

Linear Optimization Modelling for Profit Maximization in Bakery Store

Nur Syafinaz Abdul Aziz¹, Kek Sie Long^{1*}

¹ Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology,
Universiti Tun Hussein Onn Malaysia (Pagoh Campus),
84600 Pagoh, Muar, Johor, MALAYSIA

*Corresponding Author: slkek@uthm.edu.my

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Abstract

A quantitative decision is required in operating a bakery because of perishable baked food. This paper discusses linear programming in a local bakery with four bread types: sugar, blueberry, cheese, and red bean. The operating information and resource data are collected from the bakery, where selling prices, cost expenses, raw materials, rental, and workers' salaries are analysed. We define the number of types of bread as the decision variable in a batch quantity. An objective function that includes selling volume and cost expenses is introduced to maximize the bakery's profit. Also, 22 constraints from raw materials are considered in running the bakery. Hence, the linear programming model for the bakery is formulated. To solve this linear programming problem, we employ the Microsoft Excel solver to obtain the optimal solution and suggest the bakery produces one batch of sugar, blueberry and red bean bread and three batches of cheese bread. This decision maximizes a total daily profit of RM142.92, where all constraints are satisfied. In addition, a sensitivity analysis is carried out to identify a unit change of the decision variables and resource availability to the profit. In conclusion, the applicability of linear programming to the bakery is well presented.

1. Introduction

A bakery is a business that produces and sells baked goods such as bread, pastries, cakes, and cookies [1]. Efficient management and planning in a bakery are crucial due to perishable baked foods and the need to satisfy customer demands. The baking industry has a significant economic impact in various regions, such as the US, Europe, and Asia Pacific. In the US, the baking industry accounts for about \$480.47 billion in total economic output or roughly 2.46 per cent of the GDP [2]. In Europe, it is reported that 59 kilograms of bread is consumed per year [3]. In Malaysia, the bakery industry is highly dynamic, with numerous suppliers [4]. Applying scientific methods, including linear programming, can optimize various aspects of bakery management, from resource utilization to profit maximization. Bakery management can be resolved optimally by formulating mathematical models, decision variables, constraints, and the objective function. Mathematical optimization models, such as linear programming, can help bakery management address these challenges by optimizing resource allocation, ingredient usage, staffing requirements, and operating hours. The bakery's management and decision-making process can be optimized effectively using linear programming models.

Linear programming (LP) involves optimizing a linear function within a set of linear constraints. The simplex method, developed by George B. Dantzig in 1947, is a popular approach for solving LP problems [5]. It efficiently determines the optimal allocation of resources across projects or tasks while adhering to constraints [6]. The

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graphical method is another common technique in LP, allowing decision-makers to visually identify the optimal solution by plotting constraints and objective functions on a graph. These techniques help decision-makers make informed choices in resource allocation and optimization [7].

One study conducted by [8] focused on optimizing profit at Bakers Cottage by efficiently utilizing raw materials. Another case study by [9] examined the optimization of raw materials at Goretta Bakery Limited for three different bread sizes. A study at the University of Benin Bakery also utilized linear programming to determine bread's optimal daily production quantity, considering constraints and resources [10]. These studies applied linear programming techniques to maximize profit and efficiency while considering production constraints.

This study uses real-world data to focus on mathematical modelling for a local bakery model. The main objective of this study is to determine the optimal quantity of baking products in the bakery store where raw materials are fully utilized. The second objective is to maximize the profit of a bakery store by optimizing the allocation of baked products using the application of linear programming. The third objective of this study is to analyze the sensitivity analysis of the bakery model, measuring how changes in the profit per batch of each baked product may influence the optimal production strategy.

2. Methodology of Linear Programming

In this section, we construct a linear programming model to maximize the bakery's profit.

2.1 Decision Variables

The bakery has four bread types: sugar bread, cheese bread, blueberry bread and red bean bread. Define the decision variables as follows,

- the daily number of batches of sugar bread is denoted by x_1 ,
- the daily number of batches of cheese bread is denoted by x_2 ,
- the daily number of batches of blueberry bread is denoted by x_3 ,
- the daily number of batches of red bean bread is denoted x_4 ,

where a batch contains 14 pieces of each type of bread, these decision variables have values greater than or equal to zero.

2.2 Objective Function

Consider the daily profit of the bakery,

$$\text{Profit} = \text{Total Revenue} - \text{Total Expenses.} \quad (1)$$

Table 1 shows the selling prices and revenue per batch for these four types of breads. The total revenue for each type of bread is calculated as follows,

$$\text{Total Revenue (RM)} = 77x_1 + 91x_2 + 91x_3 + 70x_4. \quad (2)$$

Table 1 Selling price and revenue per batch for four types of breads

Types of Bread	Sugar Bread	Cheese Bread	Blueberry Bread	Red Bean Bread
Pieces per batch	14	14	14	14
Selling price per piece (RM)	5.50	6.50	6.50	5.00
Revenue per batch (RM)	77.00	91.00	91.00	70.00

Table 2 provides the cost price per batch for each type of bread. Additional costs need to be considered for operating the bakery. On a monthly basis, these costs include a rent of RM200, water charges of RM500, electricity charges of RM500, and labour costs of RM1,300. These costs correspond to the bakery operating for 26 days per month. So, the total daily cost expenses are calculated as follows,

$$\text{Total Daily Expenses (RM)} = 34.55x_1 + 40.41x_2 + 75.55x_3 + 40.60x_4 + 96.15. \quad (3)$$

Thus, from (1), the profit of the bakery is simplified as follows,

$$\text{Profit} = 42.45x_1 + 50.59x_2 + 15.45x_3 + 29.40x_4 - 96.15 \quad (4)$$

Table 2 Cost price per batch of breads

Bread Name	Cost Price (RM)
Sugar Bread	34.55
Cheese Bread	40.41
Blueberry Bread	75.55
Red Bean Bread	40.60

2.3 Constraints

There are 22 constraints in this linear programming modelling. Their related items and the equations are formed as below,

- Flour: $4x_1 + 4x_2 + 4x_3 + 4x_4 \leq 50.$ (5)
- Baking time: $20x_1 + 20x_2 + 20x_3 + 20x_4 \leq 120.$ (6)
- Water: $2x_1 + 1x_2 + 2x_4 \geq 1.$ (7)
- Yeast: $0.08x_1 + 0.05x_2 + 0.12x_3 + 0.08x_4 \geq 0.05.$ (8)
- Sugar: $0.8x_1 + 0.26x_2 + 0.52x_3 + 0.65x_4 \geq 0.25.$ (9)
- Condensed milk: $0.13x_2 \geq 0.10.$ (10)
- Butter: $0.65x_1 + 0.13x_2 + 0.38x_3 + 0.65x_4 \geq 0.13.$ (11)
- Powder milk: $0.42x_1 + 0.15x_4 \geq 0.10.$ (12)
- Full cream milk: $2.26x_3 \geq 1.00.$ (13)
- Bread improver: $0.04x_1 + 0.04x_2 + 0.04x_3 + 0.04x_4 \geq 0.04.$ (14)
- Bread softener: $0.04x_1 + 0.04x_2 + 0.04x_3 + 0.04x_4 \geq 0.04.$ (15)
- Blueberry: $1.82x_3 \geq 1.00.$ (16)
- Cheese: $0.43x_2 \geq 0.30.$ (17)
- Red bean paste: $0.45x_4 \geq 0.40.$ (18)
- Egg: $12x_1 + 5x_2 + 5x_3 + 20x_4 \geq 5.$ (19)
- Proofing time: $105x_1 + 105x_2 + 105x_3 + 105x_4 \geq 105.$ (20)
- Total batch of bread produced: $x_1 + x_2 + x_3 + x_4 \leq 6.$ (21)
- Batch of sugar bread produced: $1 \leq x_1 \leq 3.$ (22)
- Batch of sugar bread produced: $1 \leq x_2 \leq 3.$ (23)
- Batch of sugar bread produced: $1 \leq x_3 \leq 3.$ (24)
- Batch of sugar bread produced: $1 \leq x_4 \leq 3.$ (25)
- Non-negativity: $x_1, x_2, x_3, x_4 \geq 0.$ (26)

3. Results and Discussion

The Microsoft Excel solver is used to solve the linear programming problem of the bakery. The sensitivity analysis is also performed for the bakery.

3.1 Optimal Result

Figure 1 shows the optimal solution to the problem in the spreadsheet. While Table 3 shows the optimal quantities for each bread.

	x1	x2	x3	x4			
Decision Variable	1.00	3.00	1.00	1.00			
Profit (P)	42.45	50.59	15.45	29.40	96.15	142.92	
Constraints							
Flour (kg)	4.00	4.00	4.00	4.00	24.00	<=	50.00
Water (l)	2.00	1.00	0.00	2.00	7.00	>=	1.00
Yeast (kg)	0.08	0.05	0.12	0.08	0.43	>=	0.05
Sugar (kg)	0.80	0.26	0.52	0.65	2.75	>=	0.26
Condensed milk (l)	0	0.13	0	0	0.39	>=	0.13
Butter (kg)	0.65	0.13	0.38	0.65	2.07	>=	0.13
Powder milk (kg)	0.42	0	0	0.15	0.57	>=	0.15
Full cream milk (l)	0	0	2.26	0	2.26	>=	2.26
Bread Improver (kg)	0.04	0.04	0.04	0.04	0.24	>=	0.04
Bread Softener (kg)	0.04	0.04	0.04	0.04	0.24	>=	0.04
Blueberry (kg)	0	0	1.82	0	1.82	>=	1.82
Cheese (kg)	0	0.43	0	0	1.29	>=	0.43
Red bean paste (kg)	0	0	0	0.45	0.45	>=	0.45
Egg (p)	12	5	5	20	52.00	>=	5.00
Proofing time (min)	105	105	105	105	630.00	>=	105.00
Baking time (min)	20	20	20	20	120.00	<=	120.00
Total batches	1	1	1	1	6	<=	6
Sugar Bread	1	0	0	0	1	>=	1
Cheese Bread	0	1	0	0	3	>=	1
Blueberry Bread	0	0	1	0	1	>=	1
Red Bean Bread	0	0	0	1	1	>=	1
Sugar Bread	1	0	0	0	1	<=	3
Cheese Bread	0	1	0	0	3	<=	3
Blueberry Bread	0	0	1	0	1	<=	3
Red Bean Bread	0	0	0	1	1	<=	3

Fig. 1 Optimal solution of the problem in spreadsheet

Table 3 Optimal result

Types of Bread	Optimal Production	Optimum Profit (RM)	Total (RM)
Sugar Bread	1	42.45	42.42
Cheese Bread	3	50.59	151.77
Blueberry Bread	1	15.45	15.45
Red Bean Bread	1	29.40	29.40

The optimal solution suggests that producing one batch of sugar bread, blueberry bread, red bean bread, and three batches of cheese bread yields the maximum profit after considering the total utility costs of RM96.15. Therefore, this gives a maximum profit of RM142.92.

3.2 Sensitivity Analysis

The sensitivity analysis helps in identifying key variables and their impacts on the optimal solution. Table 4 shows sensitivity analysis report for the decision variables. The final values represent the optimal solutions for each decision variable. Since all decision variables have final values in this report, they are included in the optimal solution, and the reduced cost is 0. The objective coefficient indicates the coefficient of the decision variable in the objective function. The allowable increase represents the maximum increase in the coefficient without affecting the optimal solution. In contrast, the allowable decrease represents the maximum decrease before causing a change in the optimal solution.

Table 4 Sensitivity analysis report for decision variables

Bread Type	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
Number of Batches for Sugar Bread	1	0	42.45	8.14	13.05
Number of Batches for Cheese Bread	3	8.14	50.59	Infinity	8.14
Number of Batches for Blueberry Bread	1	-27	15.45	27	- Infinity
Number of Batches for Red Bean Bread	1	-13.05	29.4	13.05	- Infinity

Optimal profitability is achieved by producing one batch each of Sugar Bread and Blueberry Bread and one batch of Red Bean Bread, yielding profits of RM42.45, RM15.45, and RM29.40 units, respectively. An increase of up to 8.14 units or a decrease of 13.05 units in Sugar Bread's profit per batch will not change the optimal solution. The profit from Blueberry Bread could increase by up to 27 units or decrease infinitely without affecting the optimal solution. Similarly, the profit increase of 13.05 units or an infinite decrease for Red Bean Bread does not change the optimal solution. For Cheese Bread, baking three batches is optimal for a maximum profit of RM50.59 units per batch. Any increase in profit or a decrease up to 8.14 units per batch does not affect the optimal solution. These specific numbers help evaluate potential shifts in production quantities. The optimal solution suggests particular quantities of each bread type to maximize profit.

Table 5 Sensitivity analysis report for constraints

Constraints	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Flour	24	0	50	Infinity	26
Baking Time	120	2.1225	120	40	0
Water	7	0	1	6	Infinity
Yeast	0.43	0	0.05	0.38	Infinity
Sugar	2.75	0	0.25	2.5	Infinity
Condensed milk	0.39	0	0.1	0.29	Infinity
Butter	2.07	0	0.13	1.94	Infinity
Powder milk	0.57	0	0.1	0.47	Infinity
Full cream milk	2.26	0	1	1.26	Infinity
Bread Improver	0.6	0	0.04	0.56	Infinity
Bread Softener	0.6	0	0.04	0.56	Infinity
Blueberry	1.82	0	1	0.82	Infinity
Cheese	1.29	0	0.3	0.99	Infinity

Red bean paste	0.45	0	0.4	0.05	Infinity
Egg	52	0	5	47	Infinity
Proofing time	630	0	105	525	Infinity

Table 5 shows the sensitivity report for the constraints. Each row represents a different ingredient or condition; the columns reveal specific data about that element. The final value represents the utilized amount in the optimal solution. The shadow price (also known as the dual value or marginal value) is the change in the objective value (in this case, profit) resulting from a one-unit increase in the resource (given by the Constraint Right Hand Side). Constraint Right Hand Side or Constraint R.H. Side is the original value of the resource constraint as supplied in the linear programming model. "Allowable Increase" and "Allowable Decrease" signify the maximum that the ingredient or time can change without affecting the optimal solution.

The quantity of Flour can be increased without limit but decreased only to a minimum of 26 kg. At the same time, baking time can be expanded up to 40 minutes without affecting the optimal solution, but it cannot be decreased. The volume of Water can be increased by 6 litres or infinitely reduced without impacting the solution. Similar permissible limits exist for Yeast, Sugar, Condensed Milk, Butter, Powder Milk, Full Cream Milk, Bread Softener, Bread Improver, Blueberry, Cheese, and Red Bean paste whose quantities can be adjusted in grams. The number of Eggs can be increased by a maximum of 47 or decreased infinitely without changing the optimal solution. Lastly, the Proofing Time can be increased to 525 minutes or reduced to any extent without impacting the optimal solution.

By understanding the findings from the sensitivity analysis, the third objective of the study, which is to conduct a sensitivity analysis on the bakery model, is achieved. This critical insight allows for optimizing ingredient utilization. It influences decisions about modifying ingredient usage or bakery operations for the best strategy, contributing to the overall effectiveness and profitability of the bakery's operation.

4. Conclusion

The use of linear programming in a local bakery was discussed in this paper. The decision variables were defined based on the information and data collected from the bakery, and the objective function, maximizing the bakery's profit, was introduced. In addition, there were 22 constraints considered in running the bakery operation. Using the Microsoft Excel solver, the linear programming problem of the bakery was solved, and the optimal quantities of the bread types contributed to giving the bakery a daily maximum profit. The Microsoft Excel solver was used to solve the bakery's linear programming problem, and the optimal quantities decision of one batch of sugar, blueberry and red bean bread and three batches of cheese bread were suggested. This decision will maximize the bakery's daily profit of RM142.92. Furthermore, a sensitivity analysis was conducted for decision variables and constraints available. Therefore, the results provide a valuable strategy for the bakery in operating the business and increasing profits. In conclusion, the use of linear programming in the bakery is demonstrated. For future research, it is recommended to incorporate data from various bakeries to deliver a more comprehensive understanding of how linear programming can optimize bakery operations. A more comprehensive range of baked products could be explored, and the impact of different constraints should be analysed.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: **study conception and design, data collection:** Nur Syafinaz Abdul Aziz; **analysis and interpretation of results, draft manuscript preparation:** Nur Syafinaz Abdul Aziz and Kek Sie Long. All authors reviewed the results and approved the final version of the manuscript.

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