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# Solving SIR Model of HIV Using Implicit Multistep Method of Order Four and Dormand-Prince Method

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**Abstract**

This study investigated numerical methods for solving the SIR model which stands for susceptible *(S),* infectious *(I)* and recovery *(R)* in the context of HIV dynamics. The focus was on resolving the SIR model of HIV using ode45 (MATLAB), implicit multistep method of order four and the Dormand-Prince method. These methods are applied to understand and predict the spread of HIV, evaluating their performance using different step sizes. Through computational simulations using MATLAB software, comparisons were made between the two methods at step sizes of $h$ = 0.01 and $h$ = 0.1. Results indicated that the Dormand-Prince method, especially at a step size of $h$ = 0.01, gives a good approximation to the behavior of predictions of HIV cases compared to the implicit multistep method of order four. The smallest step sizes improved the performance and reduced errors, underscoring the importance of method selection and step size in HIV case predictions. The Dormand-Prince method, particularly at the smallest step sizes, had become a helpful tool for studying HIV, assisting in understanding how the disease spreads and informing public health strategies.

## 1. Introduction

The global battle against human immunodeficiency virus (HIV) and acquired immunodeficiency syndrome (AIDS) persists, claiming around 2 million lives annually. This viral epidemic not only threatens human health and economies but also profoundly impacts education [1]. Thus, the SIR model is a compartmental model commonly used in epidemiology to describe the spread of infectious diseases within a population. The common SIR model considers three distinct compartments of an active population that vary over time which are the susceptible population *S(t),* the infected population *I(t),* and the recovered population *R(t).* Central to understanding and managing HIV/AIDS are mathematical models like the foundational SIR model, refined over time to encompass behavioral aspects and prevention strategies [2]. Complementing these models are advanced numerical methods like the implicit multistep method of order four and the Dormand-Prince method, renowned for their precision and adaptability. This study navigates the performance of these tools, exploring their role in predicting transmission dynamics and shaping effective interventions against this persistent global health crisis [3].

The HIV virus led to AIDS, devastating the immune system, and causing multiple organ failures. Africa faced the highest burden, with heterosexual transmission predominant. Efforts to curb the epidemic included increasing treatment access and prevention programs, which faced substantial financial challenges for low-income countries [1]. The impact on education was significant, particularly in rural areas, yet education remained a key tool in awareness and prevention strategies. In addition, the SIR model, proposed in the 1920s, provided a framework for understanding disease spread. Mathematical models, including variations of the SIR model, contributed to understanding HIV/AIDS transmission dynamics [4]. According to [5], these models varied from basic to complex, incorporating factors like behavioral components and preventive strategies, aiding in policy formulation, and predicting infection rates.

Implicit numerical methods, particularly the implicit multistep method of order four, were effective for solving ODEs. They provided accuracy and stability, crucial for simulating complex systems like the HIV epidemic. Their efficiency in real-time simulations and adaptability made them valuable for practical applications [6]. On the other hand, the Dormand-Prince method, a variant of Runge-Kutta methods, was renowned for its precision in solving ODEs. Its adaptability, high accuracy, and efficiency made it a popular choice in modeling HIV transmission dynamics [7]. Comparative studies showcased its superiority in accuracy over other methods, albeit with higher computational demands [8].

This study aimed to compare the approximate solutions obtained by the implicit multistep method of order four and the Dormand-Prince method for solving the SIR model of HIV with ode45 (MATLAB). The focus was to develop computational tools for understanding HIV transmission, interventions, and treatment strategies by assessing the good approximation to the behavior of these numerical methods when compared to ode45 (MATLAB) solutions. However analytic solutions were hard to use for complex problems and there were some results that were unable to be obtained from analytic methods. The results of this study will contribute to developing computational tools for understanding HIV transmission, evaluating interventions, and optimizing treatment strategies.

This study focused on solving the SIR model for HIV dynamics with ode45 (MATLAB) using the implicit multistep method of order four and the Dormand-Prince method. By examining the mathematical formulation for numerical analysis, the study compared the solutions generated by these methods with ode45 (MATLAB) solutions within the SIR model framework.

## 2. Methodology

This section explained in detail the SIR model of HIV and the two methods of numerical analysis used, namely the implicit multistep method of order four and the Dormand-Prince method. The explanation included the formulas, initial conditions, and vector notation. The selected step sizes, *h* = 0.01 and 0.1[9], will be used to evaluate the performance of the SIR model for HIV with ode45 (MATLAB) by employing the implicit multistep method of order four and the Dormand-Prince method. All the calculations are performed using MATLAB software.

### 2.1 SIR Model of HIV

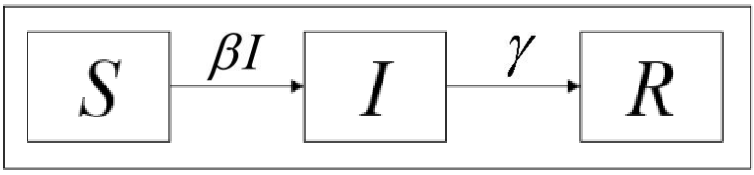


**Fig 1**: *The SIR Model compartment*

$$S(t)+I(t)+R(t)=N(t) \tag{1}$$

where;

- $N(t)$ is the total population at any time, $t$
- $S(t)$ is the susceptible population at any time, $t$
- $I(t)$ is the infectious population at any time, $t$
- $R(t)$ is the recovered population at any time, $t$.

Following is a set of ODEs that may be derived from the model that was utilized in this investigation. These equations were obtained from [10]:

$$\frac{dS}{dT}=-\frac{\beta SI}{N} \tag{2}$$

$$\frac{dI}{dT}=\frac{\beta SI}{N}-\gamma I \tag{3}$$

$$\frac{dR}{dT}=\gamma I \tag{4}$$

where;

- $\frac{dS}{dT}$ is the rate of susceptible population, $S$ with respect to any time, $t$
- $\frac{dI}{dT}$ is the rate of infected population, $I$ with respect to any time, $t$
- $\frac{dR}{dT}$ is the rate of recovered population, $R$ with respect to any time, $t$
- $\beta$ is constant rate of disease transmission.
- $\gamma$ is recovery rate.

## 2.2 Implicit Multistep Method of Order Four

Implicit multistep methods represented a category of numerical approaches used to solve differential equations describing dynamical systems. Among these methods, the fourth-order implicit multistep method stood out for its capacity to utilize past and future values in predicting the system's behavior at subsequent time points. Adams-Bashforth-Moulton predictor-corrector method is implemented to solve the SIR equations numerically. The predictor step of Adams-Bashforth of fourth order and each time step *n* from *n*=0 to *n*=*N* can be calculated as follows [11]:

$$y_{n+1}^{(p)} = y_n + \frac{h}{24}\left[55f\left(t_n, y_n,\right) - 59f\left(t_{n-1}, y_{n-1}\right) + 37f\left(t_{n-2}, y_{n-2}\right) - 9f\left(t_{n-3}, y_{n-3}\right)\right] \tag{5}$$

For corrector step of Adams-Moulton of fourth order corrects the predicted value and each time step n from n=0 to n=N using the formula:

$$y_{n+1} = y_n + \frac{h}{24}\left[9f\left(t_{n+1}, y_{n+1}^{(p)}\right) + 19f\left(t_n, y_n\right) - 5f\left(t_{n-1}, y_{n-1}\right) + f\left(t_{n-2}, y_{n-2}\right)\right] \tag{6}$$

From the general equation provided earlier, the present investigation examined the implicit multistep method of order four for the Adams-Bashforth predictor method combined with the SIR model, which is outlined as follows,

$$S_{n+1} = S_n + \frac{h}{24}\left[55f\left(t_n, S_n\right) - 59f\left(t_{n-1}, S_{n-1}\right) + 37f\left(t_{n-2}, S_{n-2}\right) - 9f\left(t_{n-3}, S_{n-3}\right)\right] \tag{7}$$

$$I_{n+1} = I_n + \frac{h}{24}\left[55f\left(t_n, I_n\right) - 59f\left(t_{n-1}, I_{n-1}\right) + 37f\left(t_{n-2}, I_{n-2}\right) - 9f\left(t_{n-3}, I_{n-3}\right)\right] \tag{8}$$

$$R_{n+1} = R_n + \frac{h}{24}\left[55f\left(t_n, R_n\right) - 59f\left(t_{n-1}, R_{n-1}\right) + 37f\left(t_{n-2}, R_{n-2}\right) - 9f\left(t_{n-3}, R_{n-3}\right)\right] \tag{9}$$

The following equations were Adams-Moulton corrector method for the SIR model:

$$S_{n+1} = S_n + \frac{h}{24}\left[9f\left(t_{n+1}, S_{n+1}\right) + 19f\left(t_n, S_n\right) - 5f\left(t_{n-1}, S_{n-1}\right) + f\left(t_{n-2}, S_{n-2}\right)\right] \tag{10}$$

$$I_{n+1} = I_n + \frac{h}{24}\left[9f\left(t_{n+1}, I_{n+1}\right) + 19f\left(t_n, I_n\right) - 5f\left(t_{n-1}, I_{n-1}\right) + f\left(t_{n-2}, I_{n-2}\right)\right] \tag{11}$$

$$R_{n+1} = R_n + \frac{h}{24}\left[9f\left(t_{n+1}, R_{n+1}\right) + 19f\left(t_n, R_n\right) - 5f\left(t_{n-1}, R_{n-1}\right) + f\left(t_{n-2}, R_{n-2}\right)\right] \tag{12}$$

## 2.3 Dormand-Prince Method

The Dormand-Prince method, known for its enhanced precision and adaptability, has replaced traditional methods such as the Runge-Kutta technique. Renowned for its accuracy, it is categorized under adaptive step methods. The specific iterative equations defining *S(t)*, *I(t)*, and *R(t)* within the framework of the SIR model are outlined as follows [12]:

The iterative equations of Dormand-Prince method for susceptible of SIR model,

$$S_{n+1} = S_n + \frac{5179}{57600}k_1^S + \frac{7571}{16695}k_3^S + \frac{393}{640}k_4^S - \frac{92097}{339200}k_5^S + \frac{187}{2100}k_6^S + \frac{1}{40}k_7^S \tag{13}$$

$$k_1^S = hf\left(t_n, S_n, I_n\right) = -h\frac{\beta S_n I_n}{N} \tag{14}$$

(16)

$$k_2^S = hf\left(t_n + \frac{1}{5}h, S_n + \frac{1}{5}k_1^S, I_n + \frac{1}{5}k_1^I\right) = -h\frac{\beta}{N}\left(S_n + \frac{1}{5}k_1^S\right)\left(I_n + \frac{1}{5}k_1^I\right) \quad (15)$$

$$\begin{aligned} k_3^S &= hf\left(t_n + \frac{3}{10}h, S_n + \frac{3}{40}k_1^S + \frac{9}{40}k_2^S, I_n + \frac{3}{40}k_1^I + \frac{9}{40}k_2^I\right) \\ &= -h\frac{\beta}{N}\left(S_n + \frac{3}{40}k_1^S + \frac{9}{40}k_2^S\right)\left(I_n + \frac{3}{40}k_1^I + \frac{9}{40}k_2^I\right) \end{aligned} \quad (16)$$

$$\begin{aligned} k_4^S &= hf\left(t_n + \frac{4}{5}h, S_n + \frac{44}{45}k_1^S - \frac{56}{15}k_2^S + \frac{32}{9}k_3^S, I_n + \frac{44}{45}k_1^I - \frac{56}{15}k_2^I + \frac{32}{9}k_3^I\right) \\ &= -h\frac{\beta}{N}\left(S_n + \frac{44}{45}k_1^S - \frac{56}{15}k_2^S + \frac{32}{9}k_3^S\right)\left(I_n + \frac{44}{45}k_1^I - \frac{56}{15}k_2^I + \frac{32}{9}k_3^I\right) \end{aligned} \quad (17)$$

$$\begin{aligned} k_5^S &= hf\begin{pmatrix} t_n + \frac{8}{9}h, S_n + \frac{19372}{6561}k_1^S - \frac{25360}{2187}k_2^S + \frac{64448}{6561}k_3^S - \frac{212}{729}k_4^S, \\ I_n + \frac{19372}{6561}k_1^I - \frac{25360}{2187}k_2^I + \frac{64448}{6561}k_3^I - \frac{212}{729}k_4^I \end{pmatrix} \\ &= -h\frac{\beta}{N}\left(S_n + \frac{19372}{6561}k_1^S - \frac{25360}{2187}k_2^S + \frac{64448}{6561}k_3^S - \frac{212}{729}k_4^S\right) \\ &\quad \left(I_n + \frac{19372}{6561}k_1^I - \frac{25360}{2187}k_2^I + \frac{64448}{6561}k_3^I - \frac{212}{729}k_4^I\right) \end{aligned} \quad (18)$$

$$\begin{aligned} k_6^S &= hf\begin{pmatrix} t_n + h, S_n + \frac{9017}{3168}k_1^S - \frac{355}{33}k_2^S - \frac{46732}{5247}k_3^S + \frac{49}{176}k_4^S - \frac{5103}{18656}k_5^S, \\ I_n + \frac{9017}{3168}k_1^I - \frac{355}{33}k_2^I - \frac{46732}{5247}k_3^I + \frac{49}{176}k_4^I - \frac{5103}{18656}k_5^I \end{pmatrix} \\ &= -h\frac{\beta}{N}\left(S_n + \frac{9017}{3168}k_1^S - \frac{355}{33}k_2^S - \frac{46732}{5247}k_3^S + \frac{49}{176}k_4^S - \frac{5103}{18656}k_5^S\right) \\ &\quad \left(I_n + \frac{9017}{3168}k_1^I - \frac{355}{33}k_2^I - \frac{46732}{5247}k_3^I + \frac{49}{176}k_4^I - \frac{5103}{18656}k_5^I\right) \end{aligned} \quad (19)$$

$$\begin{aligned} k_7^S &= hf\left(t_n + h, S_n + \frac{35}{384}k_1^S + \frac{500}{1113}k_3^S + \frac{125}{192}k_4^S - \frac{2187}{6784}k_5^S + \frac{11}{84}k_6^S\right) \\ &\quad \left(I_n + \frac{35}{384}k_1^I + \frac{500}{1113}k_3^I + \frac{125}{192}k_4^I - \frac{2187}{6784}k_5^I + \frac{11}{84}k_6^I\right) \\ &= -h\frac{\beta}{N}\left(S_n + \frac{35}{384}k_1^S + \frac{500}{1113}k_3^S + \frac{125}{192}k_4^S - \frac{2187}{6784}k_5^S + \frac{11}{84}k_6^S\right) \\ &\quad \left(I_n + \frac{35}{384}k_1^I + \frac{500}{1113}k_3^I + \frac{125}{192}k_4^I - \frac{2187}{6784}k_5^I + \frac{11}{84}k_6^I\right) \end{aligned} \quad (20)$$

The iterative equations of Dormand-Prince method for infectious of SIR model,

$$I_{n+1} = I_n + \frac{5179}{57600}k_1^I + \frac{7571}{16695}k_3^I + \frac{393}{640}k_4^I - \frac{92097}{339200}k_5^I + \frac{187}{2100}k_6^I + \frac{1}{40}k_7^I \quad (21)$$

$$k_1^I = hf\left(t_n, S_n, I_n\right) = h\frac{\beta S_n I_n}{N} - \gamma I_n \tag{22}$$

$$\begin{aligned} k_2^I &= hf\left(t_n + \frac{1}{5}h, S_n + \frac{1}{5}k_1^S, I_n + \frac{1}{5}k_1^I\right) = h\frac{\beta}{N}\left(S_n + \frac{1}{5}k_1^S\right)\left(I_n + \frac{1}{5}k_1^I\right) \\ &\quad -\gamma I_n\left(I_n + \frac{1}{5}k_1^I\right) \end{aligned} \tag{23}$$

$$\begin{aligned} k_3^I &= hf\left(t_n + \frac{3}{10}h, S_n + \frac{3}{40}k_1^S + \frac{9}{40}k_2^S, I_n + \frac{3}{40}k_1^I + \frac{9}{40}k_2^I\right) \\ &= h\frac{\beta}{N}\left(S_n + \frac{3}{40}k_1^S + \frac{9}{40}k_2^S\right)\left(I_n + \frac{3}{40}k_1^I + \frac{9}{40}k_2^I\right) - \gamma I_n\left(I_n + \frac{3}{40}k_1^I + \frac{9}{40}k_2^I\right) \end{aligned} \tag{24}$$

$$\begin{aligned} k_4^I &= hf\left(t_n + \frac{4}{5}h, S_n + \frac{44}{45}k_1^S - \frac{56}{15}k_2^S + \frac{32}{9}k_3^S, I_n + \frac{44}{45}k_1^I - \frac{56}{15}k_2^I + \frac{32}{9}k_3^I\right) \\ &= h\frac{\beta}{N}\left(S_n + \frac{44}{45}k_1^S - \frac{56}{15}k_2^S + \frac{32}{9}k_3^S\right)\left(I_n + \frac{44}{45}k_1^I - \frac{56}{15}k_2^I + \frac{32}{9}k_3^I\right) \\ &\quad -\gamma I_n\left(I_n + \frac{44}{45}k_1^I - \frac{56}{15}k_2^I + \frac{32}{9}k_3^I\right) \end{aligned} \tag{25}$$

$$\begin{aligned} k_5^I &= hf\begin{pmatrix} t_n + \frac{8}{9}h, S_n + \frac{19372}{6561}k_1^S - \frac{25360}{2187}k_2^S + \frac{64448}{6561}k_3^S - \frac{212}{729}k_4^S, \\ I_n + \frac{19372}{6561}k_1^I - \frac{25360}{2187}k_2^I + \frac{64448}{6561}k_3^I - \frac{212}{729}k_4^I \end{pmatrix} \\ &= h\frac{\beta}{N}\left(S_n + \frac{19372}{6561}k_1^S - \frac{25360}{2187}k_2^S + \frac{64448}{6561}k_3^S - \frac{212}{729}k_4^S\right) \\ &\quad \left(I_n + \frac{19372}{6561}k_1^I - \frac{25360}{2187}k_2^I + \frac{64448}{6561}k_3^I - \frac{212}{729}k_4^I\right) \\ &\quad -\gamma I_n\left(I_n + \frac{19372}{6561}k_1^I - \frac{25360}{2187}k_2^I + \frac{64448}{6561}k_3^I - \frac{212}{729}k_4^I\right) \end{aligned} \tag{26}$$

$$\begin{aligned} k_6^I &= hf\begin{pmatrix} t_n + h, S_n + \frac{9017}{3168}k_1^S - \frac{355}{33}k_2^S - \frac{46732}{5247}k_3^S + \frac{49}{176}k_4^S - \frac{5103}{18656}k_5^S, \\ I_n + \frac{9017}{3168}k_1^I - \frac{355}{33}k_2^I - \frac{46732}{5247}k_3^I + \frac{49}{176}k_4^I - \frac{5103}{18656}k_5^I \end{pmatrix} \\ &= h\frac{\beta}{N}\left(S_n + \frac{9017}{3168}k_1^S - \frac{355}{33}k_2^S - \frac{46732}{5247}k_3^S + \frac{49}{176}k_4^S - \frac{5103}{18656}k_5^S\right) \\ &\quad \left(I_n + \frac{9017}{3168}k_1^I - \frac{355}{33}k_2^I - \frac{46732}{5247}k_3^I + \frac{49}{176}k_4^I - \frac{5103}{18656}k_5^I\right) \\ &\quad -\gamma I_n\left(I_n + \frac{9017}{3168}k_1^I - \frac{355}{33}k_2^I - \frac{46732}{5247}k_3^I + \frac{49}{176}k_4^I - \frac{5103}{18656}k_5^I\right) \end{aligned} \tag{27}$$

$$k_7^I = hf\left(t_n+h, S_n+\frac{35}{384}k_1^S+\frac{500}{1113}k_3^S+\frac{125}{192}k_4^S-\frac{2187}{6784}k_5^S+\frac{11}{84}k_6^S\right)$$
$$\left(I_n+\frac{35}{384}k_1^I+\frac{500}{1113}k_3^I+\frac{125}{192}k_4^I-\frac{2187}{6784}k_5^I+\frac{11}{84}k_6^I\right)$$
$$= h\frac{\beta}{N}\left(S_n+\frac{35}{384}k_1^S+\frac{500}{1113}k_3^S+\frac{125}{192}k_4^S-\frac{2187}{6784}k_5^S+\frac{11}{84}k_6^S\right) \quad (28)$$
$$\left(I_n+\frac{35}{384}k_1^I+\frac{500}{1113}k_3^I+\frac{125}{192}k_4^I-\frac{2187}{6784}k_5^I+\frac{11}{84}k_6^I\right)$$
$$-\gamma I_n\left(I_n+\frac{35}{384}k_1^I+\frac{500}{1113}k_3^I+\frac{125}{192}k_4^I-\frac{2187}{6784}k_5^I+\frac{11}{84}k_6^I\right)$$

The iterative equations of Dormand-Prince method for recovery of SIR model,

$$R_{n+1} = R_n+\frac{5179}{57600}k_1^R+\frac{7571}{16695}k_3^R+\frac{393}{640}k_4^R-\frac{92097}{339200}k_5^R+\frac{187}{2100}k_6^R+\frac{1}{40}k_7^R \quad (29)$$

$$k_1^R = hf\left(t_n, S_n, I_n\right) = h\gamma I_n \quad (30)$$

$$k_2^R = hf\left(t_n+\frac{1}{5}h, I_n+\frac{1}{5}k_1^I\right) = h\gamma\left(I_n+\frac{1}{5}k_1^I\right) \quad (31)$$

$$k_3^R = hf\left(t_n+\frac{3}{10}h, I_n+\frac{3}{40}k_1^I+\frac{9}{40}k_2^I\right) = h\gamma\left(I_n+\frac{3}{40}k_1^I+\frac{9}{40}k_2^I\right) \quad (32)$$

$$k_4^R = hf\left(t_n+\frac{4}{5}h, I_n+\frac{44}{45}k_1^I-\frac{56}{15}k_2^I+\frac{32}{9}k_3^I\right) = h\gamma\left(I_n+\frac{44}{45}k_1^I-\frac{56}{15}k_2^I+\frac{32}{9}k_3^I\right) \quad (33)$$

$$k_5^R = hf\left(t_n+\frac{8}{9}h, I_n+\frac{19372}{6561}k_1^I-\frac{25360}{2187}k_2^I+\frac{64448}{6561}k_3^I-\frac{212}{729}k_4^I\right)$$
$$= h\gamma\left(I_n+\frac{19372}{6561}k_1^I-\frac{25360}{2187}k_2^I+\frac{64448}{6561}k_3^I-\frac{212}{729}k_4^I\right) \quad (34)$$

$$k_6^R = hf\left(t_n+h, I_n+\frac{9017}{3168}k_1^I-\frac{355}{33}k_2^I-\frac{46732}{5247}k_3^I+\frac{49}{176}k_4^I-\frac{5103}{18656}k_5^I\right)$$
$$= h\gamma\left(I_n+\frac{9017}{3168}k_1^I-\frac{355}{33}k_2^I-\frac{46732}{5247}k_3^I+\frac{49}{176}k_4^I-\frac{5103}{18656}k_5^I\right) \quad (35)$$

$$k_7^R = hf\left(t_n+h, I_n+\frac{35}{384}k_1^I+\frac{500}{1113}k_3^I+\frac{125}{192}k_4^I-\frac{2187}{6784}k_5^I+\frac{11}{84}k_6^I\right)$$
$$= h\gamma\left(I_n+\frac{35}{384}k_1^I+\frac{500}{1113}k_3^I+\frac{125}{192}k_4^I-\frac{2187}{6784}k_5^I+\frac{11}{84}k_6^I\right) \quad (36)$$

## 3. Result and Discussion

This section discussed the solution of SIR model of HIV with initial conditions and proposed to solve the SIR model of HIV with ode45 (MATLAB) by using numerical analysis, which is implicit multistep method of order four and Dormand-Prince method. This chapter explored different step sizes ($h$) and their impact on these methods when solving the SIR model. It sought to determine which step size provided a better approximation of the behavior of the results.

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## 3.1 Test Problem

Refer to the SIR model of HIV in equation (2), (3) and (4) with the initial conditions, $N(0)=1000, I(0)=1, R(0)=0$ and time interval, $0 \le t \le 40$. In this study, we adopted the values for the parameters from [13]. Specifically, the transmission rate, $\beta$ denoted as $\beta = 0.3$ and the average time of the disease course, corresponding to the recovery rate $\gamma$, denoted as $\gamma = 0.1$. This will give:

$$\frac{dS}{dT} = -\frac{(0.3)SI}{N} \tag{37}$$

$$\frac{dI}{dT} = \frac{(0.3)SI}{N} - (0.1)I \tag{38}$$

$$\frac{dR}{dT} = (0.1)I \tag{39}$$

The assessment of good approximation of the behavior of the methods in the problem involved the utilization of two different step sizes, $h$, for evaluation purposes. Specifically, the analysis considered cases where *h* = 0.1, aligned with the time interval, $t$, and compared these scenarios against a smaller step size of *h* = 0.01.

## 3.2 The Simulation of SIR Model of HIV in Malaysia

The study employed MATLAB software, utilizing the ode45 solver to simulate the spread of HIV in Malaysia using the SIR Model. Through this simulation, the ode45 solver facilitated the generation of predictions concerning potential disease transmission patterns, providing valuable insights into its future progression.

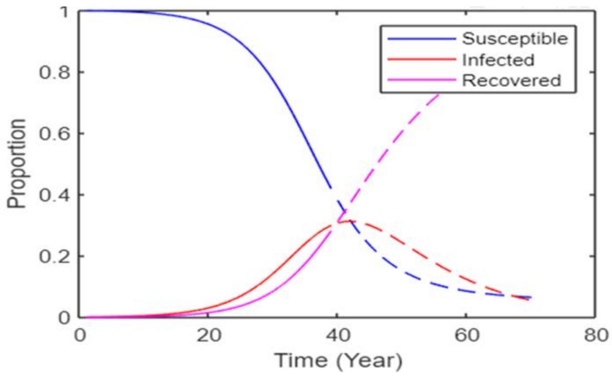



**Fig 2:** *Simulation of the SIR model of HIV in Malaysia*

Fig 2 indicated a decreasing curve for the proportion of susceptible individuals starting at 30 years. This decline mirrored Malaysia's successful efforts guided by the Ministry of Health. These efforts included widespread education programs, improved healthcare access, and proactive government interventions [14].

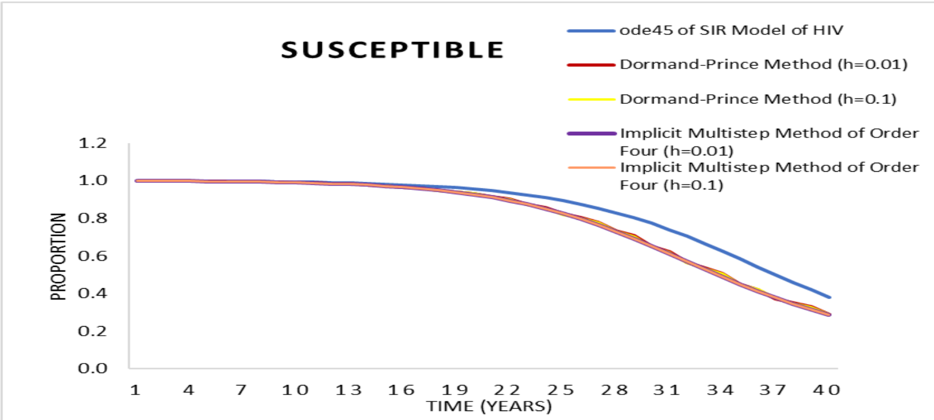
Between the years of 20 and 40, the curve representing the proportion of infections began to rise. According to [15], the first HIV case in Malaysia was reported in 1986 when a 45-year-old Chinese male of American origin fell ill while visiting the country. Since then, the highly contagious disease has continued to spread continuously. [16] highlighted that, drug abuse remained a significant issue in Malaysia which contributing to an increased number of addicts.

While the curve representing the proportion of infections was higher, the curve depicting the proportion of recoveries showed a rapid increase during the years 20 onwards, aimed at curbing the disease's spread. During this period, the Ministry of Health actively sought solutions and preventive measures to stop HIV transmission. As a result, highly active antiretroviral therapy had been provided free of charge in Malaysia by the Ministry of Health since 2006. Significantly, antiretroviral therapy slows HIV growth which can reduce the virus in the body and decrease transmission risk to others [17].

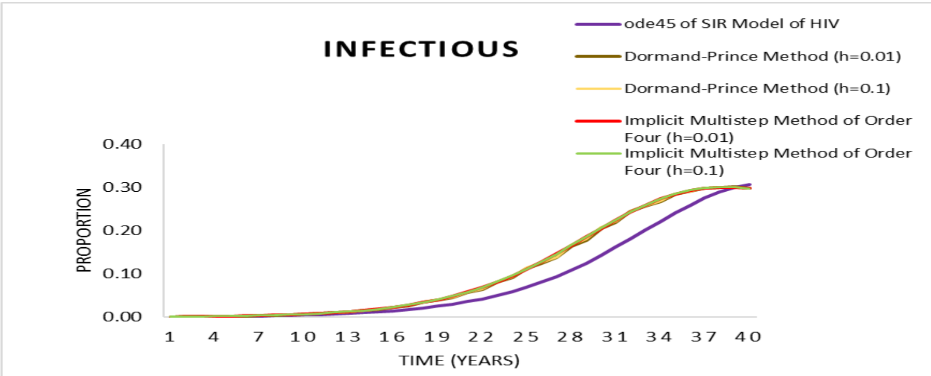
Beyond 40 years, the dotted line indicated predictions concerning the progression of HIV disease. These predictions offered an insight into the potential future trends of the disease based on existing data and patterns observed up to that point.

## 3.3 Numerical Results

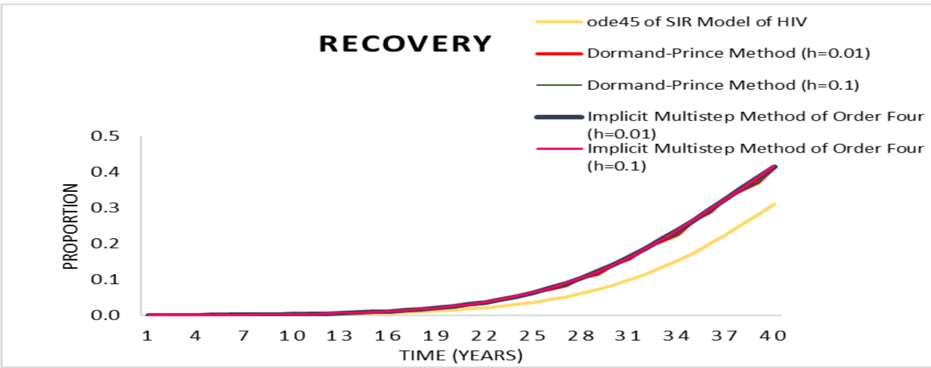
Fig 3 presented results from solving the SIR model of HIV with ode45 (MATLAB) using implicit multistep method of order four and Dormand-Prince method. Calculations were done in MATLAB. It discussed findings and graphs, testing with step sizes of $h$ = 0.01 and $h$ = 0.1.




**(a)**




**(b)**




**(c)**

**Fig 3:** *Graph depicted the numerical results of SIR model of HIV using Dormand-Prince Method and Implicit Multistep Method of Order Four with step size, h=0.01 and h=0.1 which (a) is susceptible, (b) is infectious, (c) is recovery*

## 3.4 Error Analysis

Fig 4 visually displayed error analysis results of the SIR model of HIV with ode45 (MATLAB) solved via approximate methods, implicit multistep method of order four and Dormand-Prince method. It assessed computational disparities and the impact of step sizes, $h$=0.01 and $h$=0.1 on model through graphical analysis.

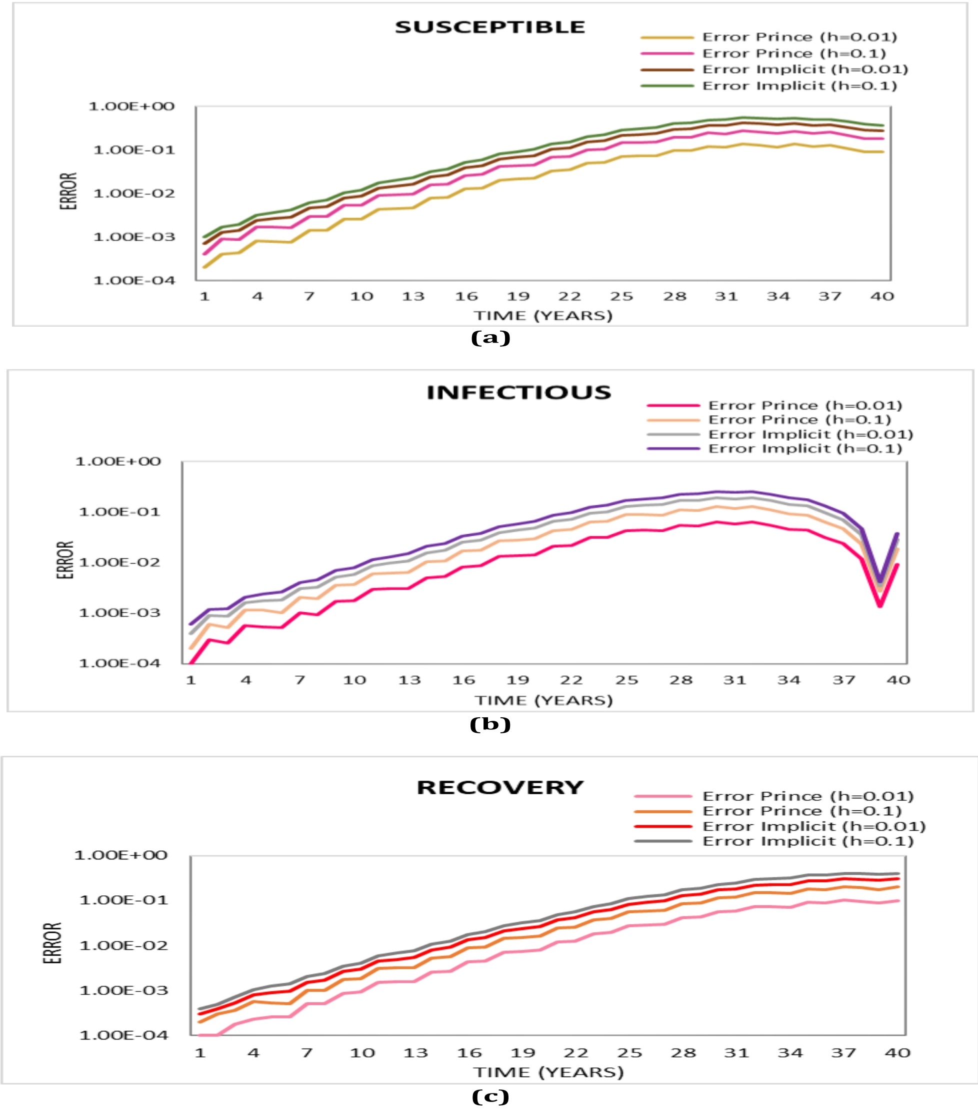



**Fig 4**: *Graphs depicting the errors for (a) is susceptible, (b) is infectious and (c) is recovery compartment for the SIR model of HIV with different step sizes*

## 3.5 Discussion

The study compared the implicit multistep method of order four and the Dormand-Prince method with ode45 (MATLAB) in solving the SIR model of HIV using different step sizes, $h$=0.01 and $h$=0.1. Fig 3 illustrated the comparison of numerical results with ode45 (MATLAB) and Fig 4 showed the error graphs of the SIR model of HIV by ode45 (MATLAB) using the implicit multistep method of order four and Dormand-Prince method with step sizes of $h$=0.01 and $h$=0.1.The results indicated that using a smaller step size, $h$=0.01 with the Dormand-Prince method gives a good approximation to the behavior compared to the implicit multistep method of order four, as reflected in slightly smaller errors and graphs closely resembling to ode45 (MATLAB) solutions. The Dormand-Prince method consistently demonstrated smaller errors, emphasizing its greater performance in approximating susceptible, infectious, and recovery proportions. Specifically, with a smaller step size, $h$=0.01, the results obtained using the Dormand-Prince method closely matched those produced by ode45 (MATLAB) solutions in Fig 3, highlighting its superior ability to reflect system dynamics precisely. In contrast, the implicit multistep method of order four, particularly with a larger step size, $h$=0.1, showed a slower approximation to the behavior and exhibited greater deviation from the values obtained with ode45 in MATLAB over time. This comparison emphasizes the importance of choosing an appropriate step size for achieving good approximations to the behavior of each method in simulating the outcomes.

## 4. Conclusion

The Dormand-Prince method performed better than the implicit multistep method of order four in solving the SIR model of HIV and demonstrated closer alignment with ode45 (MATLAB) solutions. Smaller steps size, such as $h$=0.01 significantly improved the approximation, particularly with the Dormand-Prince method. Choosing the right method and step size is crucial for predictions in HIV modelling. The Dormand-Prince method proves highly valuable for handling complex models. The results showed that the smaller the step size used, the better result obtained.

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## Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

## Author Contribution

*The authors confirm contribution to the paper as follows: **study conception and design:** Nurul Nadia Tasha Hisyam, Norzuria Ibrahim; **data collection:** Nurul Nadia Tasha Hisyam; **analysis and interpretation of results:** Nurul Nadia Tasha Hisyam, Norzuria Ibrahim; **draft manuscript preparation:** Nurul Nadia Tasha Hisyam, Norzuria Ibrahim. All authors reviewed the results and approved the final version of the manuscript.*

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