

Numerical Solution of MHD Boundary Layer Flow of Dusty Fluid Over a Stretching Surface

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Abstract

The objective of the study is to numerically solve the MHD boundary layer flow of a dusty fluid across a stretched surface. In the two-dimensional laminar boundary layer, an electrically conductive thick non-compressible dusty fluid flows across extending to infinite in one direction surface. The dust particles are disc-shaped, uniformly sized, and have a consistent density. The governing equations of problems were transformed into nonlinear ordinary differential equations using similarity transformation, which then numerically solved using the RKF45 method in Maple software. The parameters involved were computed, analyzed, and discussed. The velocity profiles is presented graphically while the skin friction coefficient is in tabular form. In addition, a comparison of present skin friction coefficient results with the existing study has achieved excellent agreement. Besides, the magnetic field parameter showed significant effects on the velocity of the fluid and dust particle phases. Furthermore, the skin friction coefficient increased dramatically as the magnetic field parameter increased.

1. Introduction

Based on [1], the field of fluid dynamics deals with a region of fluid that is nearly in contact with a solid surface where the impact of viscosity unavoidable, and the particular area is recognized as the boundary layer. The motion of fluid is decelerated when it meets a surface, like a wing, causing the molecules of the fluid touching the surface to undergo a frictional force stemming from viscosity. As the fluid molecules decreases, a narrow fluid interface emerges where the fluid velocity progressively increases from zero at the solid surface to match the free-stream velocity. The characteristics of the fluid, flow velocity, and surface roughness influence the thickness of the boundary layer. Typically, a rough surface generates a more boundary layer of air adjacent to it compared to a sleek surface. The study and analysis of the boundary layer's conduct in fluid dynamics is crucial as it greatly impacts the aerodynamics of objects in motion through a fluid.

Based on research from [2], a fluid that consists of tiny solid particles such as sand or dust is known as dusty fluid. Due to the two-phase character of this miracle, it has been the focus of current investigations. Examples of dusty fluid overflows include chemical reactions that condense solid patches into drips and the stirring of dusty air in fluidization issues. The precise analysis of colourful aspects of dusty fluid flows, including magnetohydrodynamic thick overflows over passable stretching or shrinking bodies, and dusty fluid inflow through inclined irregular channels under the influence of an obliquely applied glamorous field. These studies frequently research the goods of different parameters, similar as wall boundary conditions, temperature slants,

and glamorous fields, on the actions of dusty fluid flows. Diverse modifications, such as alterations in viscosity, thermal conductivity, and electrical conductivity, can occur in the fluid due to the influence of these particles. For instance, dusty fluids can be applied to heat transfer because they increase the fluid's thermal conductivity and result in more efficient heat transfer processes. Dusty fluids play a crucial role in several areas, including geophysics, atmospheric science, and chemical engineering. This study is vital as it enables us to comprehend the functioning of natural occurrences such as sandstorms, volcanic eruptions, and air pollution. For instance, [2] has found the behaviour of a conductive dusty fluid with the presence of magnetic field. The velocity profiles decrease as the mass concentration of dust particles and magnetic field parameter increases. [3] investigated and found that increment in liquid molecule interaction parameter contributes to extending the speed of dust phase but diminishes the speed of fluid phase. The velocity of fluid phases decreases and the velocity of dust particle phase increases as the fluid particle interaction parameter increases. [4] has studied the electrically conducting viscoelastic dusty fluid free convective unstable fluctuating MHD flow in a channel-driven by the impact of top plate motion and fluctuating pressure gradient. The increasing of magnetic field parameter reduced the velocity profiles because of Lorentz force is produced.

Magnetohydrodynamics is a special part of fluid flow. It helps us learn about how fluids move by using powerful magnets. Based on [5], the word magnetohydrodynamics originates from the term's magneto, hydro, and dynamics. These terms refer to magnetic field, liquid, and moving things. Magnetohydrodynamics or hydromagnetics is about studying fluid that conducts electricity and how it moves. The tubes, liquid essence (which is like mercury), saltwater, and electrolytes are examples of fluids that look similar to magneto. A surface that generates movement in a fluid is referred to as a stretching surface in the theoretical concept of fluid mechanics. When a fluid flow encounters a stretchy surface, the fluid is forced to stretch, which causes distortion and motion in a certain direction. Stretched surfaces can be movable or stationary, and they can also have different configurations and orientations. A stretchable surface's function is to regulate the flow of a fluid. It may control the size and direction of the liquid motion by adjusting the shape and location of the elongating surface. Surfaces that can be stretched are applied in different domains, including the formation of airplane wings, wind turbines, and fluidic arrangements. Additionally, they find utility in medical contexts such as examining the movement of blood within the circulatory system. Based on the research conducted by [6], the extent of the concentration boundary layer thickness reduces as the Brownian motion parameter and nanofluid Lewis number decrease but increases as the thermophoresis parameter increases. After that, [7] found that MHD nanofluid with the dust particles pass through a stretched surface. The skin friction coefficient increases as the mass concentration of dust particle, fluid particle interaction parameter, and magnetic field parameter increases.

There are many demands for the study and application of boundary layer flow in various fields, especially in environmental science, geology, and engineering. Numerous studies focus on fluid without impurities. However, in real life, impurities exist. Thus, this is the reason why the studies of dusty fluid are under spotlight by researchers according to [8]. The velocity of fluid phases decreases and the velocity of dust particle phase increases as the fluid particle interaction parameter increases. Next, velocity profiles decrease as the mass concentration of dust particles and magnetic field parameter increases. Skin friction coefficient increases as the mass concentration of dust particle, fluid particle interaction parameter and magnetic field parameter increases. The mathematical equations are reduced into nonlinear ordinary differential equations, formulated in Runge-Kutta Fehlberg fourth-fifth method, and numerically solved by using Maple software. [9] investigated the unsteady characteristics of fluid movement and thermal exchange when dust particles are existing amidst two parallel plates, which exhibit variable stickiness and conductivity. The velocity profiles decrease as the magnetic field parameter increases because it produces a Lorentz force to reduce the fluid flow. [10] explored the issue of composite convective motion of a dusty fluid across a stretched surface in two dimensions, considering heat generation and a heat source that varies in space. The magnetic field parameter increase reduces the velocity profiles. Furthermore, increasing the fluid particle parameter reduces the velocity of fluid phase and enhances the velocity of dust particle phase. [11] studied the MHD flow and fusion heat transmission of Casson fluid pass through a stretched surface. The reduction of velocities profile for fluid and dust particle phases as enhancement value of magnetic parameter and Casson parameter. [12] examined the free flow of each other of a two-phase fluid containing dust particles on extending to infinite in one direction vertical flat plate. The velocities of fluid and dust particles phase decreases as the mass concentration of dust particles increases. [13] investigated the flow of MHD Powell-Eyring dusty nanofluids associated with stretched surface under heat flow boundary conditions. The results concluded that an increase in magnetic field values results in a decrease in velocity profiles. Furthermore, a decrease in dust particle concentration leads to a decrease in temperature for both phases. [14] investigated the hydromagnetic flow over a stretched sheet to provide sufficient insight into how fluid-particle interaction, particle loading, and suction affect the flow behaviours. The results show the exact solution of a small value of the fluid particle interaction parameter is similar to the analytical solution. The existence of a transverse magnetic field and suction, the fluid and dust velocities decrease as the particle loading parameter, fluid-particle interaction parameter increases. [15] studied the heat transmission and momentum behaviour in a hydromagnetic movement of a fluid on an inclined stretched sheet with varying heat sources. As the fluid-particle interaction parameter

increases, velocities of fluid phases decrease and dust particle phases increase. [16] analysed the motion and transfer of heat to an exponentially stretched porous sheet when exposed to a magnetic field. The skin friction coefficient increases as the magnetic field parameter increases. [17] investigated the melting heat transfer and boundary layer flows of a Prandtl fluid across a stretched surface in the presence of varying liquid particle suspensions. Skin friction coefficient increases as the mass concentration of dust particle, fluid particle interaction parameter and magnetic field parameter increases. The increasing mass concentration of dust particle and magnetic field parameter enhances the velocity profiles. The increasing fluid particle interaction parameter reduces velocity of fluid phase and enhances velocity of dust particle phase. [18] studied the effect of chemical reaction and radiance on the flow of MHD boundary layer of an incompressible fluid across an exponential stretched surface. The velocity profiles decrease as the magnetic field parameter increases because Lorentz force produces opposite force to fluid flow. [19] used the Keller Box method to solve the boundary layer movement of nanofluids for thermal and mass transfer on stretched surface buried in a porous medium. The velocity profiles decrease as the magnetic field parameter increases because it produces Lorentz force to the fluid flow. Furthermore, skin friction coefficient increases as the magnetic field parameter increases.

Based on the existing studies as mentioned above, it is shown that the existence of dust particles within a dusty fluid may cause changes to its features, including viscosity and thermal conductivity. These modifications can lead to an impact on the fluid's flow performance. Moreover, the presence of dust particles could create a layer of boundary near the surface of an object within the dusty fluid, thereby causing an impact on the heat and mass transmission between the object and dusty fluid. The objectives of this study are to convert the current governing equations of flow and boundary layer heat transfer of dusty fluids by applying a similarity transformation technique to transform partial differential equations (PDEs) to ordinary differential equations (ODEs), solve the transformed governing equation by using Runge-Kutta Fehlberg fourth-fifth-order method (RKF45 method), and analyze the effect of magnetic field parameter, fluid particle interaction parameter, mass concentration of dust particles, stretching sheet on velocity, and skin friction coefficient.

2. Mathematical Formulation

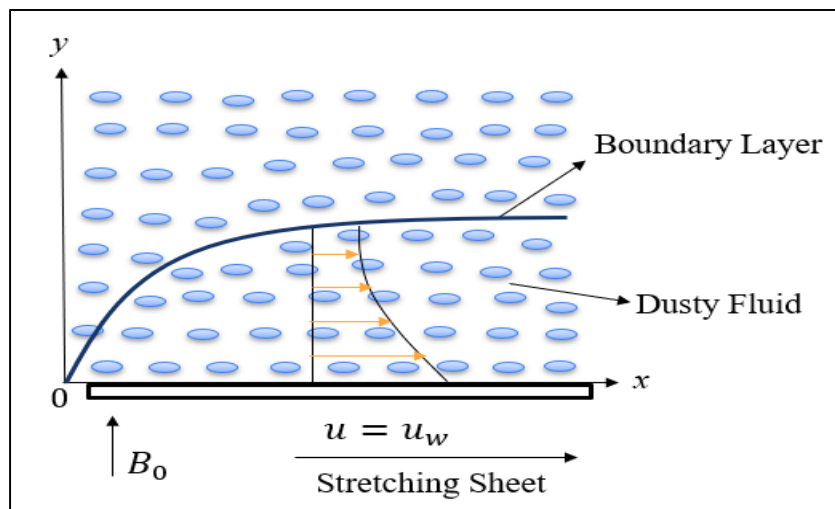


Fig. 1: Physical model of the problem

The flow of a dusty fluid passing through a stretched surface was a laminar boundary layer, a constant two-dimensional flow that was carried electrically. The surface is stretching at a positive velocity $u_w(x) = cx$ and c represents the constant rate of stretching. The leading edge is where the Cartesian Coordinate system's starting point, and the x -axes and y -axes are vertical line to the surface. The ideal characteristics of dust particles are sphericity, homogenous size, and consistent density. The equations govern the movement of a dust fluid are created with the boundary conditions under these physical hypotheses and the boundary layer approximations.

Based on the article from the [2], the governing equations are shown below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{df} \frac{\partial^2 u}{\partial y^2} + \frac{\gamma}{\tau} (u_p - u) - \frac{\sigma B_0^2}{\rho_{df}} u, \quad (2)$$

$$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0, \quad (3)$$

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = \frac{1}{\tau} (u - u_p), \quad (4)$$

corresponding to its boundary conditions,

$$\text{At } y = 0; \quad u = u_w(x), \quad v = 0,$$

$$\text{As } y \rightarrow \infty; \quad u = 0, \quad u_p = 0, \quad v_p = v, \quad (5)$$

where $\tau = \frac{m}{K}$ is the relaxation time of particle phase, $\gamma = \frac{mN}{\rho_{df}}$ is the mass concentration of dust particles, (u, v) and (u_p, v_p) are the dusty fluid components along x -axis and y -axis and the dust particles velocity components along x -axis and y -axis respectively, N is the number density of the dust particles, K is the Stokes resistance, m is the mass of the dust particles, ρ_{df} is the density of dusty fluid, σ is the fluid electrical conductivity, ν_{df} is the kinematic viscosity of dusty fluid, B_0 is the induced magnetic field.

To obtain the similarity solution for Equation (1) until (4), we consider the following similarity transformation variables:

$$\eta = \sqrt{\frac{c}{\nu_{df}}} y, \quad \psi = \sqrt{c\nu_{df}} x f(\eta) \quad (6)$$

$$u = cx f'(\eta),$$

$$v = -\sqrt{c\nu_{df}} f(\eta), \quad (7)$$

$$u_p = cx g'(\eta),$$

$$v_p = -\sqrt{c\nu_{df}} g(\eta)$$

Therefore, as the results, Equations (2) and (4) become,

$$f'''' + ff'' - f'^2 + \beta\gamma(g' - f') - Mf' = 0, \quad (8)$$

$$gg'' - g'^2 + \beta(f' - g') = 0, \quad (9)$$

and the boundary conditions become

$$f(0) = 0, \quad f'(0) = 1,$$

$$f'(\eta) = 0, \quad g'(\eta) = 0, \quad g(\eta) = f(\eta) \quad \text{as } \eta \rightarrow \infty, \quad (10)$$

where prime (') denotes differentiation with respect to η , β the fluid particle interaction parameter for velocity and M the magnetic field parameter as shown below:

$$\beta = \frac{1}{c\tau} = \frac{K}{cm},$$

$$M = \frac{\sigma B_0^2}{c\rho_{df}}. \quad (11)$$

The physical quantities of interest are the skin friction coefficient, C_f and the shear stress, τ_w are described as

$$C_f = \frac{-\tau_w}{\rho_{df} u_w^2},$$

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right) \Big|_{y=0}. \quad (12)$$

By using applying the transformation Equations (6) to (7), the dimensionless skin friction coefficient is defined by $Re_x^{-1/2} C_f = -f''(0)$, where $Re_x = \frac{cx^2}{\nu_{df}}$ is the Reynolds number.

3. Results and discussion

Numerical solutions for boundary conditions (10) of nonlinear ordinary differential equations (8) and (9) have been found by using the RKF45 method on Maple software. The non-dimensional governing parameters considered were the magnetic field parameter (M), fluid particle interaction parameter (β), and mass concentration of dust particle (γ). For mass concentration of dust particle, the value used are $\gamma = 0.2, 0.5, 1.0$. Then, the value used for fluid particle interaction parameter are $\beta = 0.2, 0.5, 1.0$. Furthermore, the value used for magnetic field parameter are $M = 2.0, 5.0, 10.0$.

The graphical and tabular results include the analysis of the dimensionless skin friction coefficient ($-f''(0)$).

| γ | β | M | $-f''(0)$ Jalil, Asghar & Yasmeen [2] | $-f''(0)$ |
|----------|---------|------|---------------------------------------|-----------|
| 0.2 | 0.5 | 1.0 | 1.437591 | 1.4375906 |
| 0.5 | | | 1.471960 | 1.4719601 |
| 1.0 | | | 1.527525 | 1.5275252 |
| 0.1 | 0.2 | 1.0 | 1.420094 | 1.4200939 |
| | 0.5 | | 1.425950 | 1.4259500 |
| | 1.0 | | 1.431782 | 1.4317821 |
| 0.1 | 0.5 | 2.0 | 1.741647 | 1.7416478 |
| | | 5.0 | 2.456284 | 2.4562845 |
| | | 10.0 | 3.321646 | 3.3216462 |

Table 1: Comparison results for the friction coefficient ($-f''(0)$)

Table 1 shows the comparison results of dimensionless skin friction coefficient ($-f''(0)$) with previous study. It shows the effects of fluid particle interaction parameter (β), mass concentration of dust particles (γ), and the magnetic field parameter (M) on the skin friction coefficient. The presence of dust particles is expected to enhance the skin friction coefficient due to their size and capability. Therefore, as mass concentration of dust particle increases, the skin friction coefficient gradually increases. Moreover, skin friction coefficient increases slowly when the fluid particle interaction parameter increases. Furthermore, skin friction coefficient increases dramatically as the magnetic field parameter increases. Fluid particle interaction parameter, mass concentration of dust particles, and magnetic field parameter produce resistances to the flow of fluid and the skin friction increases.

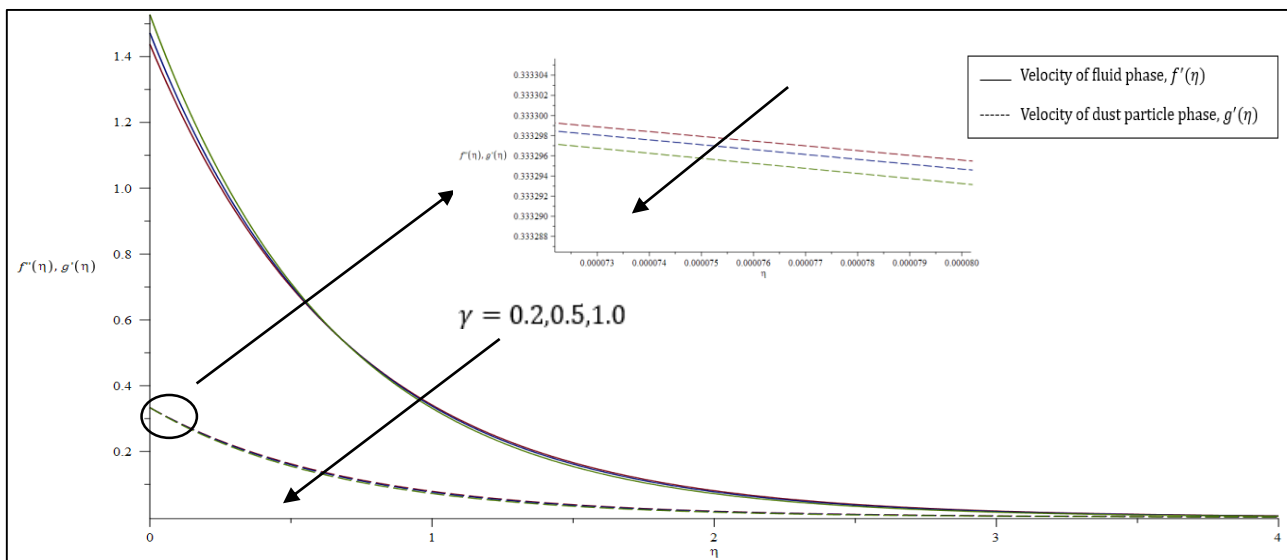


Fig. 2: Effect of mass concentration of dust particles on the velocity profiles

The effect of mass concentration of dust particles (γ) on the velocity profiles is shown in Figure 2. This implies that enhancement in the mass concentration of dust particles causes the velocity profile of the flow to slow down because of the increasing weight of the dust particle phase. The velocity boundary layers are also reduced when the mass concentration of dust particles was enhanced. This phenomenon leads to the increasing skin friction coefficient as shown in Table 1.

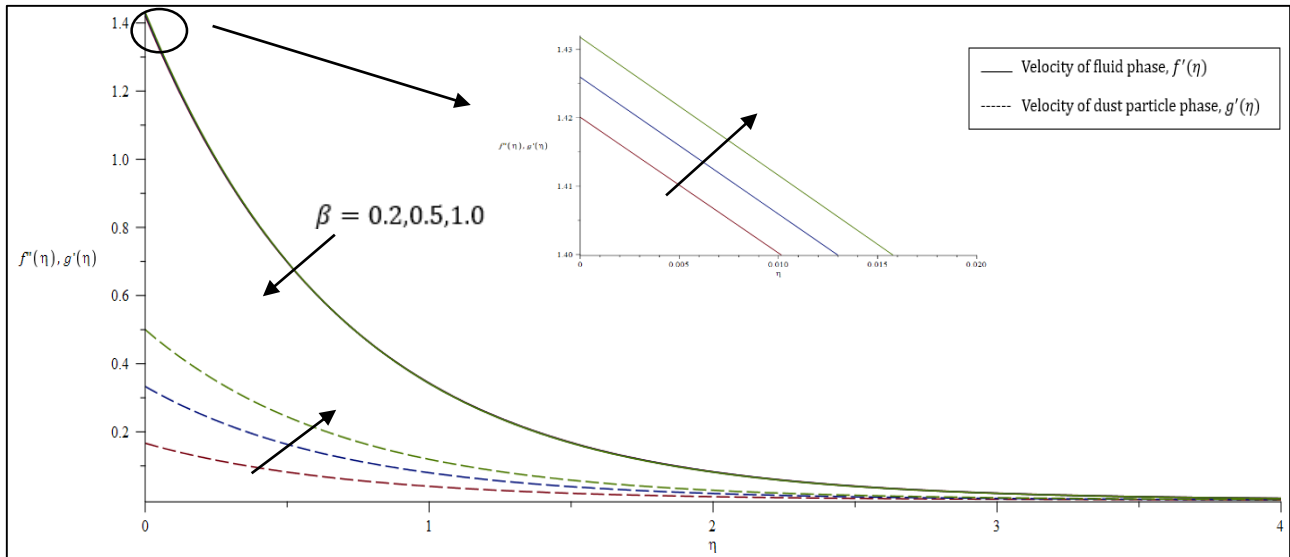


Fig. 3: Effect of fluid particle interaction parameter on the velocity profiles

The effect of fluid particle interaction parameter (β) on the velocity profiles is shown in Figure 3. From the figure, it indicates that the velocity of fluid phase decreases, velocity of dust particle phase increases as the fluid particle interaction parameter increases. This is because of the high interaction between dust particle and fluid phase, meanwhile the dust particle phase develops the counterbalance against the fluid phase until velocity of the dust particle phase attains the same velocity of the fluid phase. When the same fluid velocity is achieved, the dust particle phase reduces the fluid velocity. The dust particles have shorter relaxation time than fluid and the velocities are equal because high fluid particle interaction parameter. Hence, enhancement of dust particle phase velocity and depreciation of fluid phase velocity.

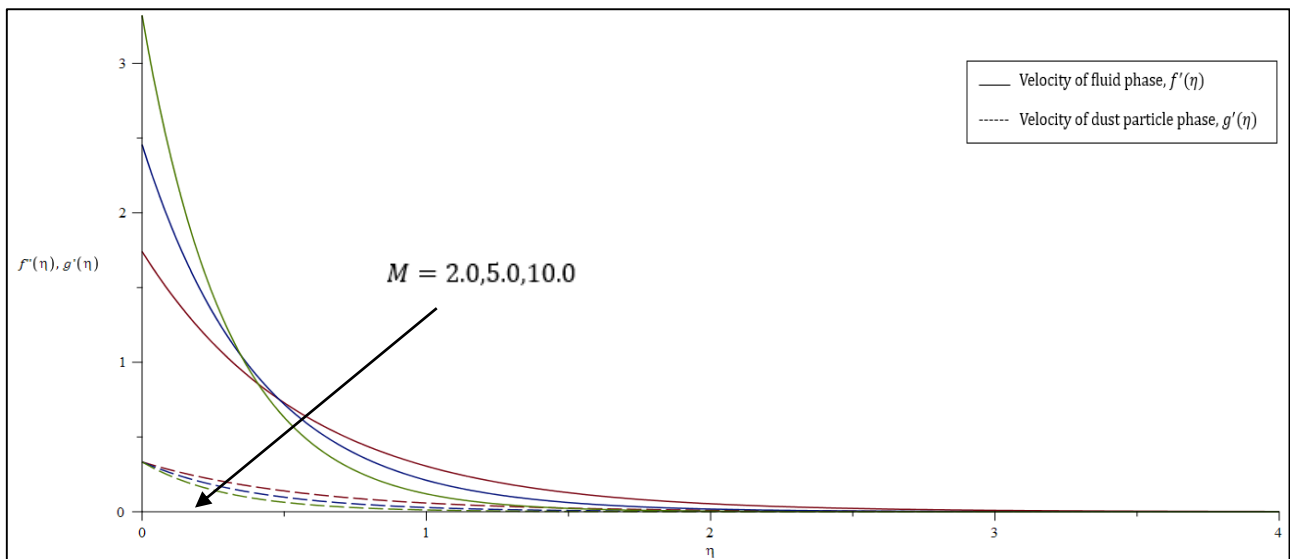


Fig. 4: Effect of magnetic field parameter on the velocity profiles

The effect of magnetic field parameter (M) on the velocity profiles is shown in Figure 4. The figure demonstrates that depreciation of fluid and dust particle phase velocity because of an increase in magnetic field parameter. When a transverse magnetic field parameter is applied, it electrically conducting dusty fluid to produce the Lorentz force which is known as resistive-type force. The motion of the charges creates a drag known as Lorentz force which lowers the velocity magnitude as a magnetic field is produced. According to the Lorentz force, enhancement in the magnetic field parameter generates the counterbalancing force that opposes the flow. It produces more resistance to the flow as Lorentz force increases because of magnetic field parameter increases. Moreover, Lorentz force tends to shrink the velocity boundary layer.

4. Conclusion

This study presents a numerical solution of MHD boundary layer flow of dusty fluid over a stretched surface. These mathematical equations are reduced into nonlinear ordinary differential equations, formulated in Runge-Kutta Fehlberg fourth-fifth method, and numerically solved by using Maple software. The effects of mass concentration of dust particles, magnetic field parameter, and fluid particle interaction parameter on the velocity profiles are presented graphically and discussed while the skin friction coefficient is presented in tabular form and discussed. The summarized of the observations are as follows:

- The increasing in the value of mass concentration of dust particles (γ) reduces the velocity profiles of the fluid and dust particle phases. Mass concentration of dust particles is enhanced causing the velocity boundary layers to reduce.
- The increasing in the value of fluid particle interaction parameter (β) reduces the velocity of fluid phase and enhance the velocity of dust particle phase. A high interaction between dust particle and fluid phase, meanwhile the dust particle phase generates the counterbalance against the fluid phase until velocity of the dust particle phase attains the same velocity of the fluid phase.
- The increasing in the value of magnetic field parameter (M) reduces the velocity of the fluid and dust particle phase because Lorentz force generates the counterbalancing force which bring resistances to the flow as enhancement in the magnetic field parameter.
- Skin friction coefficient increases gradually as the mass concentration of dust particle increases.
- Skin friction coefficient increases slowly as the fluid particle interaction parameter increases.
- Skin friction coefficient increases dramatically as the magnetic field parameter increases.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: **study conception and design:** Lee Mun Jin, Syahira Binti Mansur; **data collection:** Lee Mun Jin; **analysis and interpretation of results:** Lee Mun Jin, Syahira Binti Mansur; **draft manuscript preparation:** Lee Mun Jin, Syahira Binti Mansur. All authors reviewed the results and approved the final version of the manuscript.

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