

MHD Flow of Casson Fluid Over a Porous Medium Stretching Sheet

Ng Yee Xuan¹, Noorzehan Fazahiyah Md Shab^{1*}

¹ Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology, UTHM Kampus Cawangan Pagoh, Hab Pendidikan Tinggi Pagoh, KM 1, Jalan Panchor, 84600 Pagoh, Muar, Johor, MALAYSIA

*Corresponding Author: fazahiya@uthm.edu.my

DOI: <https://doi.org/10.30880/ekst.2024.04.02.018>

Article Info

Received: 27 December 2023

Accepted: 11 January 2024

Available online: 12 December 2024

Keywords

Magnetohydrodynamic, Casson Fluid Flow, Porous Medium, Stretching Sheet

Abstract

This study investigates the magnetohydrodynamic (MHD) flow of a Casson fluid through a porous media stretching sheet numerically. The Casson fluid model is considered to describe the non-Newtonian behaviour. The system of nonlinear ordinary differential equations is generated from the governing partial differential equations of momentum and energy conservation using the Runge-Kutta method with a shooting method. The study aims to investigate the impacts of the magnetic field parameter M , Casson fluid parameter, Prandtl number Pr , and porous medium parameter K on the velocity and thermal field profile. The purpose of this research is to analyse the properties of temperature and velocity fields by demonstrating the consequences of magnetohydrodynamics (MHD) flow of Casson fluid over a porous medium stretching sheet an increase in the magnetic parameter M values results in a decreased momentum boundary layer thickness and velocity, whereas an increase Casson fluid parameter γ values increases momentum boundary layer thickness and velocity. Furthermore, an increase in the porosity parameter K reduces the velocity profile. Greater magnetic and porous media properties correlate to higher temperature profiles. Furthermore, an increase Casson parameter γ may result in a bigger thermal boundary layer with increased thermal resistance.

1. Introduction

MHD stands for magneto-hydrodynamics is the study of magnetic properties and behaviour of electrically conducting fluid. In other words, it can be explained by the interaction of the motion of electrically conduction fluids with magnetic fields [1]. The word magneto-hydrodynamics is the combination of three words which are magneto, hydro, and dynamics. The examples of MHD are plasma, liquid metal, salt water and electrolytes. For instance, when a plasma fluid moves in the presence of a magnetic field, the charged particles in the fluid interact with the magnetic field and experience a force. These forces will produce electrical currents and result in the fluid to flow. MHD equation can be described by a set of equations that combines the fluid dynamics Navier-Stokes equations with the electromagnetism Maxwell equations. To understand MHD, these differential equations need to be solved concurrently, either through analytical or numerical methods [2]. The applications of MHD fluid can be found in geophysics, astrophysics, engineering, and so on. The Earth's magnetic field is generated by the fluid core of the Earth which is MHD fluid as a result of the motion of the molten rock. In the area of engineering, a boat was built by Mitsubishi which uses a magnetohydrodynamics drive, liquid helium-cooled superconductor was used to drive it. Casson fluid is a non-Newtonian fluid introduced by Casson in 1959. Yield stress is present in Casson fluid which there is a shear-thinning liquid with infinite viscosity at zero shear rate. The Casson fluid will

This is an open access article under the CC BY-NC-SA 4.0 license.



behave like a solid if the shear stress is lower than the yield stress [3]. However, if the shear stress is greater than the yield stress, the fluid will start to flow. Jelly, salt solutions, ketchup, paints, and shampoo are examples of Casson fluid. In short, Casson fluids are characterized by a nonlinear relationship between the shear stress and the shear rate, which means that their viscosity changes depending on the intensity of the applied force. Casson fluids are important in many industrial and biomedical applications because of their complex and nonlinear behaviour, which can be tailored to achieve specific desired properties [4]. A porous medium is a substance that has channels, voids, or interconnecting pores that allow the fluid or gases to pass through [5] The porous medium can be created by engineering materials such as ceramics, membranes, and foams or can be naturally occurring such as rocks, soils and sediments. Casson fluid with variable viscosity of Casson fluid over a stretching sheet in porous media with Newtonian heating is investigated [6]. The finite-difference method has been used to apply in this study after the partial differential equations which represent the flow motion are transformed into ordinary differential equations by a similarity transformation. MHD flow over an exponentially stretching sheet through a porous medium with heat source is studied [7] A fourth-order Runge-Kutta method with a shooting technique is used to numerically solve the governing nonlinear partial differential equations after being converted into a system of coupled nonlinear ordinary differential equations

2. Mathematical Formulation

The boundary layer of the continuous two-dimensional magnetohydrodynamic (MHD) flow of Casson fluid over porous medium stretching sheet is analysed in this study. The diagram illustrates the complex relationships of continuous two-dimensional magnetohydrodynamic flow over a stretching sheet within a porous medium, including Casson fluid behaviour, magnetic field influence, thermal boundary layer properties, and velocity field dynamics.

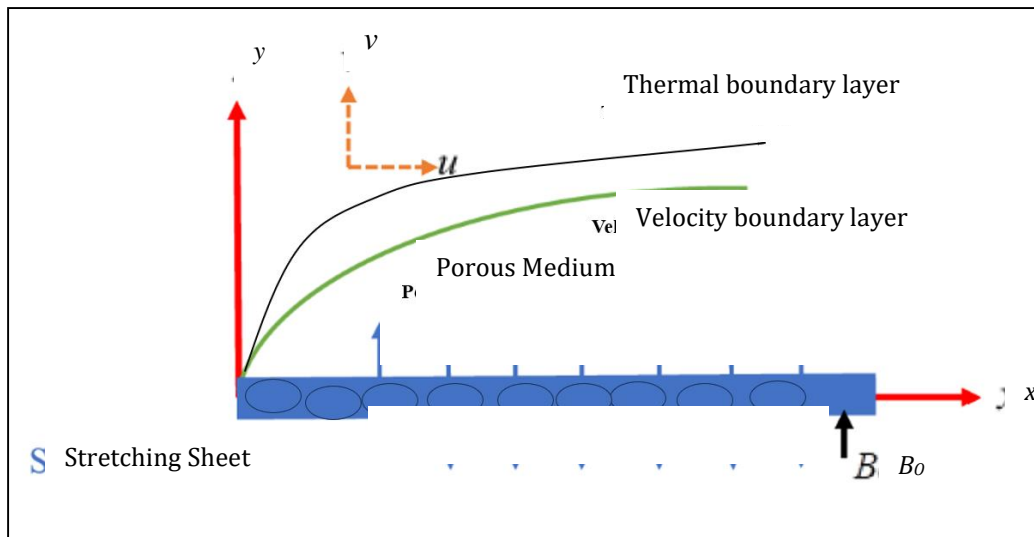


Fig. 1 Sketch of the physical model

It is assumed that U_w is the stretching velocity, and that u and v are the velocity components in the directions of x and y as measured along and normal to the sheet in cartesian coordinates, and that B_0 is the application of the magnetic field. The following equations are the two-dimensional flow expressions that fit the theory of MHD flow of Casson fluid over porous medium stretching sheet by Siddiqui & Shankar [8] which investigate the MHD Flow and Heat Transfer of Casson Nanofluid through a Porous Media over a Stretching Sheet.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \left(1 + \frac{1}{\gamma} \right) \frac{\partial^2 u}{\partial y^2} - \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{K_0} \right) u, \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

With boundary conditions

$$\begin{aligned}
 u &= u_w = bx, v = 0 \\
 T &= T_w = T_\infty + A \left(\frac{x}{l}\right)^2 \quad \text{at } y=0 \\
 u &\rightarrow 0, T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty
 \end{aligned} \tag{4}$$

where u and v are velocity components in the x and y direction respectively. ν is the kinematic viscosity, ρ is the Casson fluid density, σ is the electrical conducting of the fluid, α is the thermal diffusivity, T is the temperature and k_0 is the permeability of the porous medium. $\gamma = \mu_B \sqrt{\frac{2\pi_C}{\rho_y}}$, is the parameter of Casson fluid. μ_B considers as the plastic dynamic viscosity of the non-Newtonian fluid, π_C is a critical value of this product based on non-Newtonian model and ρ_y is the yield stress of the fluid.

$$u = \frac{\partial \psi}{\partial y}, v = \frac{\partial \psi}{\partial x}, \text{ and } \theta(n) = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

With the substitution of

$$n = y \sqrt{\frac{b}{\nu}} \tag{6}$$

$$\psi = x \sqrt{b\nu} f(n) \tag{7}$$

$$\theta(n) = \frac{T - T_\infty}{T_w - T_\infty} \tag{8}$$

After substitution, the equation is transformed to

$$\left(1 + \frac{1}{\gamma}\right) (f''') + ff'' - f'^2 - f'[M + K] = 0 \tag{9}$$

where M is magnetic parameter and K is the porosity parameter

The boundary condition are show as below

$$\begin{aligned}
 f' &= 1, f = 0, \theta = 1 \quad \text{at } \eta = 0 \\
 f &\rightarrow 0, \theta \rightarrow 0 \quad \text{as } \eta \rightarrow \infty
 \end{aligned} \tag{10}$$

3. Results and Discussion

Shooting method with RKF45 with the help of Maple 19 is used to analyze the result. This method enables to achieve the numerical solutions of the governing ordinary differential equation with the boundary condition. Physical factors such as velocity, thermal fields, skin friction, and the Prandtl number Pr will be affected by the magnetic field parameter M , Casson fluid parameter γ , and porosity parameter K . All of the parameters and numerical values are selected according to Siddiqui & Shankar [6]. Fig. 2 – Fig. 7 show a graphic representation of the numerical results.

Fig. 2 shows the velocity profiles $f'(\eta)$ for various values of M with values $\gamma = 1$, $K = 0.5$, and $Pr = 6.2$. The various parameters for M are set as $M = 0$, $M = 1$, $M = 2$, $M = 3$: The momentum boundary layer thickness decreases with an increase in the magnetic parameter M . The strengthening of Lorentz forces with increasing M is the cause for this phenomenon. The magnetic field-induced Lorentz forces work to oppose the fluid's motion, which decreases its velocity close to the surface. It gets harder for the fluid to get through viscous effects as well as the boundary layer as the velocity decreases.

Fig. 3 shows the velocity profile $f'(\eta)$ for various values of γ which $M = 1$, $K = 0.5$, and $Pr = 6.2$ with $\gamma = 0.5$, $\gamma = 1$, $\gamma = 2$, and $\gamma = 5$. Both the boundary layer velocity and thickness decrease with an increase in the Casson fluid parameter, which in non-Newtonian Casson fluids indicates a higher yield stress. The fluid close to the surface finds it more difficult to start moving because of the higher yield stress. This causes the boundary layer's velocity to decrease and its gradient to steepen, which eventually results in a thinner boundary layer.

Fig. 4 shows the cause of velocity profile with various K value such as $K = 0$, $K = 0.3$, $K = 0.7$ and $K = 1$ while the $Pr = 6.2$, $M = 1$, $\gamma = 1$. The velocity profile decreases as the porosity parameter, K increased. A lower velocity profile results from an increase in the porosity parameter, K , in a porous medium because of the increased resistance that the porous structure presents. Fluid flow is impeded by more solid obstacles in the medium as K

increases, forcing the fluid to work its way around them. Higher velocities close to the inlet gradually decrease as the fluid moves deeper into the medium, resulting in a non-uniform flow profile and decreased fluid velocity as a result of the higher-pressure gradient caused by the increased resistance.

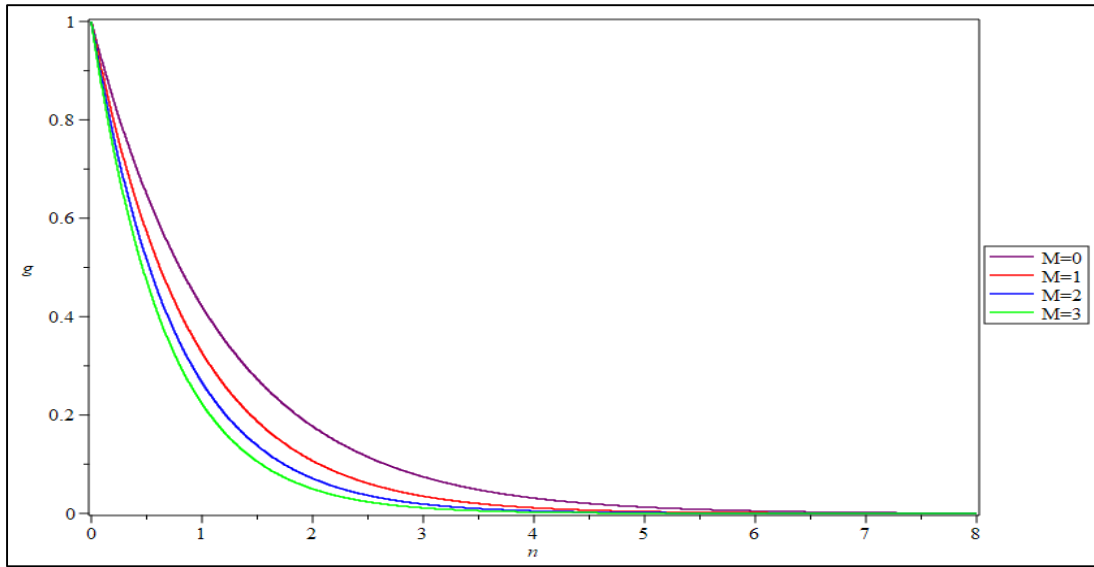


Fig. 2 The velocity profiles $f'(\eta)$ for various values of M

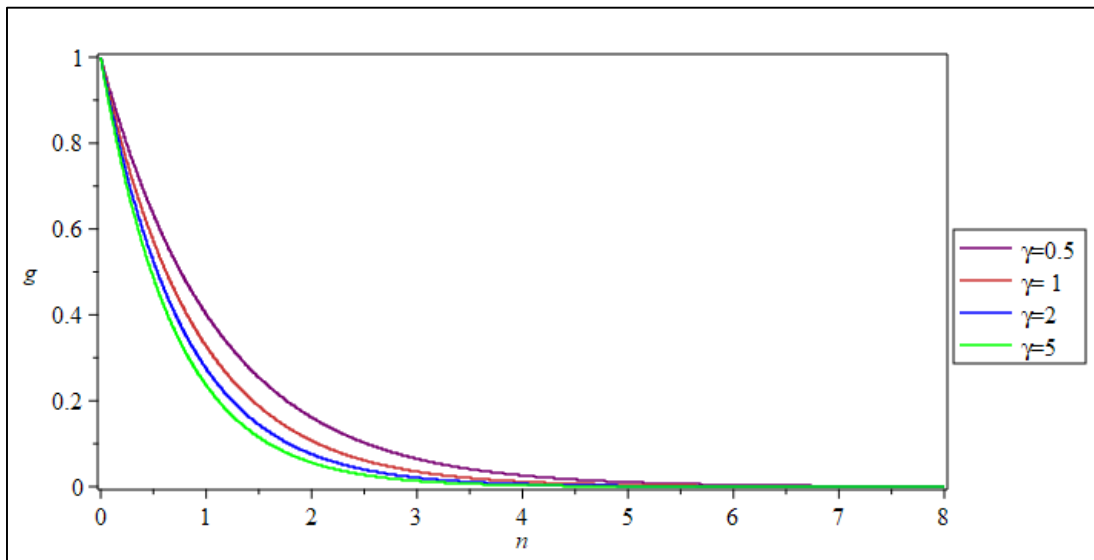


Fig. 3 The velocity profiles $f'(\eta)$ for various values of γ

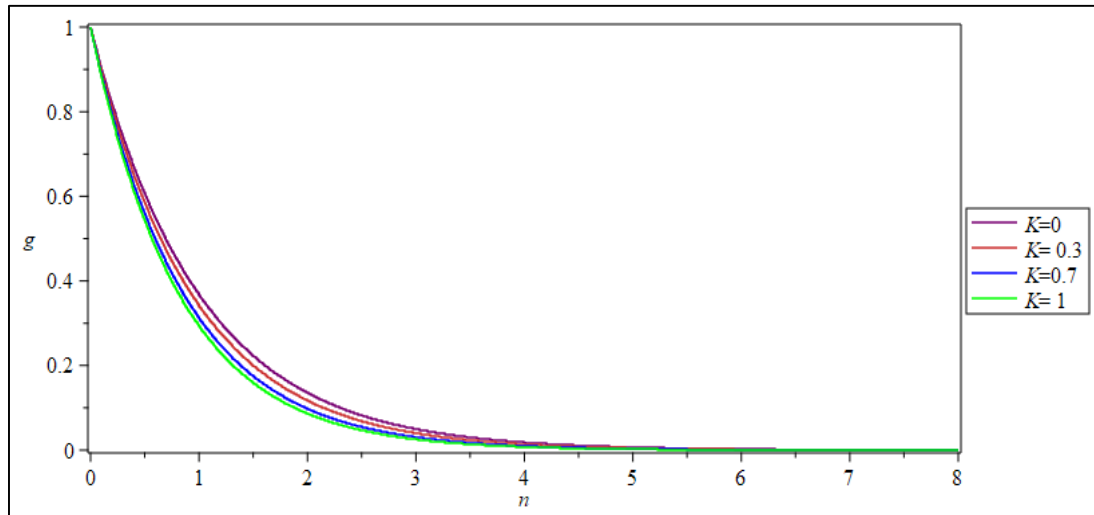


Fig. 4 Velocity profile $f'(n)$ for various values of K

Fig. 5 shows the effect of magnetic field parameter M on the temperature profile with $M = 0, M = 1, M = 2$ and $M = 3$ as the $K = 0.5, \gamma = 1$ and $Pr = 6.2$. Velocity is slowed down by the magnetic parameter while temperature profiles rise all over the surface area. The Lorentz force, which generates the force opposing fluid motion, rises as the magnetic parameter increases. The velocity in the boundary layer decreases as the Lorentz force increases.

Fig. 6 shows the effect of the temperature profile for various of K with $K = 0, K = 0.3, K = 0.7, K = 1$ as the $M = 1, \gamma = 1$ and $Pr = 6.2$. As the porous medium increases, the volume of empty spaces will increase. This leads to better heat transmission between the fluid and the solid matrix by facilitating greater fluid-solid interaction. Higher fluid temperatures and quicker heat transmission are caused by the greater porosity, which lowers thermal resistance and shortens the amount of time heat must pass through the solid material before reaching the fluid.

Fig. 7 present the impact of the temperature profile for various values of γ with $\gamma = 0.5, \gamma = 1, \gamma = 2$ and $\gamma = 5$ as the $M = 1, K = 0.5$ and $Pr = 6.2$. Increasing the Casson parameter γ might result in a larger thermal boundary layer with increased thermal resistance. This is because slower heat transmission occurs because of the Casson fluid's higher flow resistance. A greater yield stress, less convection, and lower fluid mobility as the raised Casson parameter.

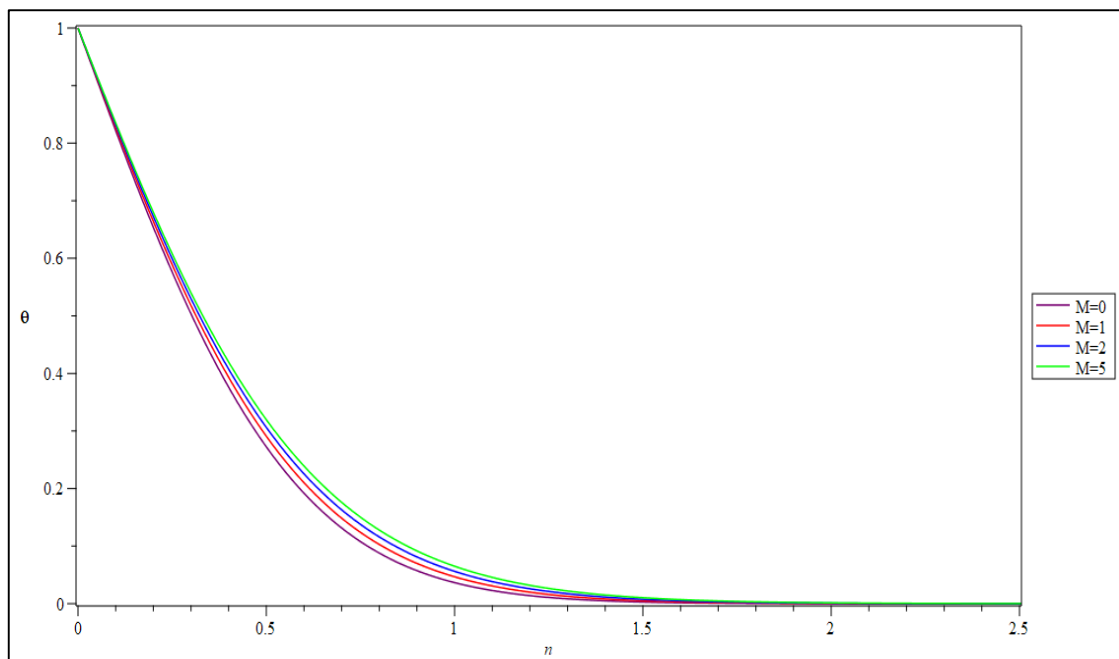


Fig. 5 Temperature profiles $\theta(n)$ for various values of M

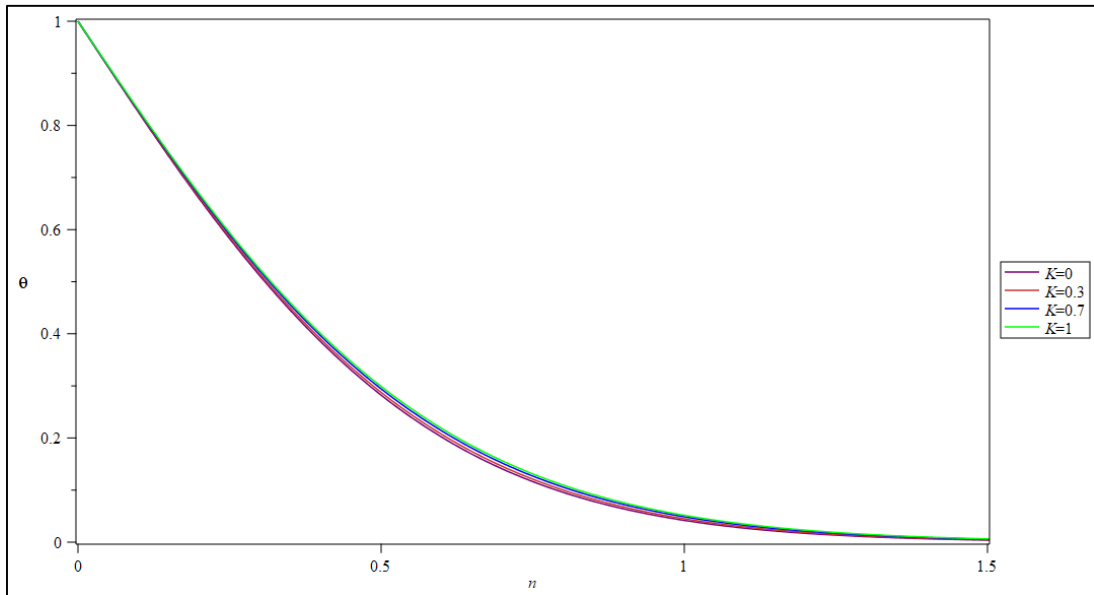


Fig. 6 Temperature profile $\theta (n)$ for various values of K

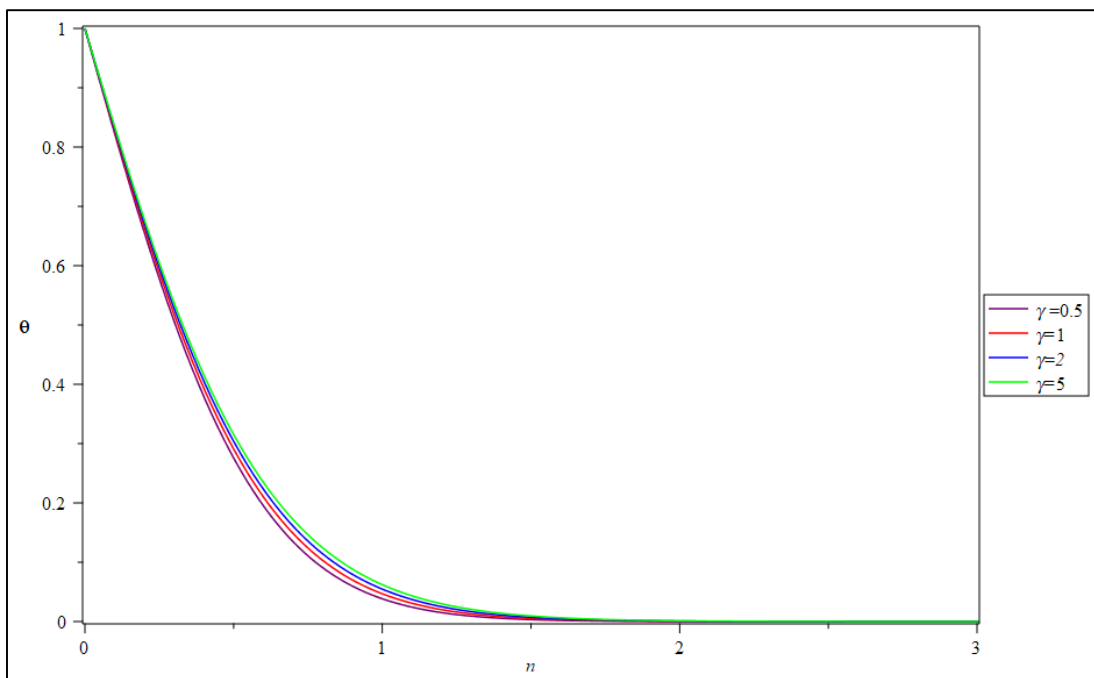


Fig. 7 Temperature profile $\theta (n)$ for various values of γ

4. Conclusion

The purpose of this research is to analyse the properties of temperature and velocity fields by demonstrating the consequences of magnetohydrodynamics (MHD) flow of Casson fluid over a porous medium stretching sheet. The problem was effectively solved by using Maple19 in. using a quantitative technique that made use of the RK45 shooting method. The objective of this research is to get numerical solutions for skin friction, velocity, thermal fields. On top of that, the research aims to investigate the effects of the Casson fluid parameter, and magnetic field parameter.

From the result obtained it can be concluded as:

- When the magnetic parameter, M increases, the momentum boundary layer thickness decreases as well as velocity.
- When the Casson fluid parameter γ increases, the boundary layer thickness and velocity decrease.

- Increasing the porosity parameter, K will lower the velocity profile.
- When the magnetic parameter M increases, the temperature profile also increases as well as the velocity in the boundary layer decreases.
- The rise of the porous medium parameter K will lead to higher temperatures of the fluid and boundary layer.
- Increasing the Casson parameter γ might result in a larger thermal boundary layer with increased thermal resistance.

The study's limitations arise from its exclusive use of the steady-state assumption, impacting accuracy. Recommendations include exploring unsteady or non-viscous flow scenarios and addressing the study's two-dimensional focus, urging further research into three-dimensional variables for improved accuracy. To enhance the model, validation with different fluids like dusty fluid or nanofluid is proposed, offering a more comprehensive understanding of Casson fluid flow through porous materials. The study is also constrained by specific parameters such as Casson fluid, magnetic field, porosity, and Prandtl number, highlighting areas for potential refinement and expansion in future research.

Acknowledgement

The authors would also like to thank the Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia for its support.

Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: **study conception and design:** NYX, NFMS; **data collection:** NFMS; **analysis and interpretation of results:** NYX, NFMS; **draft manuscript preparation:** NFMS. All authors reviewed the results and approved the final version of the manuscript.

References

- [1] Hunt, J. *Some aspects of magnetohydrodynamics*. Ph.D. Thesis. University of Cambridge;1967
- [2] Andersson, H. I. (1995). An exact solution of the Navier-Stokes equations for magnetohydrodynamic flow. *Acta Mechanica* 113(1-4),pp 241–244.
- [3] Ali, A., Farooq, H., Abbas, Z., Bukhari, Z., & Fatima, A. (2020). Impact of Lorentz force on the pulsatile flow of a non-Newtonian Casson fluid in a constricted channel using Darcy's law: a numerical study. *Scientific Reports*, 10(1), pp 10629.
- [4] Pramanik, S. (2014). Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation. *Ain Shams Engineering Journal*, 5(1), pp 205–212.
- [5] Mohamad, A. A. (2005, January 1). *11 - combustion in porous media: fundamentals and applications* (D. B. Ingham & I. Pop, Eds.). *Transport Phenomena in Porous Media III*, Page: 287-304
- [6] Ahmad, K., Halim, S. A., & Hanouf, Z. (2018). Variable Viscosity of Casson Fluid Flow Over A Stretching Sheet in Porous Media with Newtonian Heating. *Journal of Informatics and Mathematical Sciences*, 10(1-2),pp 359–370
- [7] Swain, I., Mishra, S., & Hadibandhu Pattanayak. (2015). Flow over Exponentially Stretching Sheet through Porous Medium with Heat Source/Sink. *Journal of Engineering*, 2015(2015), 1–7.
- [8] Siddiqui, A., & Shankar, B. . *MHD Flow and Heat Transfer of Casson Nanofluid through a Porous Media over a Stretching Sheet*. Sheikholeslami Kandelousi, M., Ameen, S., Shaheer Akhtar, M., & Shin, H.-S. (Eds.). *Nanofluid Flow in Porous Media*.8;2020