Enhanced Knowledge in Sciences and Technology Vol. 3 No. 2 (2023) 025-033
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EKST

Homepage: http://publisher.uthm.edu.my/periodicals/index.php/ekst
e-ISSN : 2773-6385

Mixed Convection Flow in a Trapezoidal Cavity with Sinusoidal Heating based on Finite Element Method

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DOI: https://doi.org/10.30880/ekst.2023.03.02.004
Received 15 January 2023; Accepted 19 February 2023; Available online 30 November 2023

Abstract: This study conducts mixed convection flow in a trapezoidal cavity based on finite element method. The left and right wall is considered adiabatic. The top wall moves with a velocity and the magnetic field strength are present in the cavity. A sinusoidal temperature is utilized on the top of the wall of the cavity. The governing equation are transformed to dimensionless form and then the dimensionless equation is insert to COMSOL Multiphysics to solve the mix convection problem. Results shows that when the number of *Ri* increases, the number of eddies occupied and the temperature distribution on top wall was increased while the temperature on the other wall slightly increases. When there are increases in magnetic field strength, the number of eddies formed also increases. As the Richardson number increase, the average Nusselt number also increase.

Keywords: Mixed Convection, Trapezoidal, Sinusoidal Heating, COMSOL Multiphysics

1. Introduction

Heat transmission in a cavity has become an interesting analysis in many fields such as engineering and applied science. Numerous researchers consider the heat transmission in various geometries based on finite element method. Moreover, many researchers also consider the heat transfer in a cavity with different condition such as moving lid-driven, heating, fill with non-Newtonian fluid and the presence of magnetic field [1-3]. Alesbi *et al.* [4] state that the flow influenced by buoyancy force is defined as natural convection while the flow influenced by external means is defined as forced convection. When the natural convection and forced convection occurred simultaneously, mixed convection is taken place. Alshuraiaan and Pop [5] examine the fluid flow in a lid-driven trapezoidal cavity with flexible wall.

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The results shows that when the flexible wall presence in a hollow with a clear fluid has more significant influence on the increment of heat transmission when compared to the cavity with a stiff wall. Taghikhani [6] studied the MHD mixed convection in a lid-driven square enclosure. The enclosure has two sinusoidal heating origins. As a result, when the strength of magnetic field increases, the heat transmission in the cavity decreases, which improves conduction heat transmission and decreases average Nusselt number. Cimpean *et al.* [7] studied mixed convection in a porous trapezoidal chamber with the presence of hybrid nanofluid. They found that when there is an increase in the Reynolds number, the convective circulation and energy transmission also increases. El-Shorbagy *et al.* [8] numerically study the mixed convection flow of nanofluid in two distinct aspect ratios in the present of porous media. They found that when the Richardson number decrease, the heat transmission will increase. Patel *et al.* [9] examined the flow of heat and mass in a two-sided square cavity. A variety sinusoidal heating is applied on the horizontal walls. As a result, the rate of heat transmission on horizontal walls is nearly remain unchanged in forced convection but varies greatly in natural convection. Lastly, Ibrahim and Hirpho [10] conducted mixed convection flow in a trapezoidal cavity with a variety of sinusoidal heating on the bottom wall. They conclude that when the magnetic field strength increases, the number of vortices formed also increases. Additionally, the Richardson number has positive impact on average Nusselt number.

2. Materials and Methods

2.1 Physical model

Figure 1 shows the model of trapezoidal cavity. The trapezoidal cavity is modeling by referring to Ibrahim and Hirpho [10].

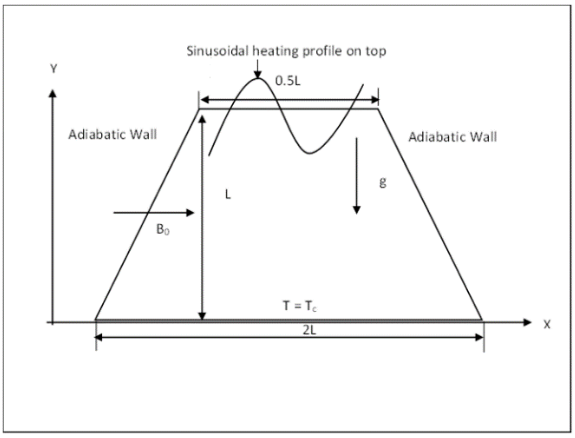


Figure 1: Schematic of the physical model for mixed convection flow in a trapezoidal cavity with sinusoidal heating

The height of the trapezoidal cavity is denoted as L while length of the bottom wall and top wall are denoted as $2L$ and $0.5L$ respectively. A variable temperature in sinusoidal form is considered on the top of the wall, while the bottom wall is set at a uniform temperature, T_c. The left and right sides of the walls are considered adiabatic. Both velocity v_0 on the top lid and the magnetic field of strength B_0 is utilize in the positive x-direction. Finally, the trapezoidal cavity is filled with Newtonian fluid and laminar flow. The boundary conditions are $\eta = \zeta = 0$ on all walls except top wall, $\frac{\partial \theta}{\partial n} = 0$ on the left and right wall, $\eta = \zeta = \theta = 0$ on the bottom wall and $\eta = 1, \zeta = 0, \theta = m \sin(2\pi x)$ on the top wall. Where η and ζ are dimensionless velocities, θ is dimensionless temperature and m is the amplitude of sinusoidal function.

2.2 Governing equation

The dimensionless governing equation are referred to the study conduct by Ibrahim and Hirpho [10]. The governing equation contains continuity equation, momentum equation and heat equation.

Continuity equation:

$$\frac{\partial \zeta}{\partial Y} + \frac{\partial \eta}{\partial X} = 0, \qquad \text{Eq. 1}$$

Momentum equation along x-axis:

$$\zeta \frac{\partial \eta}{\partial Y} + \eta \frac{\partial \eta}{\partial X} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left(\frac{\partial^2 \eta}{\partial X^2} + \frac{\partial^2 \eta}{\partial Y^2}\right), \qquad \text{Eq. 2}$$

Momentum equation along y-axis:

$$\zeta \frac{\partial \zeta}{\partial Y} + \eta \frac{\partial \zeta}{\partial X} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 \zeta}{\partial X^2} + \frac{\partial^2 \zeta}{\partial Y^2}\right) + \frac{\mathrm{Gr}}{\mathrm{Re}^2}\theta - \frac{\mathrm{Ha}^2}{\mathrm{Re}}\zeta, \qquad \text{Eq. 3}$$

Heat equation:

$$\zeta \frac{\partial \theta}{\partial Y} + \eta \frac{\partial \theta}{\partial X} = \frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right). \qquad \text{Eq. 4}$$

Subject to:

On the inclined walls:

$$\eta = \zeta = \frac{\partial \theta}{\partial n} = 0$$

where n shows direction the normal to a plane

On the top walls:

$$\eta = 1, \zeta = 0, \theta = m\sin(2\pi x),$$

where m is the amplitude of sinusoidal temperature.

On the bottom walls:

$$\eta = \zeta = \theta = 0$$

2.3 COMSOL Multiphysics 5.5

COMSOL Multiphysics 5.5 is a simulation software that based on advanced numerical method. The latest version COMSOL Multiphysics 5.5 provided in the Matlab of University Tun Hussein Onn Malaysia is chosen in this study to ensure that the service provided by COMSOL Multiphysics 5.5 are latest, bugs that occurred in older versions are fixed, new features requested by users and more.

2.4 Flow chart of compute fluid dynamics problems in this study

Figure 2 shows the procedure of how to solve the mixed convection problem with the assist of COMSOL Multiphysics.

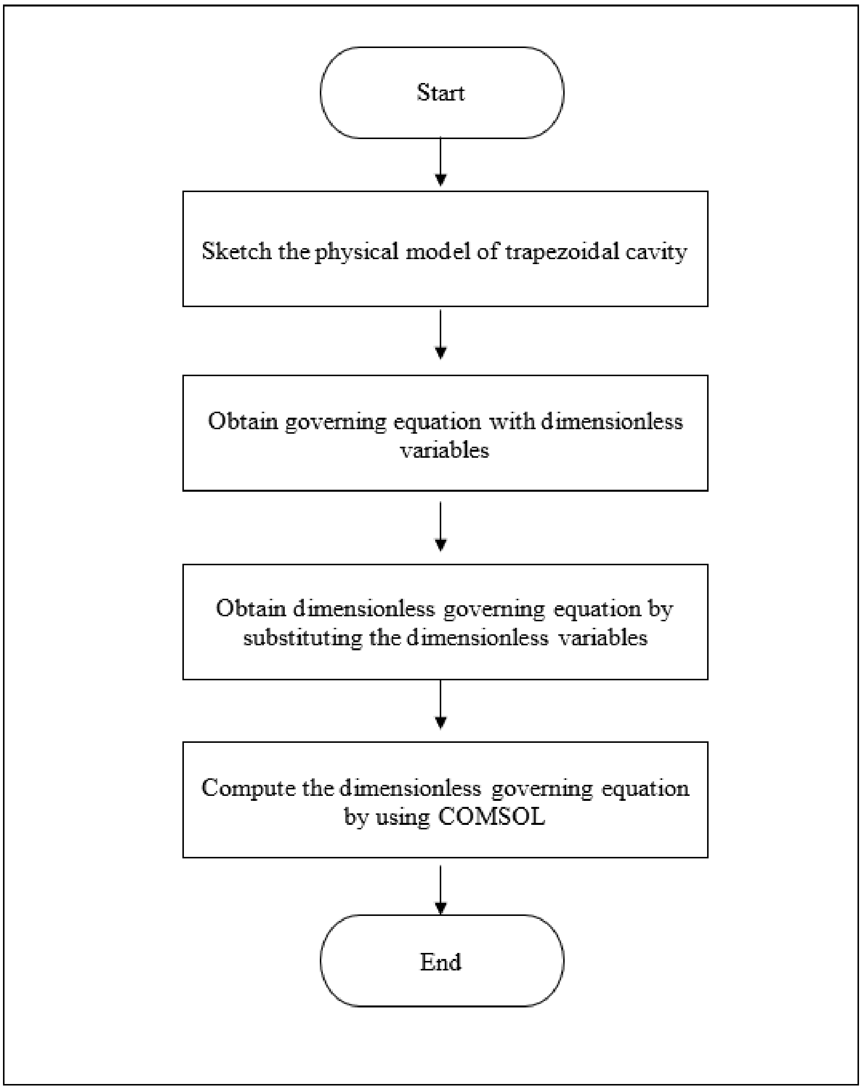
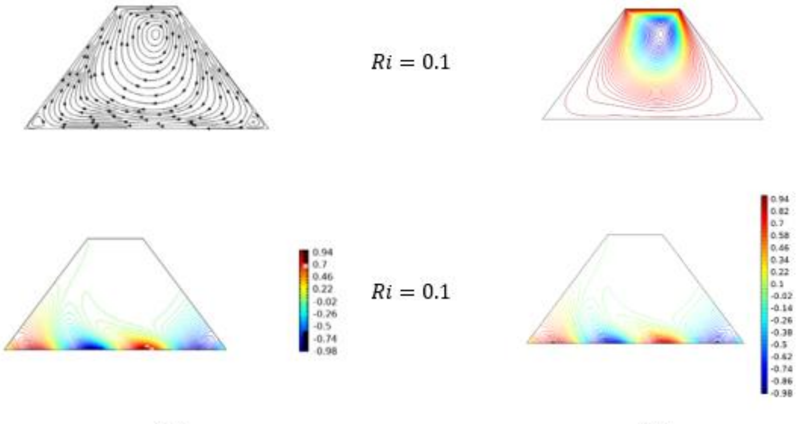


Figure 2: Flow chart of solving mixed convection problem

3. Results and Discussion

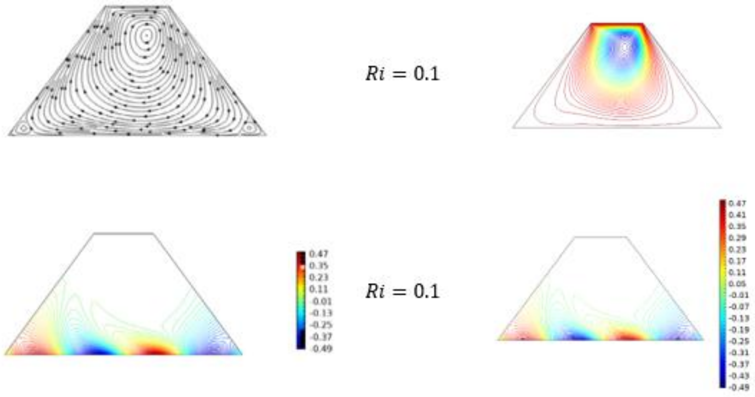
3.1 Streamlines and isotherms plot compared with Ibrahim and Hirpho [10]

The results obtained by COMSOL Multiphysics is compared with the results obtained by Ibrahim and Hirpho [10] to show that the results compute by COMSOL Multiphysics is convincing. The comparison of the results has the same model and boundary conditions.



(a) (b)

Figure 3: Streamlines on the top and isotherms on the bottom for $m = 1, Ha = 0$ and $Ri = 0.1$. (a) Ibrahim & Hirpho [10] (b) present



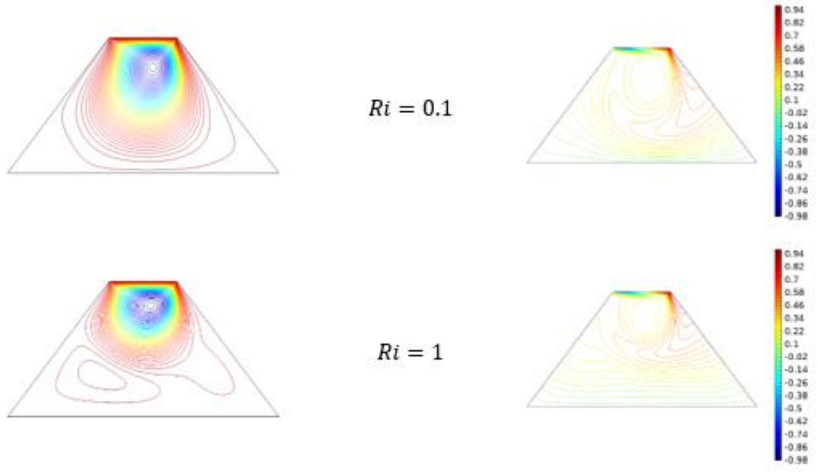
(a) (b)

Figure 4: Streamlines on the top and isotherms on the bottom for $m = 0.5, Ha = 0$ and $Ri = 0.1$. (a) Ibrahim & Hirpho [10] (b) present

From streamlines of Figure 3, it can observe that the difference between the results compute by previous study and COMSOL Multiphysics is the formation of eddies at the corner left-bottom and right-bottom wall. The pattern of isotherm plots is almost the same. This situation is same for the Figure 4.

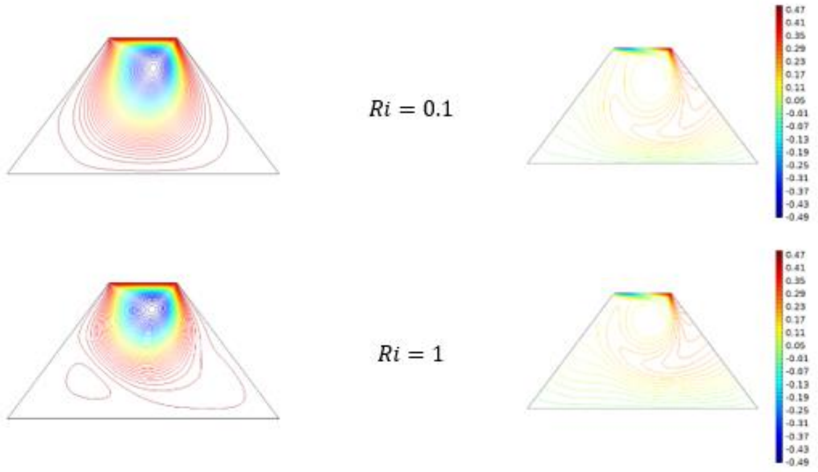
3.2 Streamlines and isotherms in this study

In this study, the boundary conditions of the top wall and bottom wall is contradiction with the boundary conditions of previous research where the sinusoidal temperature is utilized on the top wall while the bottom wall remain constant cold temperature. The results obtained by COMSOL Multiphysics are shown at below.



(a) (b)

Figure 5: (a) Streamlines (b) Isotherms $(m = 1, Ha = 0)$

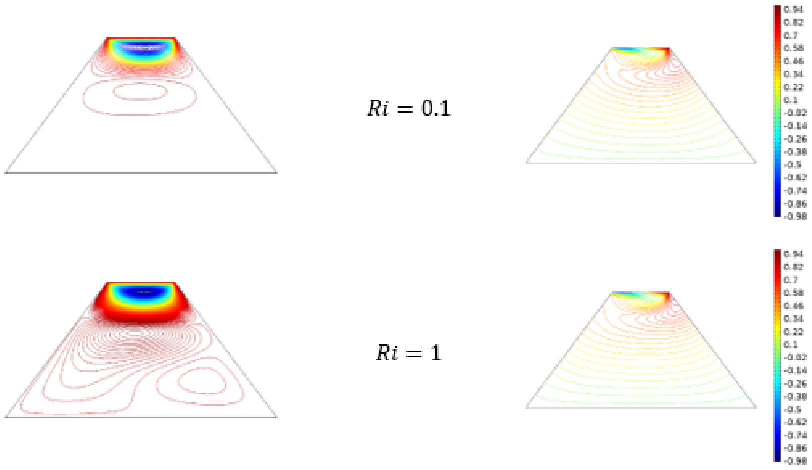


(a) (b)

Figure 6: (a) Streamlines (b) Isotherms $(m = 0.5, Ha = 0)$

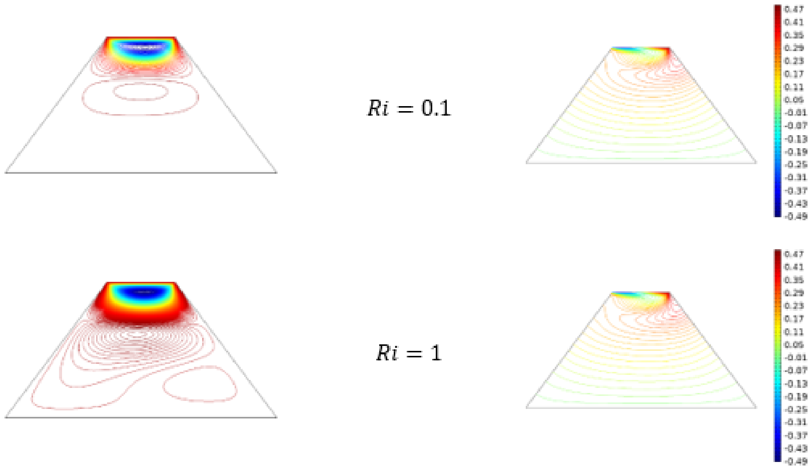
Streamlines are the trajectories taken by fluid particles in steady flow situations while an eddy is a circular current of water. Based on the streamlines of Figure 5 and Figure 6, a large clockwise circulation eddy occupied the cavity that near the top wall of the cavity. When *Ri* increased from 0.1 to 1, there is one eddy occupied in the direction of left-bottom side in anticlockwise direction. The increment of Richardson number related with the increases in the buoyancy force as contrasted to the force created by the lid driven. Therefore, the number of eddies in the cavity increases.

Isotherms are the lines that link two points which matched the temperature in the cavity. Based on isotherms of Figure 5 and Figure 6, when the *Ri* increase from 0.1 to 1, the temperature distribution on top wall increases while the temperature distribution on the other wall slightly increases. The top wall which exposed to the sinusoidal temperature has receive more heat than the wall contrasted to steady temperature. The isotherms in the centre of the cavity move a little bit upward.



(a) (b)

Figure 7: (a) Streamlines (b) Isotherms $(\boldsymbol{m = 1, Ha = 50})$



(a) (b)

Figure 8: (a) Streamlines (b) Isotherms $(\boldsymbol{m = 0.5, Ha = 50})$

Based on Figure 7 and Figure 8, when the magnetic field strength is increased, the location and direction of the eddies are different as compared to Figure 5 and Figure 6. This is because the Lorenz force has a greater influence than lid driven force, which has an adverse influence on eddy formation. Additionally, the increases in Hartmann number also increase the formation of eddy. The increases in formation of eddy indicate that the velocity of fluid flow in the cavity increases. When the Hartmann number set to 50, the isotherms show significant in the top wall only. The isotherms in Figure 7 and Figure 8 almost have the same pattern.

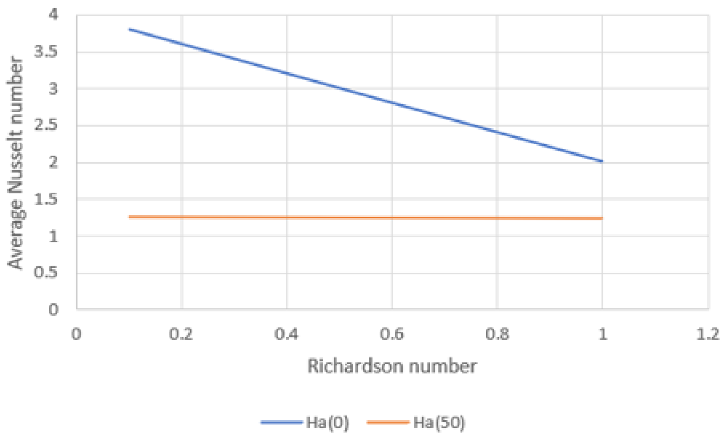


Figure 9: Variation of average Nusselt number along the top wall

Based on Figure 9, it can be observed that the average Nusselt number decrease not obviously when the Hartmann number has increased from 0 to 50. This situation show that the magnetic field strength has negative relationship with the average Nusselt number. When the Richardson number increase, the average Nusselt number decreases which indicates that the convection inside the cavity is decrease.

4. Conclusion

In conclusion, the mixed convection problems in a trapezoidal cavity are solved via finite element method with the assist of COMSOL Multiphysics. The dimensionless governing equation was insert to COMSOL Multiphysics to simulate the mixed convection problems. Based on the results compute from COMSOL Multiphysics, it can be concluded that, the increases of Richardson number and the Hartman number will increase the formation of eddies. Lastly, the Richardson number have negative impact on the average Nusselt number. The streamlines isotherms, and the graph of average Nusselt number versus Richardson number provided in this study can be a reference for future studies.

Acknowledgement

The authors would like to thank the Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia for fully support on my research.

References

[1] M. S. Astanina,, M. A. Sheremet, H. F. Oztop,, & N. Abu-Hamdeh. (2018). MHD natural convection and entropy generation of ferrofluid in an open trapezoidal cavity partially filled with a porous medium. *International Journal of Mechanical Sciences*, *136*, 493–502.

[2] M. Hamid, M. Usman, Z. H. Khan., R. U. Haq., & W. Wang. (2019). Heat transfer and flow analysis of Casson fluid enclosed in a partially heated trapezoidal cavity. *International Communications in Heat and Mass Transfer*, *108*, 104284.

[3] F., Selimefendigil., & H. F., Öztop. (2018). Modeling and optimization of MHD mixed convection in a lid-driven trapezoidal cavity filled with alumina–water nanofluid: Effects of electrical conductivity models. *International Journal of Mechanical Sciences*, *136*, 264–278

[4] I., Alesbe., S. H., Ibrahim., & S., Aljabair. (2021). Mixed convection heat transfer in multi-Lid-driven trapezoidal annulus filled with hybrid nanofluid. *Journal of Physics: Conference Series*, *1973*(1), 12065.

[5] B., Alshuraiaan., & I., Pop. (2021). Numerical simulation of mixed convection in a lid-driven trapezoidal cavity with flexible bottom wall and filled with a hybrid nanofluid. *The European Physical Journal Plus*, *136*(5), 580.

[6] M. A., Taghikhani. (2019). Cu–Water Nanofluid MHD Mixed Convection in a Lid-Driven Cavity with Two Sinusoidal Heat Sources Considering Joule Heating Effect. *International Journal of Thermophysics*, *40*(4), 44.

[7] D. S., Cimpean., M. A., Sheremet, & I., Pop. (2020). Mixed convection of hybrid nanofluid in a porous trapezoidal chamber. *International Communications in Heat and Mass Transfer*, *116*, 104627.

[8] M. A., El-Shorbagy., F., Eslami., M., Ibrahim., P., Barnoon., W. -F., Xia, & D., Toghraie, (2021). Numerical investigation of mixed convection of nanofluid flow in a trapezoidal channel with different aspect ratios in the presence of porous medium. *Case Studies in Thermal Engineering*, *25*, 100977.

[9] A., Patel, M., Kumar., & S., Bagai. (2021). Heat and mass transfer in a two-sided lid-driven square cavity with non-uniform sinusoidal heating on horizontal walls. *The European Physical Journal Plus*, *136*(10), 1034.

[10] W., Ibrahim., & M., Hirpho. (2021). Finite element analysis of mixed convection flow in a trapezoidal cavity with non-uniform temperature. *Heliyon*, *7*(1).