ENHANCED KNOWLEDGE IN SCIENCES AND TECHNOLOGY
e-ISSN: 2773-6385

EKST

Vol. 4 No. 2 (2024) 155-160
https://publisher.uthm.edu.my/periodicals/index.php/ekst

MHD Flow of a Casson Fluid Over an Exponentially Shrinking Sheet

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DOI: https://doi.org/10.30880/ekst.2024.04.02.019

Article Info

Received: 27 December 2023
Accepted: 11 January 2024
Available online: 12 December 2024

Keywords

MHD, Casson Fluid, Exponentially Shrinking Sheet, Bvp4c

Abstract

In this study, the MHD flow of a Casson fluid over an exponentially shrinking sheet is investigated. The governing partial differential equations were transformed to a system of nonlinear ordinary differential equations using a similarity transformation. Next, the governing equation was solved using different method, shooting method using bvp4c technique in Matlab to obtain the results with various parameters such as Casson fluid parameter, γ, shrinking parameter, s and Hartmann number, M. The profiles act as a result are analysed and discussed. It is found that an increase in the Casson fluid parameter γ reduces the velocity profile because the flow stress decreases as γ increases. With increasing magnetic interaction parameter M and suction parameter s, the magnitudes of velocity decrease significantly.

1. Introduction

Magnetohydrodynamics (MHD) is the study of how fluids that conduct electricity act when a magnetic field is around. The way fluids move, magnetic fields, and electric currents all work together to create a variety of fascinating phenomena. MHD flows are used in astrophysics, geophysics, plasma physics, and engineering systems like MHD power generators and MHD pumps [1].

One of the studies of MHD flow was from [2] where they studied how thermal radiation affects the movement of an electrically conductive fluid when it flows past a plate without moving. [3] obtained an analytical solution to the famous Falkner-Skan equation for the MHD flow. The Falken-Skan equation has been used to study how magnetic fields affect the boundary layer in MHD flow. Furthermore, [4] carried out a separate study on the magnetic flow of liquids over a surface that undergoes curved motions. in closed-form solutions with hyperbolic solutions. By employing diverse measurements and calculation approaches, while considering various controllable factors, the individual impacts of each factor on the final result has been analysed. [5] researched MHD flow solution under the time-varied external magnetic field which the MHD flow equations are generated for cases where the applied magnetic field varies overtime. The effects of a time-varying applied magnetic field on the solution are investigated using several function definitions.

Casson fluid is a type of non-Newtonian fluid with shear-thinning behaviour. As stated by [6] in their study, unlike Newtonian fluids, Casson fluids do not have a linear relationship between shear stress and shear rate. It is said to have extremely high thickness when there is no shearing force, and a minimum force is needed to start the flow. This fluid becomes very thin when there is a lot of shearing force applied to it. This means that if the force applied to an object is not strong enough to cause it to break, the object will remain solid. However, when the force on a Casson fluid becomes stronger than its resistance to flowing, the fluid will start to flow. [7] discussed the steady boundary layer stagnation-point flow of Casson fluid and heat transfer towards a shrinking/stretching sheet. [8] studied Casson fluid flow over an unsteady stretching surface. The investigation focused on the characteristics of Casson fluid, a non-Newtonian liquid, as it passed over a surface subjected to stretching and maintained a constant temperature. A study was conducted by [9] to investigate the behaviour of Casson fluid when it flows past a surface that either increases (stretching) or decreases (shrinking) in size. Moreover, they explored how this behaviour is affected by the presence of a magnetic field. [10] performed numerical simulations of mixed Casson fluid convection with a heated bottom wavy wall in a trapezoidal enclosure where the impact of non-dimensional parameters such as the Richardson number, Casson fluid parameters, and the number of oscillations on flow and thermal fields, as well as the heat transfer rate, has been investigated.

Understanding how fluids move over a shrinking surface is important in different areas of engineering and industry. An exponentially shrinking sheet is defined as a sheet that continually reduces in size the more distant you are from a particular point [11]. This type of sheet geometry is frequently encountered in heat transfer, boundary layer flow, and nanofluid dynamics research. [12] considered a study of the steady MHD stagnation flow due to a shrinking sheet. They researched how suction, magnetism, and velocity ratio affect the flow and heat transfer. Unsteady magnetohydrodynamic stagnation point flow of a nanofluid past a permeable shrinking sheet was studied by [13]. The effects of magnetic fields and thermal radiation are investigated. With the exception of radiation, all relevant parameters have dual solutions in a larger domain.

This study employs the shooting approach to address MHD flow of a Casson fluid over an exponentially shrinking sheet, aiming to comprehend the mathematical formulation. The solution is sought using the bvp4c in Matlab. Governing equations, derived from conservation laws and complemented by proper boundary conditions, are transformed into ordinary differential equations via similarity transformation. Adopting the boundary layer assumptions by [14], the study focuses on the graphical analysis of the effects of MHD flow of a Casson fluid over an exponentially shrinking sheet.

Inspired by previous literature, this study specifically investigates the graphical solution of the magnitude of the velocity profile with various parameters. The research aimed to address two crucial questions of the effects of the parameters;

- Does the magnitude of velocity profile will decrease as the Casson fluid parameter increase?
- Do the magnitudes of the velocity profiles decrease significantly as the magnetic interaction parameter M and suction parameters, s, increase?

2. Mathematical Formulation

The investigation centered around understanding the dynamics of an incompressible liquid known as a Casson fluid as it undergoes flow on a progressively shrinking surface. As illustrated in Figure 1, we assume that $U_w(x)$ denotes the velocity of the shrinking sheet and, $V_w(x)$ is the variable wall mass transfer velocity.

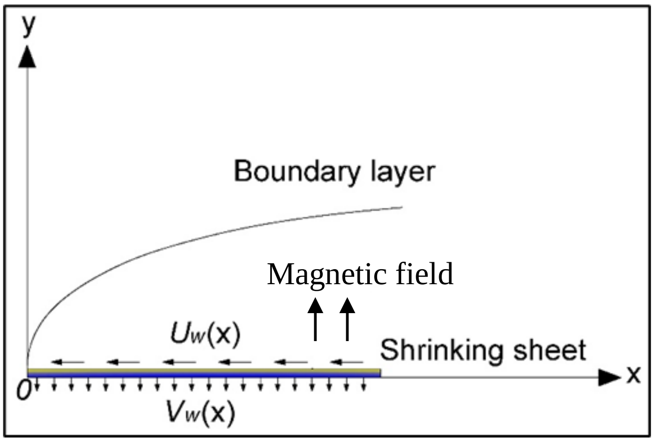

Fig. 1 The physical model

Following Nadeem et al [14] boundary layer assumptions, the governing equations are;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\beta^2}{\rho}u, \tag{2}$$

subject to the boundary conditions that can be expressed as:

$$
\begin{aligned}
&u = U_w(x) = -U_0 e^{\left(\frac{x}{l}\right)}, &&\text{at} \quad y = 0,\\
&\nu = V_w(x) = V_0 e^{\left(\frac{x}{2l}\right)}, &&\text{at,} \quad y = 0,\\
&u = 0 &&\text{as} \quad y \to \infty,
\end{aligned} \tag{3}
$$

where u and ν are the corresponding velocities in x and y-directions, respectively. μ is the Casson viscosity coefficient, v is the kinematic fluid viscosity, and ρ is the fluid density. The Casson fluid parameter is γ. σ is the electrical conductivity of the Casson fluid. l is the characteristic length, and β denotes the magnetic field.

2.1 Similarity Transformation

The similarity transformation is used to convert the non-linear partial differential equations of continuity equation (1), and momentum equation (2) into ODEs. The similarity variables are as follows:

$$
\begin{aligned}
&u = U_0 x e^{\frac{x}{l}} f'(\eta) .\\
&\nu = -\left[\sqrt{2\nu l U_0} e^{\frac{x}{2l}} f(\eta) + \sqrt{2\nu l U_0} \frac{x}{2l} e^{\frac{x}{2l}} f(\eta) + U_0 x y \frac{1}{2l} e^{\frac{x}{l}} f'(\eta)\right],\\
&\eta = y \sqrt{\frac{U_0}{2\nu l}}\, e^{\left(\frac{x}{2l}\right)},
\end{aligned} \tag{4}
$$

where η is the independent similarity variable, $f(\eta)$ is the dimensionless stream function. The continuity equation (1) is obeyed by the transformation in Eq. (4). Using the similarity transformations, the nonlinear partial equations (2) with boundary conditions (3) are reduced to the following nonlinear ordinary differential equations:

$$
\left(1+\frac{1}{\gamma}\right) f''' - M^2 f' + f f'' - 2 f'^2 = 0. \tag{5}
$$

with the boundary conditions

$$
\begin{aligned}
&f = s, \quad f' = -1 &&\text{at} \quad \eta = 0,\\
&f' \to 0 &&\text{as} \quad \eta \to \infty .
\end{aligned} \tag{6}
$$

where M is the Hartmann number, s is the shrinking parameter and γ is the Casson fluid parameter. Meanwhile $f''(0)$ is the skin friction coefficient.

3. Result and Discussion

The resulting of the governing equations (5) subject to the boundary condition (6), is obtained using bvp4c implemented in Matlab. The results are meticulously analyzed and presented in the form of tables (Table 1) and graphical representations (Figs. 2-4). It can be seen that the comparison results from the Table 1 shows a good agreement.

Table 1: Comparison with previous published data for values $f''(0)$

$f''(0)$					
s = 1, M = 3 and γ = 1		s =0.5, M = 2 and γ = 1		s =1, M = 2 and γ = 1	
Present Result	Nadeem [14]	Present Result	Nadeem [14]	Present Result	Nadeem [14]
2.184183	2.184183	1.215503	1.215503	1.36668	1.36668

Fig. 2 depicts the velocity profiles for various fluid parameter values. As γ increases, the magnitude of the velocity profile decreases. The thickness of the velocity boundary layer decreases as γ increases. This is due to the fact that as γ increases, the yield stress decreases, causing the velocity to be reduced to a Newtonian case. With increasing γ, the decreasing nature of the momentum boundary layer thickness appears. In Figure 3, the magnitude of velocity decreases significantly as mass suction increases, resulting in a decrease in boundary layer thickness. The increased influence of viscosity slows the flow. The maximum velocity in the boundary layer is suppressed by this effect. Meanwhile, as *M* increases, the magnitude of velocity in the boundary layer is suppressed because the force of the magnetic field opposes fluid motion as shown in Figure 4.

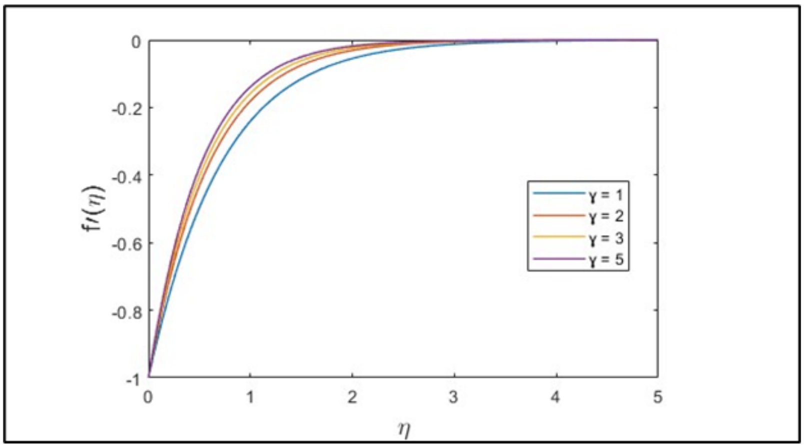

Fig. 2 Variation of velocity and boundary layer thickness for various Casson fluid parameter values γ for s = 1 and M = 2.

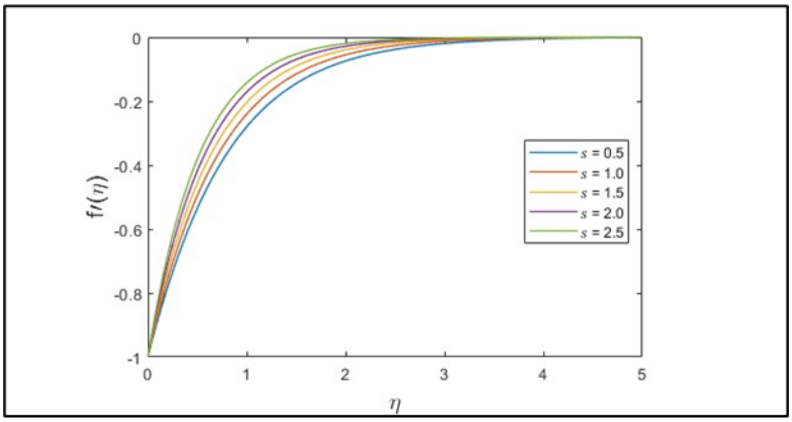

Fig. 3 Variation of velocity and boundary layer thickness for different values of shrinking parameter s for γ = 1 and M = 2.

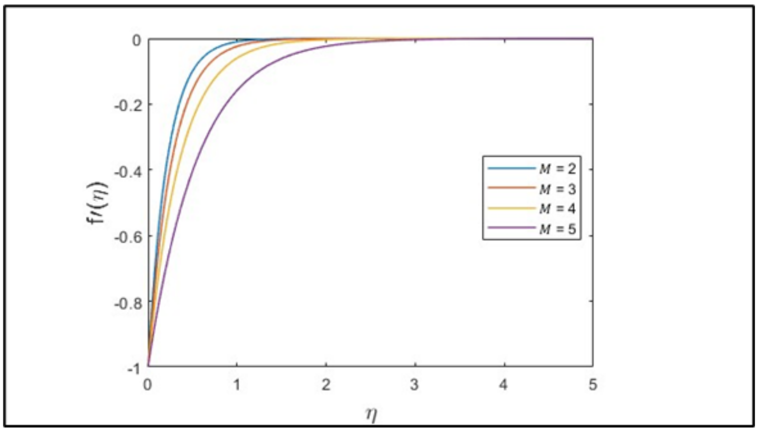

Fig. 4 Variation of velocity and boundary layer thickness for various values of Hartmann number M for s = 1 and γ = 3.

4. Conclusion

A meticulous comparative analysis was conducted on the MHD flow of Casson fluid over an exponentially shrinking sheet. The transformed ODEs were efficiently solved using bvp4c in Matlab software. Validation of the precision of these numerical solutions was achieved by comparing them with the findings obtained by Nadeem et al. [14], revealing a commendable level of agreement.

The study addresses the research questions outlined in the introduction, leading to the following key conclusions:

- As the Casson fluid parameter γ increases, the magnitude of the velocity profile $f'(n)$ decreases due to decreased yield stress.
- The magnitudes of velocity, $f'(\eta)$ decrease significantly with increases in the magnetic interaction parameter M and suction parameter s.
- The velocity and thermal boundary layer thicknesses decrease as M and s increase because the presence of a magnetic field force opposite to the velocity and suction reduces the momentum and thermal thickness of the boundary layer.

The limitation of this research is the method used to find the numerical solution in Nadeem [14] only show one value for some used parameters as shown in the result and discussions. So, the comparisons for the numerical values are limited. For the next research, different methods should be picked to get the numerical solutions.

Acknowledgement

The authors would thank the Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia for its support.

Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Nurul Aini Nabilah Rasol;* ***data collection:*** *Nurul Aini Nabilah Rasol;* ***analysis and interpretation of results:*** *Nurul Aini Nabilah Rasol;* ***draft manuscript preparation:*** *Nurul Aini Nabilah Rasol, Noorzehan Fazahiyah Md Shab. All authors reviewed the results and approved the final version of the manuscript.*

References

[1] Ahammad, M. U., & Mollah, M. SH. (2011). Numerical study of MHD free convection flow and mass transfer over a stretching sheet considering Dufour & Soret effects in the presence of magnetic field. International Journal of Engineering and Technology, 11(05), pp. 4-11. Paper ID: 112803-05-7474-IJET-IJENS

[2] Raptis, A. Perdikis, C. & Takhar, H. S. (2004). Effect of thermal radiation on MHD flow. Applied Mathematics and computation, 153(3), pp. 645-649. https://doi.org/10.1016/S0096-3003(03)00657-X

[3] Zhang, J., Fang, T. & Zhong, Y.F (2011). Analytical solution of magnetohydrodynamic sink flow. Applied Mathematics and Mechanics, 32, pp.1221–1230. https://doi.org/10.1007/s10483-011-1495-9

[4] Fang, T. (2014). Magnetohydrodynamic viscous flow over a nonlinearly moving surface: Closed-form solutions, The European Physical Journal Plus, 129. https://doi.org/10.1140/epjp/i2014-14092-4

[5] Kaya, E. E., & Tezer-Sezgin, M. (2020). DRBEM Solution of MHD flow in a rectangular duct with time-varied external magnetic field. Engineering Analysis with Boundary Elements, 117, 242-250. https://doi.org/10.1016/j.enganabound.2020.03.021

[6] Dash, R.K., Mehta, K.N., & Jayaraman, G. (1996). Casson fluid flow in a pipe filled with a homogeneous porous medium. International Journal of Engineering Science, 34(10), pp. 1145-1156. https://doi.org/10.1016/0020-7225(96)00012-2

[7] Bhattacharyya, K. (2013). Boundary layer stagnation-point flow of Casson fluid and heat transfer towards a shrinking/stretching sheet. Frontiers in Heat and Mass Transfer, 4. https://doi.org/10.5098/hmt.v4.2.3003

[8] Mukhopadhyay, S., De, P. R., Bhattacharyya, K., & Layek, G. C. (2013). Casson fluid flow over an unsteady stretching surface. Ain Shams Engineering Journal, 4(4), pp.933-938. https://doi.org/10.1016/j.asej.2013.04.004

[9] Hussain, A., Afzal, S., Rizwana, R., & Malik, M. Y. (2020). MHD stagnation point flow of a Casson fluid with variable viscosity flowing past an extending/shrinking sheet with slip effects. Physica A: Statistical Mechanics and Its Applications, 553, 124080. https://doi.org/10.1016/j.physa.2019.124080

[10] Hafidzuddin, E. H., Nazar, R., Arifin, N. M., & Pop, I. A. N. (2015). Stability analysis of unsteady three-dimensional viscous flow over a permeable stretching/shrinking surface. Journal of Quality Measurement and Analysis, 11(1), pp. 19-31. https://doi.org/10.1063/1.4882520

[11] Hirpho, M. (2021). Mixed convection of Casson fluid in a differentially heated bottom wavy wall. Heliyon, 7(6). https://doi.org/10.1016/j.heliyon.2021.e07361

[12] Yian, L. Y., Ishak, A. & Pop, I. (2011). MHD Stagnation Point Flow with Suction Towards a Shrinking Sheet. Sains Malaysiana, 40(10), pp. 1179-1186. http://journalarticle.ukm.my/2729/1/16_Lok_Yian.pdf

[13] Roy, N. C., & Pop, I. (2022). Unsteady magnetohydrodynamic stagnation point flow of a nanofluid past a permeable shrinking sheet. Chinese Journal of Physics, 75, 109-119. https://doi.org/10.1016/j.cjph.2021.12.018

[14] Nadeem, S., Haq, R. U., & Lee, C. (2012). MHD flow of a Casson fluid over an exponentially shrinking sheet. Scientia Iranica, 19(6), pp. 1550-1553. https://doi.org/10.1016/j.scient.2012.10.021