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Solving MHD Hybrid Nanofluid Flow Over a Shrinking Sheet with Quadratic Velocity using Shooting Technique

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Abstract

This research presents analysis on the steady MHD flow of an incompressible, electrically conducting Al_2O_3-Cu/H_2O towards a two-dimensional stretching/shrinking sheet with quadratic velocity. The base fluid is water, whereas the two nanoparticles in the fluid are alumina and copper. A similarity transformation is used to convert the governing equation from partial differential equations to ordinary differential equations using a set of similarity variables, and then the solutions are obtained using the shooting technique with RKF45 method in Maple software. The results of these findings dependents on the different value of parameters such as volume fraction nanoparticles, magnetic parameters, and transpiration parameters. It is found that the skin friction coefficient increases as the volume of Cu nanoparticle magnetic parameter and transpiration parameter increase.

1. Introduction

Nanofluid is a fluid that contains suspended nanoparticles ranging in size from 1 to 100 nanometers. When these nanoparticles are introduced to a base fluid, the viscosity and thermal conductivity of the fluid alter. [1] at the American Argonne National Laboratory was the first that introduced the concept of nanofluid. This fluid has new features that are particularly useful in a variety of applications requiring heat transfer phenomena, such as hybrid powered engines, automotive thermal management, solar water heating, heat exchangers, nuclear system cooling, freezers, and microelectronics. The extension of nanofluid that consists of two separate nanoparticles dispersed in the primary fluid is called hybrid nanofluid. The behavior of these fluids at changing flow conditions is investigated in the research of hybrid nanofluid flow. Researchers have recently looked at hybrid nanofluids, which are made by combining many nanoparticles with the cutting fluid in order to make use of the tribological principles of each addition.

Magnetohydrodynamics (MHD) is the study of the dynamics of electrically conducting materials. It consists of three words magneto, hydro, and dynamics, which stand for magnetic effects, water, and movement. According to [2] MHD is the study of electrically conducting fluids in the presence of a magnetic field. Most of the studies on electrically conducting flows demonstrate the addition of a magnetic field considerably changes their heat transfer characteristics. Power generators, nuclear reactors, and heat exchangers are a few real-world examples of fluid movement in a magnetic field. In addition, [3] studied the MHD focuses on how an electrically conducting fluid moves in a magnetic field, which may be used to regulate how the system transfer's heat. Theoretically, magnetic fields may cause a drag force called the Lorentz force to act on a fluid in motion, delaying the flow and raising the fluid's temperature and concentration. However, the addition of the magnetic field effectively delays the boundary layer's separation. A sampling of the numerous sectors that employ MHD include metal casting, crystal growth, adjustable optical fibre filters, optical grafting, MHD generators, and the polymer industry, including the metallurgical process and stretching of plastic sheets [4-6].

Moreover, [7] investigated the deduction of a quadratic velocity field and its application to rolling force of extra-thick plate. In this research, the kinematically admissible criterion is met by constructing a quadratic velocity field. Using the programme ANSYS/LS-DYNA, the change law of metal flow in the deformation zone is examined to verify the accuracy of the suggested velocity field. A three-dimensional rolling force model for extra-thick plate is created using this velocity field as the foundation. In the analysis, the issue of integral difficulty brought on by the nonlinear Mises yield criteria is resolved, and the internal power of deformation is calculated using the inner product approach and cumulative summation of vector components.

Aside from that, due to its widespread use, the study of boundary layer flows caused by stretching or shrinking surfaces has been extensively investigated. The flow and heat transfer characteristics produced by stretching and contracting surfaces are also frequently used in engineering processes like lamination, melt-spinning, continuous casting, spinning of fibres, aerodynamic extrusion of plastic sheets, material-handling conveyors, and condensation techniques. The first researcher who established the distinction between the boundary layer on a surface of limited length and a solid surface was [8]. In order to solve the two-dimensional Navier-Stokes equations beyond a stretched sheet, [9] expanded the work and found a precise solution. In research on the viscous flow caused by a decreasing sheet, [10] discovered the precise solutions, both in numerical and closed form. Additionally, they note that in order to maintain the flow behaviour, there must be enough suction on the surface. Thus, when comparing the behaviour of the boundary layers on a surface of limited length and constant velocity, [11] explored the characteristics of laminar flow in a boundary layer on an expanding sheet. The study argued that the validity of the linear stretching assumption may only be partial and application specific.

Therefore, the literature contains a review of nanofluid and hybrid nanofluid for further study. Thus, the purpose of this research is to examine the MHD hybrid nanofluid flow over a shrinking sheet with quadratic velocity. The issue is to understand the mathematical formulation and try to solve the problem by using the shooting technique with Runge-Kutta-Fehlberg (RKF45) method in Maple software. In a prior investigation, [3] employed the bvp4c solver to address a similar issue. Consequently, in the present study, we will compare our results with those obtained by [3] to validate the findings.

2. Research Method

A steady MHD flow towards a two-dimensional shrinking sheet with quadratic velocity of incompressible conductive Al_2O_3-Cu/H_2O is considered. As shown in Figure 1, x and y are measured towards and normal to the shrinking sheet, corresponding to the plane $y = 0$, the flow is restricted to $y \geq 0$. In the positive y- direction of the sheet, a transverse uniform magnetic field B_0 is applied. The velocity of the shrinking sheet is taken as $u_w(x) = ax + bx^2$ and $v_w(x) = 0$ respectively, whereas both $u_w(x)$ and $v_w(x)$ represent the fluid velocity but at the different locus. In this case, $u_w(x)$ denotes the fluid velocity far away from the shrinking sheet, while $v_w(x)$ symbolizes the velocity of the mass that penetrates the sheet's surface. T_∞ represents the temperature of hybrid nanofluid far from the shrinking sheet, while T_w represents the temperature of hybrid nanofluid at the sheet surface.

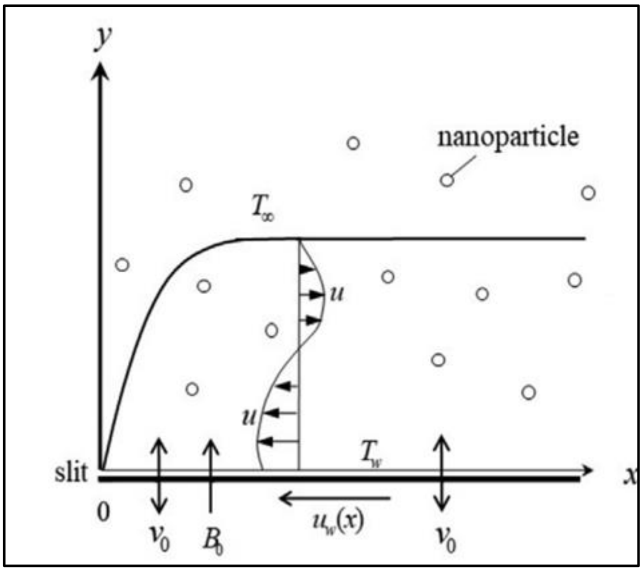


Fig. 1 The physical model and coordinate system

The governing equation of the hybrid nanofluid are [3]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_{hnf}}{\rho_{hnf}}B_0^2u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{hnf}}{(\rho C_p)_{hnf}}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

subject to the boundary conditions that can be expressed as:

$$u=u_w(x)=(ax+bx^2)\lambda,\ \ v=v_w(x),\ \ T=T_w\ \text{at}\ y=0,$$
$$u\to 0,\ \ T\to T_\infty\ \text{as}\ y\to\infty, \tag{4}$$

where u and v are the velocity component on x-axis and y-axis, a and b are shrinking constant, λ is the shrinking parameter and T is the hybrid nanofluid temperature. Further, μ_{hnf} is the dynamic viscosity of hybrid nanofluid, ρ_{hnf} is the hybrid nanofluid density, k_{hnf} is the conductivity of thermal hybrid nanofluid, σ_{hnf} is the electrical conductivity of hybrid nanofluid and $(\rho C_p)_{hnf}$ is the hybrid nanofluid heat capacity. For instance, $v_w(x)$ represents the velocity of the mass that penetrates the sheet's surface, whereas $u_w(x)$ represents the fluid velocity far from the shrinking sheet. Hybrid nanofluid temperature at the sheet surface is represented by T_w, whereas hybrid nanofluid temperature further from the shrinking sheet is represented by T_∞. Further, Table 1 presents the thermophysical correlations of hybrid nanofluid. Meanwhile, Table 2 displays the properties of nanoparticles and water. Note that alumina oxide and copper are the nanoparticles, and their volume fractions are symbolized by ϕ_1 and ϕ_2, respectively.

The similarity transformation variables are as follows:

$$u=axf'(\eta)+bx^2g'(\eta),\qquad v=-\sqrt{av_f}f(\eta)-\frac{2bx}{\sqrt{\frac{a}{v_f}}}g(\eta),$$
$$\theta(\eta)=\frac{T-T_\infty}{(T_w-T_\infty)},\qquad \eta=y\sqrt{\frac{a}{v_f}} \tag{5}$$

where η is the independent similarity variable, $f(\eta)$ and $g(\eta)$ is the dimensionless stream function.
So that:

$$v_w=-\sqrt{av_f}S_1-\frac{2bx}{\sqrt{\frac{a}{v_f}}}S_2 \tag{6}$$

Here S_1 and S_2 are transpiration parameters with $(S_1,S_2)>0$ for suction and $(S_1,S_2)<0$ are for injection or blowing parameter.

By using the similarity variables (5), equation (2)-(3) are converted to:

(7)

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} f''' + f f'' - f'^2 - \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} M f' = 0 ,$$

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} g''' + f g'' - 3f'g' + 2f''g - \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} M g' = 0 , \tag{8}$$

$$\frac{1}{Pr} \frac{k_{hnf}/k_f}{(\rho C_p)_{hnf}/(\rho C_p)_f} \theta'' - 2f'\theta + f\theta' = 0 , \tag{9}$$

$$g'\theta - g\theta' = 0 , \tag{10}$$

Subject to the boundary conditions:

$$f(0) = S_1 ,\ g(0) = S_2 ,\ f'(0) = \lambda ,\ g'(0) = \lambda ,\ \theta(0) = 1 ,$$
$$f'(\infty) \to 0 ,\ g'(\infty) \to 0 ,\ \theta(\infty) \to 0 , \tag{11}$$

Integrating equation (10) with the boundary conditions (11), we get:

$$g(\eta) = S_2 \theta(\eta) \tag{12}$$

Therefore, the ordinary differential equations are,

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} f''' + f f'' - f'^2 - \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} M f' = 0 , \tag{13}$$

$$\frac{1}{Pr} \frac{k_{hnf}/k_f}{(\rho C_p)_{hnf}/(\rho C_p)_f} g'' + f g' - 2f' g = 0 , \tag{14}$$

subject to boundary condition,

$$f(0) = S_1 , \quad g(0) = S_2 , \quad f'(0) = \lambda ,$$
$$f'(\eta) \to 0 , \quad g'(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \tag{15}$$

where the Prandtl number Pr and *M* appears to be the magnetic parameter are defined as

$$Pr = \frac{v_f}{a_f}, \qquad M = \frac{\sigma_f}{\rho_f} \frac{B_0^2}{a} \tag{10}$$

The physical quantities of interest are the skin friction coefficients C_f which is defined as

$$C_f = \frac{\frac{\mu_{hnf}}{\mu_f}}{\rho_f (ax)^2} \left(\frac{\partial u}{\partial y}\right)_{y=0} , \tag{17}$$

using (5) and (17), we get

$$Re_x^{1/2} C_f = \frac{\mu_{hnf}}{\mu_f} [f''(0) + \beta x g''(0)] , \tag{18}$$

where $Re_x = (ax)x/v_f$ and β is a dimensionless parameter defined as $\beta = b/a$.

Table 1: Thermophysical properties of hybrid nanofluid by [3]

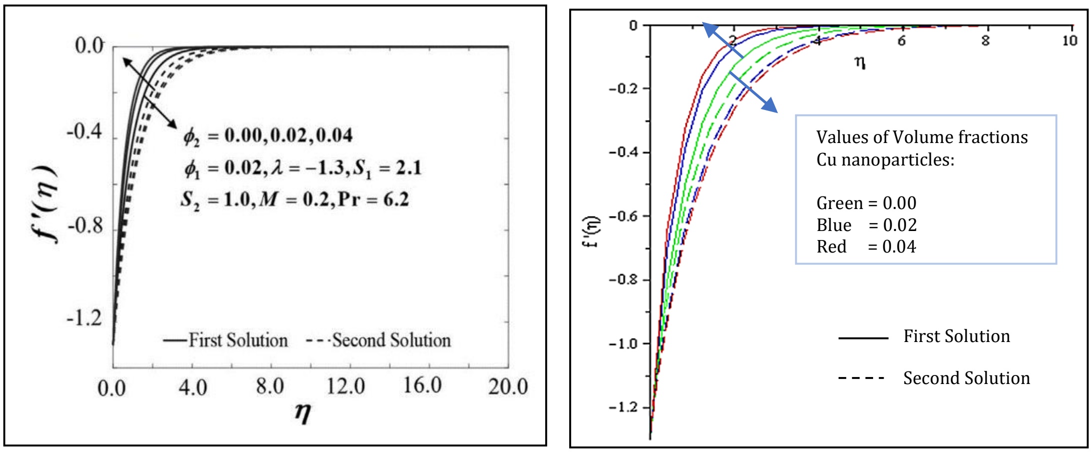
Properties	Hybrid nanofluid
Heat capacity	$(\rho C_p)_{hnf} = (1-\phi_2)\left[(1-\phi_1)(\rho C_p)_f + \phi_1(\rho C_p)_{S1}\right] + \phi_2(\rho C_p)_{S2}$
Dynamic viscosity	$\mu_{hnf} = \dfrac{\mu_f}{(1-\phi_1)^{25}(1-\phi_2)^{25}}$
Thermal Conductivity	$\dfrac{k_{hnf}}{k_{nf}} = \dfrac{k_{s2} + 2k_{nf} - 2\phi_2(k_{nf} - k_{s2})}{k_{s2} + 2k_{nf} + 2\phi_2(k_{nf} - k_{s2})}$, where, $\dfrac{k_{hnf}}{k_{nf}} = \dfrac{k_{s1} + 2k_f - 2\phi_2(k_f - k_{s1})}{k_{s1} + 2k_f + 2\phi_2(k_f - k_{s1})}$,
Density	$\rho_{hnf} = (1-\phi_2)\left[(1-\phi_1)\rho_f + \phi_1\rho_{s1}\right] + \phi_2\rho_{s2}$

Table 2: Thermophysical of nanoparticles and water by [3]

Physical properties	Al_2O_3	Cu	H_2O
(kg/m^3)	3970	8933	997.1
c_p(J/kgK)	765	385	4179
k(W/mK)	40	400	0.613
$\beta \times 10^{-5}$(mK)	0.85	1.67	21

3. Result and Discussion

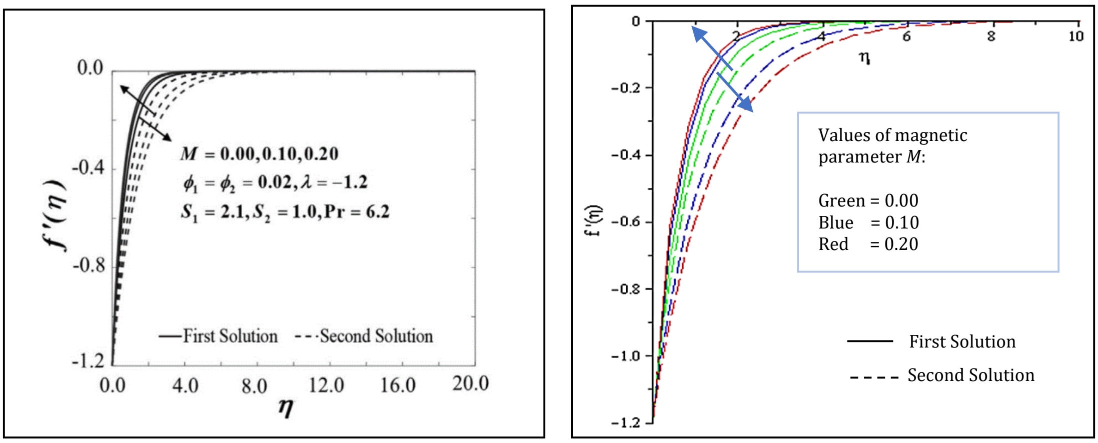
To validate the accuracy of this study, the obtained results are compared to the existing study by [3] for several graphs. The effect on velocity profiles, $f'(\eta)$ for various values of ϕ_2 are shown in Figure 2. In this figure, the volume fractions of Cu nanoparticles are set as ϕ_2 = 0.00, 0.02 and 0.04 with fixed values of shrinking parameter $\lambda = -1.3$, volume fractions of Al_2O_3 nanoparticle ϕ_1 =0.02, magnetic parameter M = 0.2, transpiration parameter S_1= 2.1 and S_2= 1.0, and Prandtl number Pr = 6.2. The comparison results from the graphs shows good agreement. The presence of the second solution relates to solution bifurcations, which are caused by the nonlinearity of the ordinary differential equations system and variations in the shrinking parameter or other governing factors. Fig. 2 also shows that when the volume fraction of nanoparticles ϕ_2 increases, the fluid velocity increases due to an increase in fluid viscosity. As the volume fraction of nanoparticles increases, the thickness of the momentum boundary layer in this figure decreases, leading to a heightened velocity gradient and fluid velocity.



a) **Zainal et al. [3]** **b)** **Present results**

Fig. 2 Velocity profiles at three different values of ϕ_2.

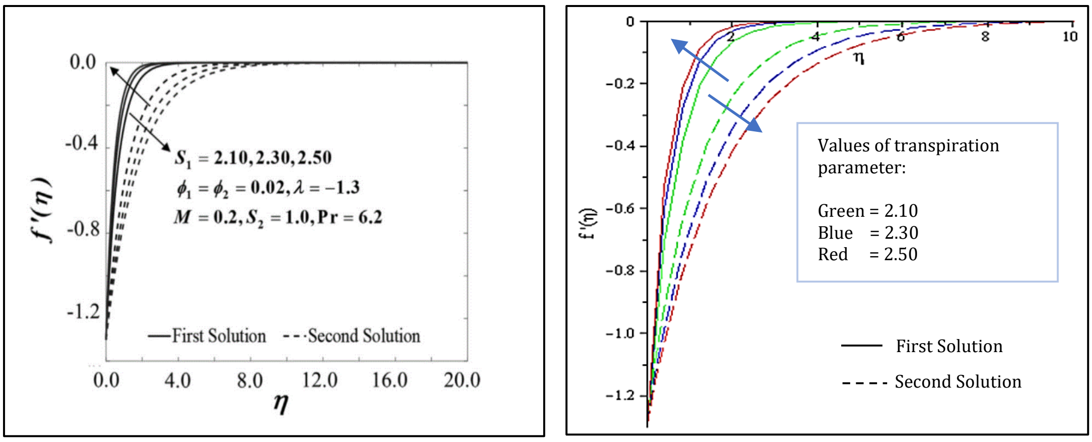
The graph in Fig. 3 shows the comparison of velocity profiles with different values of magnetic parameter M = 0.00, 0.10 and 0.20. The values of shrinking parameter $\lambda = -1.2$, volume fractions of nanoparticles ϕ_1 =0.02 and ϕ_2 =0.02, transpiration parameter S_1 = 2.1 and S_2 = 1.0, and Prandtl number Pr = 6.2 are used. In an electrically conducting fluid, the presence of a magnetic field encourages the Lorentz force, which is a resistive force. Because of the resistance this force creates to the fluid particle's motion, the fluid's velocity is decreased. The Lorentz force establishment causes the magnetic and electrical fields to synchronize, which slows down the conducting fluid's motion close to the boundary layer. As a result, the increment of M increase $f''(0)$, which then result in the increment of the friction drag exerted on the sheet surface. This figure also shows good agreement between the previous and the present results.



a) Zainal et al. [3] **b) Present results**

Fig. 3 Velocity profiles at three different values of M.

Lastly, Fig. 4 present comparison of velocity profiles for various values of transpiration parameter S_1 which are set as 2.1, 2.3 and 2.5 with fixed values of shrinking parameter λ = −1.3, volume fractions of nanoparticles ϕ_1 =0.02 and ϕ_2 =0.02, magnetic parameter M = 0.2, transpiration parameter S_2= 1.0, and Prandtl number Pr = 6.2. Good agreement is also observed from both previous and present results. The increment of S_1 increase the permeability of the sheet, which assists in trapping the low-speed molecules. Eventually, this increases the fluid velocity past the shrinking sheet. The velocity profile in this figure show that increasing sheet permeability S_1 reduces the thickness of the momentum boundary layer, increases fluid velocity, and accentuates the velocity gradient.



a) Zainal et al. [3] **b) Present results**

Fig. 4 Velocity profiles at three different values of $\boldsymbol{S_1}$.

4. Conclusion

The MHD hybrid nanofluid flow over a shrinking sheet with quadratic velocity has been analyzed in this study. The objectives of this research have been achieved. From this study, the first objective that is to transform the governing equation of the MHD hybrid nanofluid by employing the usual boundary layer, from PDEs to ODEs using the similarity transformation technique has been reached. The second objective also has been obtained since we have solved the transformed governing equations using shooting method with RKF45 technique in Maple software. The last objective is to compare and analyze the effect on velocity profile with various parameters such as volume fraction of Cu nanoparticles, magnetic parameter and transpiration parameter also has been discussed as follows:

•As the volume fraction of nanoparticles increases, the thickness of the momentum boundary layer decreases, leading to a heightened velocity gradient and fluid velocity.

•The increment of magnetic parameters, M increase the skin friction $f''(0)$, which then result in the increment of the friction drag exerted on the sheet surface.

•Increasing sheet permeability S_1 reduces the thickness of the momentum boundary layer, increases fluid velocity, and accentuates the velocity gradient.

Lastly, we compare the results obtained in this study with results of the [3] in order to prove the validity of the results obtained and the comparison shows a good agreement.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *NIN;* ***solve the governing equation:*** *NIN;* ***analysis and interpretation of results:*** *NIN, FA;* ***draft manuscript preparation:*** *NIN, FA. All authors reviewed the results and approved the final version of the manuscript.*

References

[1] Choi, S U.S., and Eastman, J A. (1995). Enhancing thermal conductivity of fluids with nanoparticles. International mechanical engineering congress and exhibition, San Francisco, C.A.

[2] Wahid, N.S., Arifin, N.M., Khashi'ie, N.S., Pop, I., Bachok, N., Hafidzuddin, M.E.H. (2022).Unsteady MHD mixed convection flow of a hybrid nanofluid with thermal radiation and convective boundary condition. *Chinese Journal of Physics, 77,* pp. 378–392.

[3] Zainal, N.A., Nazar, R., Naganthran, K., and Pop, I. (2021). Stability analysis of MHD hybrid nanofluid flow over a stretching/shrinking sheet with quadratic velocity. *Alexandria Engineering Journal, 60(1), pp*. 915-926.

[4] Y.S. Daniel, Z.A. Aziz, Z. Ismail, F. Salah. (2017). Double stratification effects on unsteady electrical MHD mixed convection flow of nanofluid with viscous dissipation and Joule heating, *J. Appl. Res. Technol. 15 (5), pp*. 464–476.

[5] R.A. Van Gorder, K. Vajravelu,. (2011). Multiple solutions for hydromagnetic flow of a second-grade fluid over a stretching or shrinking sheet, *Quart. Appl. Math. 69 (3), pp*. 405–424.

[6] A. Malvandi, M.R. Safaei, M.H. Kaffash, D.D. Ganji. (2015). MHD mixed convection in a vertical annulus filled with Al2O3–water nanofluid considering nanoparticle migration*, J. Magn. Magn. Mater. 382, pp*. 296–306.

[7] Zhang, S.H., Deng, L., Tian, W.H., Che, L.Z., Li. Y.(2022). Deduction of a quadratic velocity field and its application to rolling force of extra-thick plate. *Computers & Mathematics with Applications, 109*, pp. 58-73.

[8] B.C. Sakiadis. (1961). Boundary-layer behavior on continuous solid surfaces: I. Boundary-layer equations for two-dimensional and axisymmetric flow, *AIChE J. 7, pp.* 26–28.

[9] Crane, L.J. (1970). Flow past a stretching plate. *Journal of Applied Mathematics and Physics, 21*, pp 645–647.

[10] Miklavčič, M and Wang, C.Y. (2006). Viscous flow due to a shrinking sheet. *Quarterly of Applied Mathematics,64,* pp. 283-290.

[11] Vleggaar, J. (1977).Laminar boundary layer behaviour on continuous, accelerating surfaces. *Chemical Engineering Science, 32,* pp. 15-17.