ENHANCED KNOWLEDGE IN SCIENCES AND TECHNOLOGY
e-ISSN: 2773-6385

EKST

Vol. 4 No. 2 (2024) 141-147
https://publisher.uthm.edu.my/periodicals/index.php/ekst

Numerical Study of Two-Phase Dusty Casson Fluid Flow over a Stretching Sheet with Aligned Magnetic Field and Newtonian Heating

Muhammad Shafirudin Fitri Saidis[1], Noorzehan Fazahiyah Md Shab[1]*

[1] *Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology, UTHM Kampus Cawangan Pagoh, Hab Pendidikan Tinggi Pagoh, KM 1, Jalan Panchor, 84600 Pagoh, Muar, Johor, MALAYSIA*

*Corresponding Author: fazahiya@uthm.edu.my
DOI: https://doi.org/10.30880/ekst.2024.04.02.017

Article Info

Received: 27 December 2023
Accepted: 11 January 2024
Available online: 12 December 2024

Keywords

Magnetic field, Dusty Casson fluid, Stretching Sheet, Newtonian Heating, bvp4c Method

Abstract

The purpose of this study is to numerically investigated the steady flow of dusty Casson fluid and heat transfer over a stretching sheet subjected to an aligned magnetic field and Newtonian heating. Similarity transformation technique is employed to convert the governing partial differential equations into a system of ordinary differential equations. The effects of key parameters, including the aligned magnetic field strength, Casson parameter, fluid-particle interaction parameter, Prandtl number, and conjugate parameter, are analysed numerically in detail. Key findings revealed that the aligned magnetic field and Casson parameter effectively reduced the fluid velocity but have minimal impact on the particle velocity.

1. Introduction

The dusty Casson fluid model combines the Casson fluid model with the presence of dust particles, making it a valuable tool in various engineering and industrial applications. The particles can have a significant impact on the flow properties of the fluid, such as viscosity, yield stress, and heat transfer. [1] who first introduced Casson fluid, which is a type of non-Newtonian fluid that exhibits shear thickening behaviour. In this research, exposure has been given related to the idea that the flow of a suspension is governed by two forces which are viscous force and yield stress. Various studies have investigated the effect of dust particles on the flow characteristics, including viscosity, yield stress, and shear-thinning behaviour. [2] investigated the effects of particle concentration on the viscosity of dusty Casson fluids. They found that the viscosity increased with increasing particle concentration. [3] studied the effects of particle size on the yield stress of dusty Casson fluids and found that the yield stress increased with increasing particle size.

[4] investigated how inclined Lorentz forces influence the flow and heat transfer of a Casson fluid boundary layer over a non-absorbent stretched sheet. They discovered that the Lorentz forces had a considerable impact on the fluid's temperature and velocity characteristics. Exploring the impact of magnetized flow and dust particles on heat transfer, [5] investigated the behaviour of a dusty Casson fluid over a stretching sheet. Their findings revealed a notable increase in heat transfer rate with the presence of a magnetic field.

Aside from that, [6] investigated the behaviour of a dusty Casson fluid over a stretching sheet with presence of aligned magnetic field and Newtonian heating. The study's findings demonstrated that the dusty Casson fluid's flow and heat transfer characteristics can be greatly influenced by the presence of a magnetic field and Newtonian heating. In their study of radiative heat transfer in dusty Casson fluids moving past an undulating surface, [7] revealed a surprising boost in heat exchange due to the presence of dust particles.

Newtonian heating refers to the phenomenon where the fluid temperature increases linearly with the distance from a heated surface. Several studies have examined the effect of Newtonian heating on fluid flow and heat transfer characteristics. Examining the combined effects of Newtonian heating, thermal radiation, and the Casson parameter on fluid flow over a flat plate, [8] observed a decrease in both velocity and temperature as the Casson parameter increased. It shows that the Newtonian heating effect has a significant impact on the flow and heat transfer characteristics of a Casson fluid past a flat plate. [9] employed an experimental approach to probe the influence of Newtonian heating on the mass flow conditions of a Casson fluid flowing over a stretched surface. Their findings revealed a substantial impact of the heating strategy on both the fluid's flow characteristics and its heat transfer capabilities.

Therefore, the purpose in this research is to numerically investigate and create a mathematical description of the effects of several parameters which are fluid-particle interactions (β), Casson parameter (A), magnetic field parameters (M), and aligned angle (α_1) on the velocity and temperature profiles of the fluid and dust phases. In contrast to the previous research by [10], this study employs the bvp4c method in MATLAB R2022a providing a distinct analytical perspective and potentially refining the accuracy of numerical results.

2. Materials and Methods

A steady two-dimensional dusty Casson fluid boundary layer flow is applied to a stretching sheet at a stretching linear velocity $u_w(x)=ax$. The flow is subjected to an aligned magnetic field with an acute angle α_1 as shown in Fig. 1.

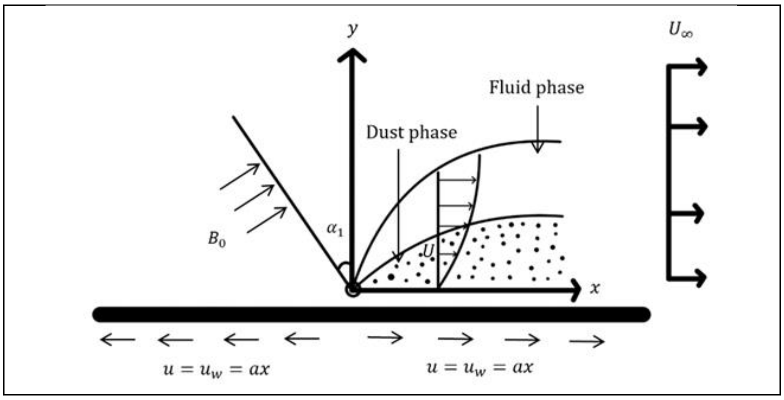

Fig. 1 Flow Configuration by Ariffin et al. [10]

The governing equation for fluid and dust phases of this flow by refering to Arifin *et al.* [10] are:
Fluid phase:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\left(1+\frac{1}{A}\right)\left(\frac{\partial^2 u}{\partial y^2}\right)+\frac{\rho_p}{\tau_v}\left(u_p-u\right)-\sigma u B_0{}^2\sin^2\alpha_1, \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{\rho_p c_s}{\gamma_T}\left(T_p-T\right), \tag{3}$$

Dust phase:

$$\frac{\partial}{\partial x}\left(u_p\right)+\frac{\partial}{\partial y}\left(v_p\right)=0, \tag{4}$$

$$\rho_p\left(u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)=\frac{\rho_p}{\tau_v}\left(u-u_p\right), \tag{5}$$

$$\rho_p c_s\left(u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}\right)=-\frac{\rho_p c_s}{\gamma_T}\left(T_p-T\right), \tag{6}$$

subject to the boundary conditions:

$$u = u_w(x) = ax, v = 0, \frac{\partial T}{\partial y} = -h_s T \quad \text{as } y = 0,$$
$$u \to 0, u_p \to 0, v_p \to v, T \to T_\infty, T_p \to T_\infty \quad \text{as } y \to \infty. \tag{7}$$

Where (u, v) and (u_p, v_p) are the fluid and particle phase's velocities. μ is the coefficient of viscosity of the fluid, ρ and ρ_p are the density of the fluid and dust phase, α_1 is the aligned angle, $\tau_v = 1/k$ is the relaxation time or particles phase, α is a positive constant, k is the Stoke's resistance, c_p and c_s are specific heat of fluid and dust particle, T and T_p are the temperature of fluid and particle phase, γ_T the thermal relaxation time, B_0 is the magnetic field strength, A=$\mu_B\sqrt{(2\pi_c)}/p_y$ is the non-Newtonian (Casson) parameter and h_s is the heat transfer parameter.

These non-linear partial differential equations are transformed into ordinary differential equations by using similarity equations

$$u = axf'(\eta), v = -\sqrt{av}\, f(\eta), \eta = \sqrt{\frac{a}{v}}y, \theta(\eta) = \frac{T - T_\infty}{T_\infty},$$
$$u_p = axF'(\eta), v_p = -\sqrt{av}F(\eta), \theta_p(\eta) = \frac{T_p - T_\infty}{T_\infty}. \tag{8}$$

The transformation of equations (1)-(6) can be experessed as:

$$(1+\frac{1}{A})f'''(\eta) + f(\eta)f''(\eta) - (f'(\eta))^2 + \beta N(F'(\eta) - f'(\eta)) - M\sin^2\alpha_1 f'(\eta) = 0 \tag{9}$$

$$\theta''(\eta) + Pr\, f(\eta)\theta'(\eta) + \frac{2}{3}\beta N\left(\theta_p(\eta) - \theta(\eta)\right) = 0, \tag{10}$$

$$\left(F'(\eta)\right)^2 - F(\eta)F''(\eta) + \beta\left(F'(\eta) - f'(\eta)\right) = 0, \tag{11}$$

$$\theta_p'(\eta)F(\eta) + \frac{2}{3}\frac{\beta}{\Pr\lambda}[\theta(\eta) - \theta_p(\eta)]. \tag{12}$$

Along with the boundary conditions

$$f(0) = 0,\ f'(0) = 1,\ \theta'(0) = -\gamma\left(1 + \theta(0)\right) \ \text{as}\, \eta = 0,$$
$$f'(\eta) \to 0,\ F'(\eta) \to 0,\ F(\eta) \to f(\eta),\ \theta(\eta) \to 0,\ \theta_p(\eta) \to 0 \ \text{as}\, \eta \to \infty \tag{13}$$

Where mass concentration of particle phase is $N=\rho_p/\rho$, fluid-particle interaction parameter is $\beta=1/\alpha\tau_v$, magnetic field paramater is $M=\sigma B_0^2/\rho a$, Prandtl number is Pr=$\mu c_p/k$, and $\gamma=-h_s(v/a)^{1/2}$ is the conjugate parameter for Newtonian heating.

3. Results and Discussions

The the numerical computation for governing ordinary differential equations is obtained by using bvp4c method on MATLAB R2022a. The current problem's flow and heat transfer are graphically evaluated through several physical parameters which are fluid-particle interactions (β), Casson parameter (A), magnetic field parameters (M), and aligned angle (α_1). The numerical results for the velocity and temperature profile for both fluid and dust phases are computed for several fixed pertinent parameter which are α_1= $\pi/6$, M=2, β=1, γ=0.5, A=1, N=12.6, Pr=16.7 and λ=0.25. Table 1 showed the comparison of skin friction coefficient $f''(0)$ when the Casson parameter and fluid particle interaction parameter is absent. The numerical solution of Ariffin *et al*,[10] were discovered to be consistent with the current research.

Fig. 2 and 3 illustrate the Casson parameter's effects on fluid flow, with and without dust particles. On Fig. 2, both the dusty Casson fluid (β=1) and the Casson fluid (β=0) exhibit a decreasing pattern of fluid velocity with increasing A values. It is important to note that when there are dust particles present, fluid movement tends to slow down. Same as in Fig. 3, as A increases, both phases show a significant decreasing trend. As A increases, the plastic dynamic viscosity of the fluid (μ_B) also rises. This viscosity governs the internal friction within the fluid and influences its resistance to flow. Higher viscosity causes the fluid molecules to have more intermolecular cohesiveness, which makes it more difficult for them to pass one another thus reduces the flow velocity.

The velocity profile for different values of of magnetic field, M and aligned angle, α_1 are captured in Fig 4 and 5 respectively. The different values of M and α_1 are sets as M=0.0, 1.0, 3.0, 5.0 and α_1=0, $\pi/6$, $\pi/4$, $\pi/3$. $\pi/2$. It can be noticed that the increasing value of M and α_1 resulting the velocity profile to decrease for fluid and dust phases. When a fluid particle moves through a magnetic field, it experiences the Lorentz force which acts perpendicular to both the magnetic field and the flow direction, essentially causing a drag on the particles and fluid in motion. When α_1 increases, the component of the magnetic field acting perpendicular to the flow also increases. As a result, there is a greater drag and a stronger Lorentz force, which lowers the flow velocity of the fluid and dust phases.

The effect of fluid-particle interaction parameters, β on the temperature and velocity profiles for both phases is shown in Fig. 6 and 7. β is set as β=0.2, 0.8, 1.2 and 1.5. As β increases, the fluid phase showed a decreasing trend meanwhile dust phase showed an increasing trend. Increasing β generally causes the fluid's phase velocity to drop due to the drag exerted produced by the opposing behaviour of the dust particle to the flowing stream. Conversely, this force also drags the dust particles along with the flow, increasing their velocity results in equilibrium velocities where both phases move at the same speed.

From Fig. 7, an increasing value of β resulting in increasing heat transfer between the fluid and particle phases due to more frequent collisions and interactions between the phases facilitate thermal energy exchange. The dust particles tend to be hotter than the fluid making them act as heat sinks. This indicates that during their interactions and collisions, they quickly absorb heat from the fluid thus the temperature profile of the fluid will gradually decrease.

Table 1 Comparison result for $f''(0)$, when $\beta = N = 0,\quad \lambda \to \infty$.

M	(Ariffin *et al.,*) [10]	Present results
0.0	-1.0000	-1.0000
0.2	-1.0954	-1.0954
0.5	-1.2247	-1.2247
1.0	-1.4141	-1.4141
1.2	-1.4832	-1.4832
1.5	-1.5811	-1.5811
2.0	-1.7320	-1.7320

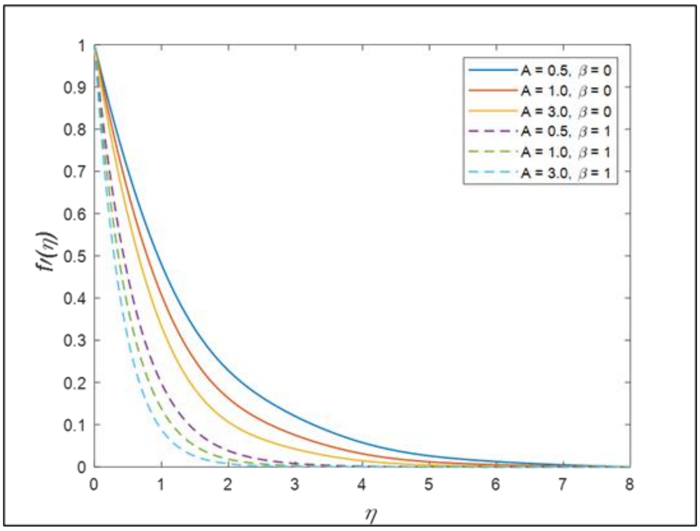

Fig. 2 Velocity profile for several values of A with and without dust particle

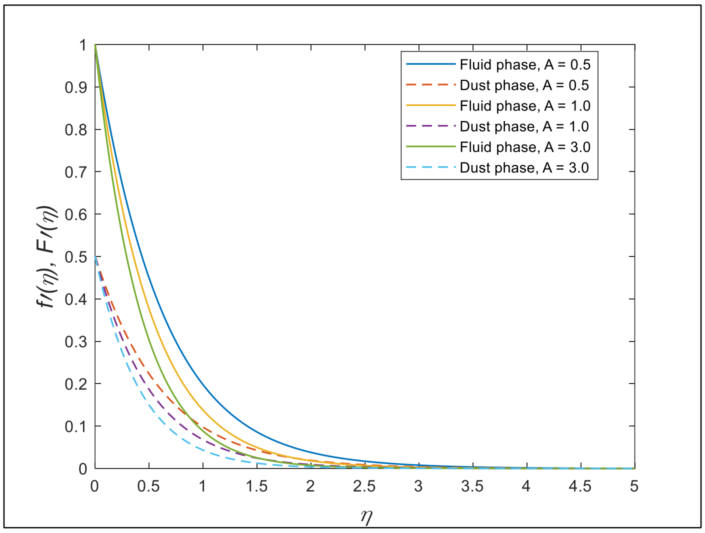

Fig. 3 Velocity profile for several values of A

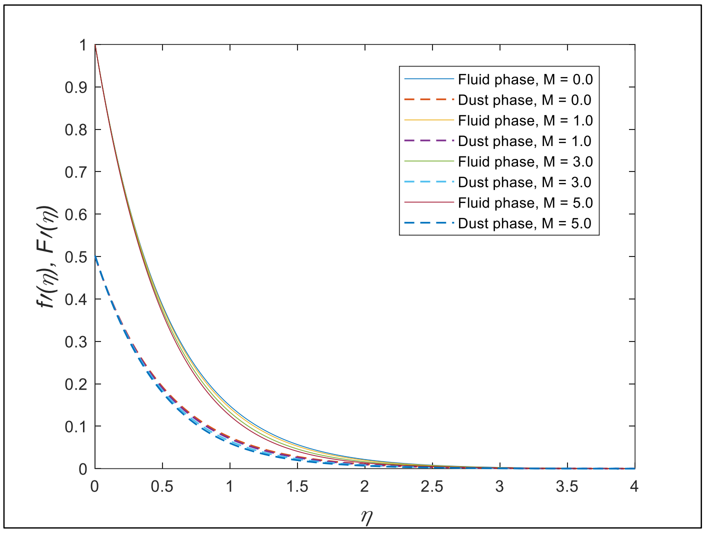

Fig. 4 Velocity profile for different values of M

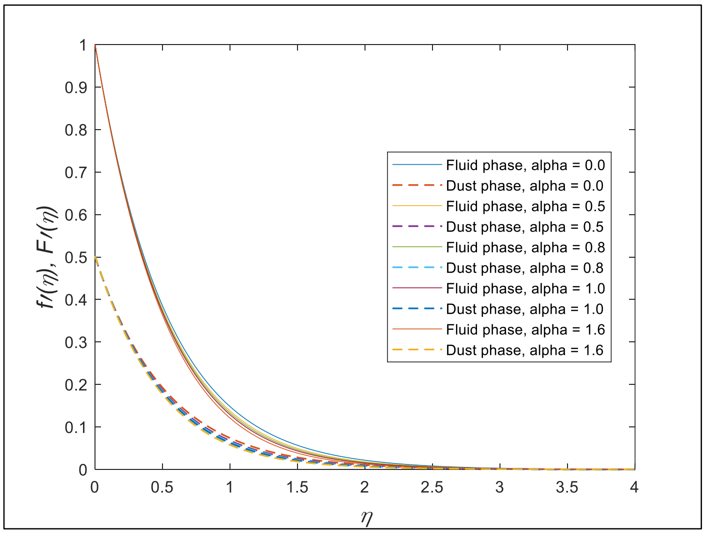

Fig. 5 Velocity profile for different values of $\alpha 1$

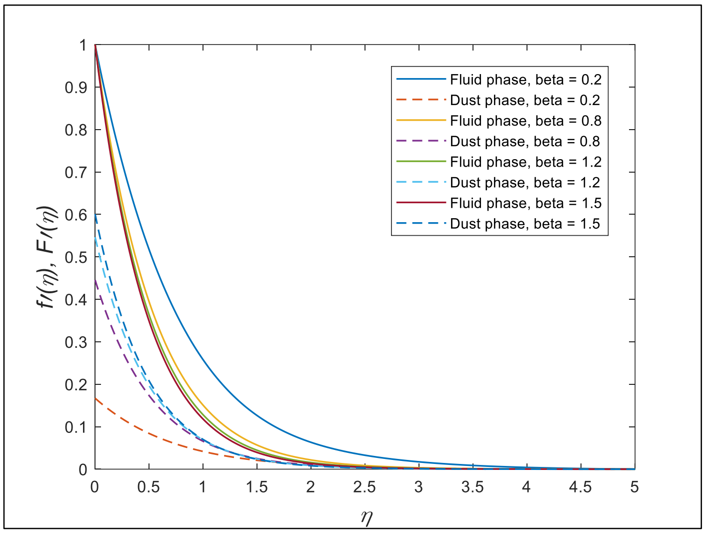

Fig. 6 Velocity profile for different values of β

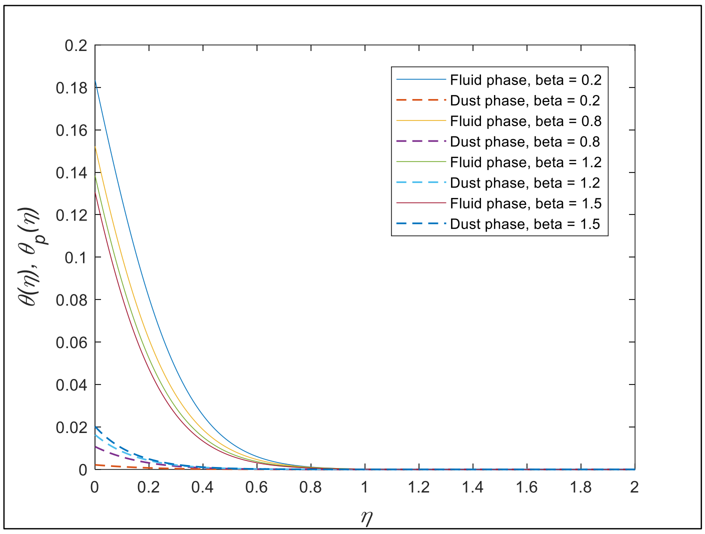

Fig. 7 Temperature profile for different values of β

4. Conclusion

The numerical solution for this problem has been solved by utilizing the bvp4c technique in MATLAB R2022a, we unveiled intricate interactions between these parameters, significantly impacting both the fluid and particle phases. The effects of several parameters were analysed and revealed that the Casson parameter (A) acted as potent brakes, drastically reducing the velocity of fluid and dust phases due to heightened internal friction with dust particles further impeded the flow of the fluid. Besides that, a greater magnetic force (M) and aligned angle (α_1), generating a drag-inducing Lorentz force. Similarly reduced the velocity of both phases. Fluid-particle interaction (β) showed opposing effects, increasing particle velocity while reducing the fluid velocity through drag, which might result in equilibrium velocities at certain values. As β increased, there was an increase in heat transmission because more frequent collisions promoted phase-to-phase thermal exchange. The fluid's temperature profile decreased because of dust particles absorbing heat from the fluid and serving as heat sinks.

It is advised that further research investigates the effects of magnetic fields at different angles in the flow direction to provide a more comprehensive understanding of their influence. Besides, examine the additional flow conditions, such as an unstable flow or viscous or non-viscous flow or analysing the behaviour of the system under turbulent conditions would be relevant for various practical applications.

Acknowledgement

The authors would thank the Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia for its support.

Conflict of Interest

This thesis represents the culmination of independent inquiry, free from any influence that could compromise its scientific merit. Authors declare no conflicts of interest and affirm our commitment to the pursuit of knowledge without bias.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Muhammad Shafirudin Fitri Saidis, Noorzehan Fazahiyah Md Shab;* ***data collection:*** *Muhammad Shafirudin Fitri Saidis;* ***analysis and interpretation of results:*** *Muhammad Shafirudin Fitri Saidis;* ***validation of results****: Noorzehan Fazahiyah Md Shab;* ***draft manuscript preparation:*** *Muhammad Shafirudin Fitri Saidis. All authors reviewed the results and approved the final version of the manuscript.*

References

[1] Casson, N. (1959) A Flow Equation for Pigment-Oil Suspensions of the Printing Ink Type. *Rheology of disperse systems,* 84-104.

[2] Akbar, S., & Khan, M. A. (2014). Effects of particle concentration on the viscosity of dusty Casson fluids. *Journal of Colloid and Interface Science,* 418(1), 237-243.

[3] Nadeem, M., Khan, M. A., & Akbar, S. (2015). Effects of particle size on the yield stress of dusty Casson fluids. *Journal of Rheology,* 59(5), 1173-1186.

[4] Hakeem, A. A., Renuka, P., Ganesh, N. V., Kalaivanan, R., & Ganga, B. (2016). Influence of inclined Lorentz forces on boundary layer flow of Casson fluid over an impermeable stretching sheet with heat transfer. *Journal of Magnetism and Magnetic Materials*, 401, 354-361. https://doi.org/10.1016/j.jmmm.2015.10.026

[5] Kumar, A., & Kumar, R. (2017). Experimental investigation of peristaltic flow of a Casson fluid in an asymmetric channel with a magnetic field. *International Journal of Heat and Mass Transfer*, 114, 1246-1258.

[6] Akbar, Noreen. (2015). Influence of magnetic field on peristaltic flow of a Casson fluid in an asymmetric channel: Application in crude oil refinement. *Journal of Magnetism and Magnetic Materials*. 378. 463-468. https://doi.org/10.1016/j.jmmm.2014.11.045

[7] Siddiqa, S., Begum, N., Hossain, M. A., Shoaib, M., & Gorla, R. S. R. (2018). Radiative heat transfer analysis of non-Newtonian dusty Casson fluid flow along a complex wavy surface. *Numerical Heat Transfer, Part A: Applications,* 73(4), 209-221. https://doi.org/10.1080/10407782.2017.1421741

[8] Das, M., Mahato, R., & Nandkeolyar, R. (2015). Newtonian heating effect on unsteady hydromagnetic Casson fluid flow past a flat plate with heat and mass transfer. *International Journal of Heat and Mass Transfer*, 89, 720-729. https://doi.org/10.1016/j.aej.2015.07.007

[9] Hayat, Tasawar & Khan, M. & Waqas, Muhammed & Alsaedi, A. (2017). Newtonian heating effect in nanofluid flow by a permeable cylinder. *Results in Physics*, 7, 256-262. https://doi.org/10.1016/j.rinp.2016.11.047

[10] Arifin, N. S., Zokri, S. M., Kasim, A. R. M., Salleh, M. Z., Yusoff, W. N. S. W., Mohammad, N. F., & Shafie, S. (2017). Aligned magnetic field on dusty Casson fluid over a stretching sheet with Newtonian heating. *Malaysian Journal of Fundamental and Applied Sciences*, 13(3), pp.245-248. https://doi.org/10.11113/mjfas.v13n3.592