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MHD Hybrid Nanofluid Flow Over a Permeable Surface

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Abstract

This investigation focused on the two-dimensional steady MHD hybrid nanofluid flow over a moving permeable surface. The base fluid is water, whereas the two nanoparticles in the fluid are Molybdenum Disulfide (MoS_2) and Silicon Dioxide (SiO_2). This study transformed the governing equation of hybrid nanofluid flow from partial differential equations (PDEs) to ordinary differential equations (ODEs) by using similarity variables. Shooting method is used to obtain the numerical results using Maple software. The results of this findings depends on the parameter of the hybrid nanofluid such as heat source parameterand radiation parameter. It is found that the temperature and heat transfer rate on the surface of $SiO_2 - MoS_2 / water$ hybrid nanofluid are enhanced by higher radiation parameters. Additionally, an increased heat source parameter signifies heat generation, raising the temperature of the hybrid nanofluid flow in the system.

1. Introduction

The hybrid nanofluid gain better rate of heat transfer, as it has higher thermal conductivity compared to plain fluid. There are many studies investigate about enhance the rate of heat transfer and to improve thermal conductivity of the fluid, and nanoparticles should be outstanding potentials. The nanoparticles in nanofluid significantly heighten the rate of heat transfer due to wide surface area per volume for its size. In return, the fluid may absorb more heat from the higher temperature. Magneto-hydrodynamics (MHD) simulates in several branches of physics, such as magnetoconvection, magnetohydrodynamics turbulence and hydromagnetic dynamo action. Refer on the studies by [1] and [2], MHD is an abbreviation of Magnetohydrodynamics with the word magneto- means magnetic, hydro- means water (or fluid) and -dynamics is addressed as the movement of an object by either external or internal or both forces. Basically, MHD is referred to as a study an electrically conducting fluid dynamics and characterize the bonding between fluid and magnetic field. Examples of fluid are salt water or electrolytes, plasma and liquid metals. Research by [3] stated, the issues related to nanofluids on the plate and heat transfer with magnetic field received various applications in engineering science, including MHD pumps, MHD power generators, heat exchangers, electronics and more.

MHD is the physic-mathematical framework that discusses the magnetic field of its dynamics in conducting the fluid electrical current for instance, liquid metals. The field of research of hybrid nanofluid was first made by [4]. The research stated that different nanofluid contains only unique type of nanoparticle. By owning the unique type of nanoparticle in different type of fluids, the nanofluid can enhance the properties of fluid such as the thermal conductivity, electrical conductivity, density, specific heat capacity, and viscosity. Additionally, [5] stated in the recent study, the nanofluid properties become extra complex as well as effective in some cases due to nanofluids require more convective of heat transmission. The combined presence of more than two subsidiary nanoparticles that are solid in fluid-based is known as hybrid nanofluid, since the nanoparticles are formed via combination two different nanoparticles. Then, the research had been dug by [2]. The research stated that magnetohydrodynamics (MHD) will conduct an electricity when both hybrid nanofluid and magnetic field have their implications. In MHD systems, electrical conductivity plays an important role such as metallic nanoparticles (copper, aurum or 'gold') as they casually have high electrical conductivity and can transfer the heat and electricity with the base of fluid. Unlike the oil and pure water are typically have lower rate of heat transfer. The study stated by [6] that the heat transfer of MHD and natural convection flow in a closed space significantly cooled and heated either with heat source or sink also were supplied with a $Cu-Al_2O_3/water$ in hybrid nanofluid.

The interpretation of magnetic field with heat transfer has always been the primary focus in this investigation as it can be implemented to plenty of things to facilitate daily life less difficult especially in science and engineering. For instance, MHD turbines, control over sand boundary layers and more. The highest consensus of this fluid behaviour can assist and optimize the systems including magnetics fields and nanofluids, such as heat exchangers, cooling systems and energy conversion devices.

Other than that, [7] and [8] have been conducted a study referring an unsteady stretching permeable surface within wall temperature prescribed and eventually shows the properties of heat transfer. The study shows that the stretching permeable surface will affect on the features of the heat transfer and there is a thermal convective and a relative movement of the fluid and its boundary. Nanofluid has numerous types of surfaces with different geometry and orchestrations. Therefore, those were applied and perceived as there are reliable duality of solutions. The duality in MHD system assists researchers to be more precise in order to analyse the studies. Thus, it can interpret the studies with better understanding in the presence of magnetic field and produce the convection of heat transfer. In the meantime, [9] studied the time independent of the heat transfer of the based hybrid nanofluid with different properties of nanoparticles by a non-linear permeable surface, resulting of radiation and the surface of the velocity was examined. The heated fluid will affect the viscosity of fluid as the viscosity of the fluid will low when the temperature of the fluid is high. As a result, the flow of hybrid nanofluid will flow faster.

Furthermore, [10] studied that the shooting methods requires initial value and final value to predict the final point. That initial slope was pursued which results in the trajectory 'hitting' the destination point, that is the final value. Shooting methods can be utilized as problem solving in which it is well investigate in a certain sense and conventional shooting methods are not proper. This study about shooting technique has been discovered and examined that the boundary conditions are divided into two points, the initial and the terminal values of the independent variable.

Therefore, the purpose of this research is to examine the MHD hybrid nanofluid flow past a permeable moving surface. The matter is to understand the mathematical formulation and try to solve the problem by using the shooting technique with Runge-Kutta-Fehlberg (RKF45) method in Maple software. Previous research by [2] used bvp4c solver to solve the same problem. Hence, in this current study, the results comparison with outcome from [2] will be made to confirm the validity of the finding.

2. Mathematical Formulation

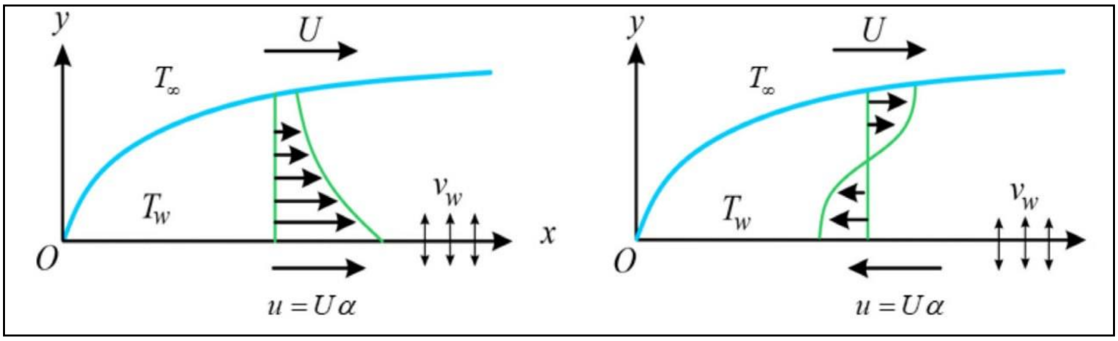


Fig. 1 Physical Configuration Problem by [2]

Based on Fig. 1, v_w shows the permeable moving surface of MHD hybrid nanofluid. In compliance of the physical problem above, the continuity equation, momentum equation and energy equation are as below by [2]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2}+\frac{\sigma_{hnf}B^2}{\rho_{hnf}}\left(u_\infty-u\right)+\frac{(\rho\beta^*)_{hnf}g}{\rho_{hnf}}\left(T-T_\infty\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{hnf}}{(\rho C_p)_{hnf}}\frac{\partial^2 T}{\partial y^2}+\frac{1}{(\rho C_p)_{hnf}}\left(\frac{16\sigma^* T_\infty^3}{3K_{hnf}^*}\right)\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{Q_0^*\left(T-T_\infty\right)}{(\rho C_p)_{hnf}}, \tag{3}$$

and subject to boundary conditions as follows:

$$\begin{aligned}&u=U_\alpha+k'^*\frac{\partial u}{\partial y},\quad v=v_w(x),\quad -k_{hnf}\frac{\partial T}{\partial y}=h^*{}_f\left(T_w-T\right)\ \text{at}\ y=0\\&u=u_\infty\to U,\quad T\to T_\infty\ \text{as}\ y\to\infty\end{aligned} \tag{4}$$

where (u,v) are the components of the flow velocity in the (x,y), whereas these are function of x ($Q_0^*=x^{-1}Q_0$, $h_f^*=x^{-1/2}$, $k'^*=x^{1/2}k'$ and $\beta^*=x^{-1}\beta$). In this study, the subscripts hnf stands for SiO_2-MoS_2/water hybrid nanofluid while f stands for base fluid and nf stands for the nanofluid.

The similarity variables below are used to fulfill transformation equation:

$$u=Uf'(\eta),\quad v=\sqrt{\frac{Uv_f}{2x}}\left(\eta f'(\eta)-f(\eta)\right),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \eta=y\sqrt{\frac{U}{2xv_f}} \tag{5}$$

Eqns. (2) and (3) are transformed by using eqn. (5) respectively as follows:

$$ff''+\frac{1}{AB}f'''+\frac{C}{B}M\left(1-f'\right)+2D\lambda\theta=0, \tag{6}$$

$$\left(\frac{k_{hnf}}{k_f}+\frac{4}{3}R_d\right)\theta''+E\,Pr\,f\,\theta'+2Pr\,Q\theta=0 \tag{7}$$

and the boundary conditions are transformed to the following form:

$$\begin{aligned}&f(\eta)=S,\quad f'(0)=\alpha+\gamma f''(0),\quad -\frac{k_{hnf}}{k_f}\theta'(\eta)=Bi(1-\theta(\eta))\quad at\ \eta=0\\&f'(\eta)\to 1,\quad \theta(\eta)\to 0\quad as\ \eta\to\infty\end{aligned} \tag{8}$$

where M $\left(=\frac{\sigma_f B_0^2}{\rho_f U}\right)$ is magnetic field parameter, Gr_x $\left(=\frac{gx^2\beta_f\left(T_w-T_\infty\right)}{v_f^2}\right)$ is local Grash of number, $\lambda\left(=\frac{Gr_x}{Re_x^2}=\frac{g\beta_f\left(T_w-T_\infty\right)}{U^2}\right)$ is mixed convection parameter, $R_d\left(=\frac{4\sigma^* T_\infty^3}{K_{hnf}^* k_f}\right)$ is thermal radiation parameter, $\Pr\left(=\frac{\mu_f(C_p)_f}{k_f}\right)$ is Prandtl number, $Q\left(=\frac{Q_0}{U(\rho C_p)_f}\right)$ is heat source/sink parameter, $\gamma\left(=k'\sqrt{\frac{U}{2v_f}}\right)$ is velocity slip parameter, $A=\frac{\mu_f}{\mu_{hnf}}$, $B=\frac{\rho_{hnf}}{\rho_f}$, $C=\frac{\sigma_{hnf}}{\sigma_f}$, $D=\frac{\beta_{hnf}}{\beta_f}$ and lastly, $E=\frac{(\rho C_p)_{hnf}}{(\rho C_p)_f}$

Table 1 Thermophysical properties of hybrid nanofluid by [2]

Properties	Hybrid Nanofluid
Dynamic Viscosity	$\mu_{hnf} = \dfrac{\mu_{water}}{\left(1-\varphi_{SiO_2}\right)^{2.5}\left(1-\varphi_{MoS_2}\right)^{2.5}}$
Density	$\rho_{hnf} = \left[\varphi_{MoS_2}\rho_{MoS_2} + \left(1-\varphi_{MoS_2}\right)\left\{\varphi_{SiO_2}\rho_{SiO_2} + \left(1-\varphi_{SiO_2}\right)\rho_{water}\right\}\right]$
Thermal Conductivity	$k_{hnf} = \dfrac{k_{MoS_2} + 2k_{nf} - 2\varphi_{MoS_2}\left(k_{nf} - k_{MoS_2}\right)}{k_{MoS_2} + 2k_{nf} + \varphi_{MoS_2}\left(k_{nf} - k_{SiO_2}\right)} \times k_{nf}$
	$where\ k_{nf} = \dfrac{\left(2k_{water} + k_{SiO_2}\right) - 2\varphi_{SiO_2}\left(k_{water} - k_{SiO_2}\right)}{\left(2k_{water} + k_{SiO_2}\right) + \varphi_{SiO_2}\left(k_{water} - k_{SiO_2}\right)} \times k_{water}$
Electrical Conductivity	$\sigma_{hnf} = \sigma_{nf}\left[\dfrac{\sigma_{MoS_2}\left(1+2\varphi_{MoS_2}\right) + 2\sigma_{nf}\left(1-\varphi_{MoS_2}\right)}{\sigma_{MoS_2}\left(1-\varphi_{MoS_2}\right) + \sigma_{nf}\left(2+\varphi_{MoS_2}\right)}\right]$
	$where\ \sigma_{nf} = \sigma_{water}\left[\dfrac{\sigma_{SiO_2}\left(1+2\varphi_{SiO_2}\right) + 2\sigma_{water}\left(1-\varphi_{SiO_2}\right)}{\sigma_{SiO_2}\left(1-\varphi_{SiO_2}\right) + \sigma_{water}\left(2+\varphi_{SiO_2}\right)}\right]$
Thermal expansion coefficient	$\beta_{hnf} = \left[\varphi_{MoS_2}\beta_{MoS_2} + \left(1-\varphi_{MoS_2}\right)\left\{\varphi_{SiO_2}\beta_{SiO_2} + \left(1-\varphi_{SiO_2}\right)\beta_{water}\right\}\right]$
Heat Capacitance	$\left(\rho C_p\right)_{hnf} = \left[\left(1-\varphi_{MoS_2}\right)\left\{\varphi_{SiO_2}\left(\rho C_p\right)_{SiO_2} + \left(1-\varphi_{SiO_2}\right)\left(\rho C_p\right)_{water}\right\} + \varphi_{MoS_2}\left(\rho C_p\right)_{MoS_2}\right]$

The physical quantity of interest in this investigation are the coefficient for skin friction (Cf) and the local Nusselt number (Nux), which are elaborated as [2]:

$$C_f = \frac{\tau_w}{\rho_f U^2} \quad \text{and} \quad Nu_x = \frac{x\left(q_w + q_r\right)}{k_f\left(T_w - T_\infty\right)}, \tag{9}$$

where the sum of heat flux on the surface and radiative heat flux $\left(q_w + q_r\right)$ and shear stress of the surface τ_w are defined as below:

$$q_w + q_r = -\left(k_{hnf}\frac{\partial T}{\partial y} + \frac{4\sigma^*}{3K^*_{hnf}}\frac{\partial T^4}{\partial y}\right)\Bigg|_{y=0} \quad \text{and} \quad \tau_w = \mu_{hnf}\left(\frac{\partial u}{\partial y}\right)\Bigg|_{y=0}. \tag{10}$$

By using equation (9) and (10), the quantities can be described in equation (11) below:

$$\sqrt{Re_x}\, C_f = \frac{1}{\sqrt{2}A} f''(0), \quad \frac{Nu_x}{\sqrt{Re_x}} = -\frac{1}{\sqrt{2}}\left\{\left(\frac{k_{hnf}}{k_f} + \frac{4}{3}R_d\right)\right\}\theta'(0), \tag{11}$$

which $\sqrt{Re_x} = \dfrac{U_x}{v_f}$, is local Reynolds number.

3. Results and Discussion

The ordinary differential equations with boundary conditions are solved numerically by using shooting technique with RKF45 in Maple software. This study choose assisting case ($\lambda = 1$) to observe. Furthermore, the variety of parameters such as suction/injection parameter, thermal radiation parameter and hybrid nanofluid flow impacts in the current model are studied.

The results comparison between this study and results of recent study by [2] are reviewed. Table 1 below shows the comparison of the values of skin fraction coefficient, ($\sqrt{Re_x}C_f$) and heat transfer coefficient, ($(Re_x)^{-0.5}Nu_x$) for the hybrid nanofluid of $SiO_2 - MoS_2$/ water.

Table 2 Comparison values of skin friction and heat transfer coefficient

S	λ	Results by Yaseen et al. [2]		Present Results	
		$\sqrt{Re_x}C_f$	$(Re_x)^{-0.5}Nu_x$	$\sqrt{Re_x}C_f$	$(Re_x)^{-0.5}Nu_x$
1	-1	-0.024956446	0.465692735	-0.024956446	0.465692735
	0.2	0.004976567	0.465739898	0.004976567	0.465739898
	1.4	0.034739308	0.465787061	0.034739308	0.465787061
-1	-1	-0.221132672	0.304844218	-0.221132672	0.304844218
	0.2	0.039495041	0.319642837	0.039495041	0.319642837
	1.4	0.254945519	0.329486748	0.254945519	0.329486748

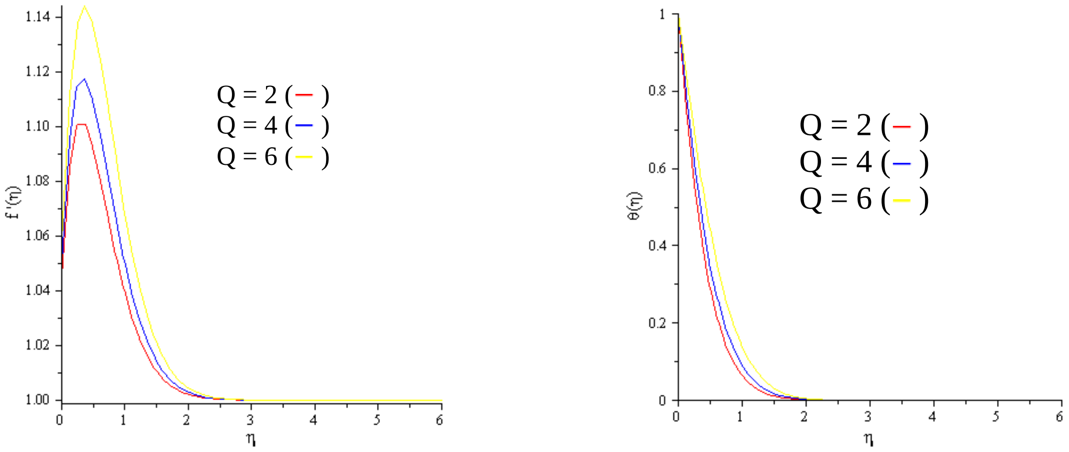
Table 1 shows that S = 1 is the case of suction while S = — 1 refers to the case of injection. These two types of cases influencing the characteristic of fluid, this is why the different value of S would control the types of fluids. For example, the values of $\sqrt{Re_x}C_f$ and $(Re_x)^{-0.5}Nu_x$ are depending on the values of the mixed convection parameter, (λ) and the suction and injection cases (S).

Regarding the observation on the table above, it shows that the higher value of λ, the bigger values of both coefficients. However, it is clearly seen in the table that (S = — 1) got higher for both values of coefficient rather than (S = 1) is applied. This is because the suction condition makes the fluid goes to the source of energy such as magnetic field whereas injection is adding the new substances to the fluid such as external forces. We can observe that the comparison results from the table shows both values of coefficient are in excellent agreement.

As indicated by [2] the values of parameters are fixed as follows:

$$\varphi_{SiO_2} = 0.01, \varphi_{MoS_2} = 0.01, S = 1, M = 2, Pr = 6.2, Q = -0.1, R_d = 2, \lambda = 1, \gamma = 0.1, \text{ and } \alpha = 1 \qquad (12)$$

Figure 2 (a), (b) below show the influence of heat source/sink parameter, Q on velocity profile ($f'(\eta)$) and temperature profile ($\theta(\eta)$). The positive values of Q are chosen and the rest are same and fixed as mentioned in (12).

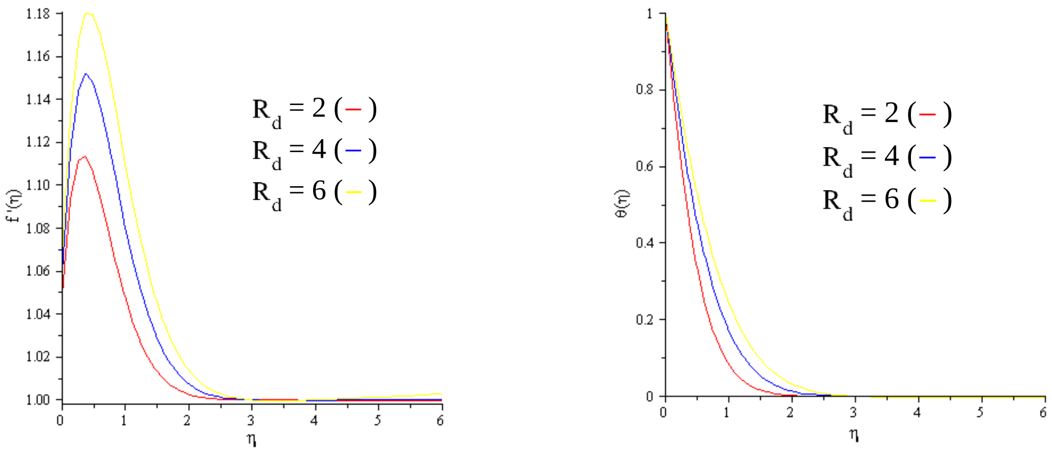


(a) **(b)**

Fig. 2 (a) Impacts of Q in velocity profile; (b) Impacts of Q in temperature profile

A positive heat source parameter (Q) value relates to heat generation in the flow field and indicates that heat is produced within the system during the flow. As the parameter (Q) value increases, the velocity and temperature are observed to rise. The positive increase in the Q parameter signifies heat generation, and this generated heat raises the temperature of the flow of hybrid nanofluid within the system.

The influenced of radiation, Rd velocity profile ($f'(\eta)$) and temperature profile ($\theta(\eta)$) present on Fig. 3 (a), (b) below. The values of Rd are chosen to be manipulated and the rest are similar and fixed as mentioned in (12).



(a) **(b)**

Fig. 3 (a) Impacts of Rd in velocity profile; (b) Impacts of Rd in temperature profile

The obtained results elucidate that in the case of assisting flow ($\lambda = 1$), the velocity increases in the radiation parameter. An increasing radiation parameter causes temperature to rise, showing radiation's dominance over conduction in hybrid nanofluid flow. The heightened radiation results in increased heat transfer in the boundary layer, leading to elevated temperature in the hybrid nanofluid.

4. Conclusion

This study explores the magneto-hydrodynamics (MHD) in hybrid nanofluid flow in a permeable moving surface by using shooting technique by Runge-Kutta Method (RKF45) method. Assisting flow is considered for heat source parameter and thermal radiation. The objectives of this study have been achieved successfully. The first objective of this study is to transform the governing equation from partial differential equation (PDE) to ordinary differential equation (ODE) by using similarity variables has been achieved. Another ordinary differential equation (ODE) by using similarity variables. Another goals that had been achieved in this research is solving the modified governing equation PDE to ODE by applying the shooting method with RKF45 by using Maple software. On top of that, the outcome various parameters such as heat source, thermal radiation parameter and suction/injection parameter on the temperature and velocity profiles on heat transfer and MHD hybrid nanofluid flow have been analyzed in this study. It is found that the temperature and heat transfer rate of the surface in the $SiO_2 - MoS_2/\text{water}$ hybrid nanofluid flow are boosted by the increased radiation parameter. Moreover, a higher heat source parameter indicates heat generation, elevating the temperature of the hybrid nanofluid within the system.

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Conflict Interest

Authors declare that there is no conflict of interest regarding the publication of the paper

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design****: Auna Mohd Azhar;* ***solve the governing equation****: Auna Mohd Azhar;* ***data collection****: Auna Mohd Azhar;* ***analysis and interpretation of results****: Auna Mohd Azhar, Fazlina Aman;* ***validation of results****: Auna Mohd Azhar, Fazlina Aman;* ***draft manuscript preparation****: Auna Mohd Azhar. All authors reviewed the results and approved the final version pf the manuscript.*

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