustrated the comparison of numerical results with ode45 (MATLAB R2024b) and Fig. 4 showed the error graphs of the SIR model of yellow fever by ode45 (MATLAB R2024b) using the Dormand-Prince method and Euler method with step sizes of h=0.01 and h=0.05. The results indicated that using a smaller step size, h=0.01, with the Dormand-Prince method gave a good approximation to the behaviour compared to the Euler method. The Dormand-Prince method produced the smallest error, which is 312.55, while the Euler method with h=0.05 had the largest error, which is 90,418,534.98. The Dormand-Prince method gives a good approximation to the behaviour compared to the Euler method, as reflected in slightly smaller errors and graphs closely resembling ode45 (MATLAB R2024b) solutions. The Dormand-Prince method consistently demonstrated smaller errors, emphasizing its greater performance in approximating susceptible, infectious, and recovery proportions. Specifically, with a smaller step size, h=0.01, the results obtained using the Dormand-Prince method closely matched those produced by ode45 (MATLAB R2024b) solutions in Fig. 3, highlighting its superior ability to reflect system dynamics precisely. In contrast, the Euler method, particularly with a larger step size, h=0.05, showed a slower approximation to the behaviour and exhibited greater deviation from the values obtained with ode45 in MATLAB R2024b over time. This comparison emphasizes the importance of choosing an appropriate step size for achieving good approximations to the behaviour of each method in simulating the outcomes.

5. Conclusion

The Dormand-Prince method performed better than the Euler method in solving the SIR model of yellow fever and demonstrated closer alignment with ode45 (MATLAB R2024b) solutions. Smaller steps size, such as h = 0.01 significantly improved the approximation, particularly with the Dormand-Prince method. Choosing the right method and step size is crucial for predictions in yellow fever modelling. The Dormand-Prince method proves highly valuable for handling complex models. The results showed that the smaller the step size used, the better result obtained.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Muhammad Irham Hisham, Norzuria Ibrahim;* ***data collection:*** *Muhammad Irham Hisham;* ***analysis and interpretation of results:*** *Muhammad Irham Hisham, Norzuria Ibrahim;* ***draft manuscript preparation:*** *Muhammad Irham Hisham, Norzuria Ibrahim. All authors reviewed the results and approved the final version of the manuscript.*

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