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Solving MHD Flow over a Shrinking Wedges Using Shooting Technique

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Abstract: The study investigates MHD flow over a shrinking wedge using shooting technique. A similarity transformation is used to transform the governing partial differential equations of the flow and heat transfer into a system of ordinary differential equations. These equations are then solved numerically using shooting technique with the Runge-Kutta Fehlberg method in Maple software. The issues are quantitatively addressed in this study. The findings are compared to those obtained by previous researchers. For the shrinking case, the existence of the solution depends on the shrinking strength, λ and the angle of the wedge, Ω. The range of λ (for which the solution exists) increases as the angle of the wedge, Ω increases.

Keywords: Magnetohydrodynamic, Shrinking Wedges

1. Introduction

Magnetohydrodynamics (MHD) is the study of the interaction of electrically conducting fluids and electromagnetic forces. The combination of hydrodynamics with electromagnetic waves is known as MHD. There are many studies on MHD flows toward a stretching sheet. For instance, [1] studied about the MHD viscous fluid flow over a stretching sheet and reported an exact solution for the problem. They found that the fluid flow and shear stress are greatly affected by partial slip, magnetic parameter and mass transfer. There are only a few studies regarding such flows toward a shrinking sheet. They extended the work to a shrinking sheet and included the injection or suction effect at the boundary. They reported that the velocity decreases as the injection effect is increased, but it increases as the suction strength is increased [2].

The stagnation point flow and heat transfer over a nonlinear shrinking sheet with slip effects has been studied by [3]. Meanwhile, [4] investigated the heat transfer characteristics on MHD Powell-

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Eyring fluid flow across a shrinking wedge with non-uniform heat source/sink. The curves of velocity reduce when the shrinking parameter, magnetic field parameter, and material fluid parameter are increased. The heat transfer performance is also influenced by non-uniform heat source/sink factors. Furthermore, [5] studied the flow over a shrinking sheet containing hybrid nanoparticles with nonlinear thermal radiation and magnetohydrodynamic effects. It thickens the thermal barrier layer and slows the rate of heat transfer. However, as the Lorentz force grows stronger, increasing the magnetic parameter causes the friction factor to rise, which immediately enhances the efficiency of heat transfer. The solutions for the shrinking sheet must also be produced with sufficient suction strength. The first solution is reported as stable and therefore physically trustworthy over the long term according to the temporal stability study, whereas the second solution is unstable.

As a result, the purpose of this research is to identify an alternative way to solve the MHD flow problem across contracting wedges. The problem is quantitatively explored using the shooting strategy in Maple software with Runge-Kutta Fehlberg (RKF45) technique.

2. Methodology

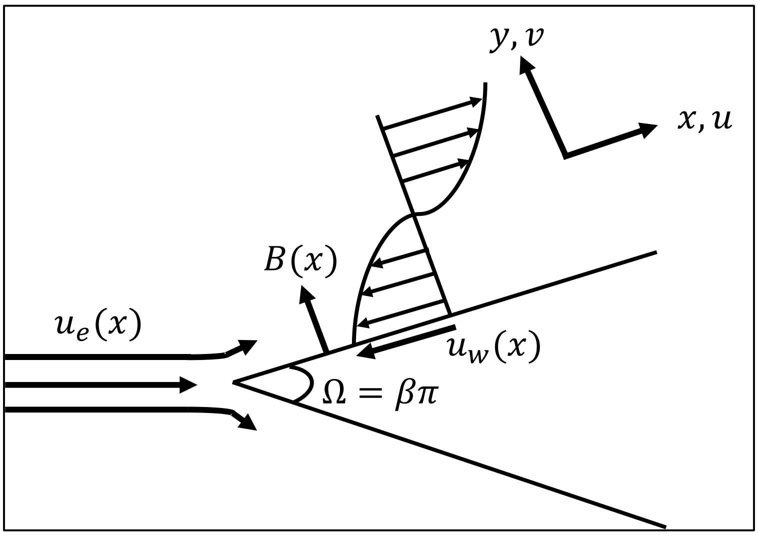

Figure 1: Physical model and coordinate system for shrinking wedge.

Figure 1 illustrates the physical model and coordinate system for the shrinking wedge where *x* and *y* are respectively the Cartesian coordinate measured along the surface, while *u* and *v* are the velocity component along the Cartesian coordinate *x* and *y* respectively [1]. Assumed that the velocity of the shrinking wedge is $u_w(x) = U_w x^m$, where U_w < 0 corresponds to shrinking where m and U_∞ are positive constants [6]. In the positive direction of the y-axis, a variable magnetic field of intensity $B(x)$ is applied. The induced magnetic field is assumed to be negligible and therefore is not taken into consideration. β is the Hatree pressure gradient parameter which corresponds to β = Ω/π for a total angle Ω of the wedge [1]. 0 ≤ m ≤ 1 with m = 0 for the boundary-layer flow over a stationary flat plate (Blasius problem) and m = 1 for the flow near the stagnation point on an infinite wall.

$$\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}} + \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{y}} = \boldsymbol{0}, \qquad \boldsymbol{Eq.}\,1$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} + v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho}(u - u_e), \qquad Eq.\,2$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \qquad Eq.\,3$$

while the boundary conditions are:

$$v = v_w, u = u_w(x), T = T_w \text{ at } y = 0,$$
$$u \to u_e(x), T = T_\infty \text{ as } y \to \infty, \qquad Eq. 4$$

where v is kinematic viscosity, T is the fluid temperature, T_w is the uniform surface temperature, σ is the electrical conductivity, α is the thermal diffusivity of the fluid, ρ is the fluid density and $y_w(x)$ is the mass flux velocity with $v_w(x) < 0$ for suction and $v_w(x) > 0$ for injection.

The similarity variables which are:

$$\psi = \sqrt{\frac{2vxu_e}{1+m}}f(\eta), \eta = \sqrt{\frac{(1+m)u_e(x)}{2vx}}y, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \qquad Eq. 5$$

where ψ is the stream function, which is defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ which satisfy the continuity *Eq.* 1 and are obtained as:

$$\psi = u_e(x)f'(\eta), v = -\sqrt{\frac{(1+m)vu_e(x)}{2x}}\left[f(\eta) + \frac{m-1}{m+1}\eta f'(\eta)\right], \qquad Eq. 6$$

where prime denotes differentiation with respect to η. At the boundary where $\eta = 0$, the transpiration rate is given by

$$v_w = -\sqrt{\frac{(1+m)vu_e(x)}{2x}}s, \qquad Eq. 7$$

where $s = f(0)$, a constant parameter with $s > 0$ for suction and $s < 0$ for injection. Similarity solution can be obtained when all the parameters are constant. We take $B(x) = B_0 x^{m-1/2}$, where B_0 is a positive constant [7]. Substitute *Eq.* 5 and *Eq.* 6 into *Eq.* 2 and *Eq.* 3 to obtain the ordinary differential equations as follows:

$$f''' + ff'' + \beta\left(1 - f'^2\right) + M^2(1 - f') = 0, \qquad Eq. 8$$

and

$$\frac{1}{Pr}\theta'' + f\theta' = 0, \qquad Eq. 9$$

which is subject to the boundary conditions:

$$f(0) = s, f'(0) = \lambda, \theta(0) = 1,$$
$$f'(\eta) \to 1, \theta(\eta) \to 0 \; as \; \eta \to \infty, \qquad Eq. 10$$

where λ is the shrinking parameter, β is the pressure gradient parameter, M is the magnetic parameter (Hartmann number), Pr is the Prandtl number and s is the suction or injection parameter, which are defined as:

$$\lambda = \frac{U_w}{U_\infty}, \beta = \frac{2m}{m+1}, M = \sqrt{\frac{2\sigma}{(m+1)\rho U_\infty}}B_0, Pr = \frac{v}{\alpha}, \qquad Eq. 11$$

where $\lambda < 0$ is for shrinking case. Magnetic field is absent, so $M = 0$. When $\beta = 0$ and $\beta = 1$, the *Eq.* 8 reduces to the classical Blasius equation and Hiemenz equation respectively. There are two physical quantities of interest in this study, which are skin friction coefficient C_f and the local Nusselt number, Nu_x which can be expressed as:

$$C_f = \frac{\tau_w}{\rho u_e^2(x)}, Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \qquad Eq. 12$$

where τ_w is the wall shear stress along the shrinking surface and q_w is the surface heat flux, which are defined as:

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}. \qquad Eq. 13$$

Substituting *Eq.* 6 into *Eq.* 13 to obtain

$$Re_x^{\frac{1}{2}}C_f = \left(\frac{1+m}{2}\right)^{\frac{1}{2}} f''(0), Re_x^{-\frac{1}{2}}Nu_x = -\left(\frac{1+m}{2}\right)^{\frac{1}{2}} \theta'(0), \qquad Eq. 14$$

where $Re_x = u_e(x)x/v$ is the local Reynolds number.

3. Results and Discussion

The results are achieved by using shooting method with Runge-Kutta Fehlberg (RFK45) in Maple and the analysis of the results are presented in the table and graphs.

Table 1: The outcome of skin friction coefficient, $f''(0)$ when $\lambda = 0, s = 0$ and $M = 0$ for different values of β

β	M	Skin friction coefficient, $f''(0)$	
		Awaludin et al. [8]	Present study
0	0	0.469600	0.469600
0.5	0	0.927680	0.927680
0.7	0	1.059808	1.059808

From Table 1, it can be seen that the result shows a good agreement. The result of Nusselt number also can be obtained as shown in Table 2 below.

Table 2: The outcome of the Nusselt number, Nu_x when $\lambda = 0, s = 0$ and $M = 0$ for different values of β

β	M	Nusselt number $-\theta''(0)$
0	0	-0.469600
0.5	0	-0.538978
0.7	0	-0.553660
1	0	-0.570465

3.1 The effect on velocity profiles

Figures below illustrate the velocity profiles, $f'(\eta)$ for different values of shrinking parameter λ when $s = 1$ and $M = 0.2$ with Hatree pressure gradient parameter $\beta = 0.1$ and $\beta = 0.33$ as in Figure

2 and Figure 3 respectively. Based on these figures, the infinity boundary conditions ($f \to 1$ and $\theta \to 0$ as $\eta \to \infty$) are satisfied asymptotically which support the validity of the numerical results obtained. The increases in λ contributes to the increment of $f\,'(\eta)$ as shown in Figure 2 because of the Lorentz force grows stronger, increasing the magnetic parameter causes the friction factor to rise, which immediately enhances the efficiency of heat transfer [9], while the boundary layer thickness is high and decreases the value of λ contributes to the increment of $f\,'(\eta)$ in Figure 3.

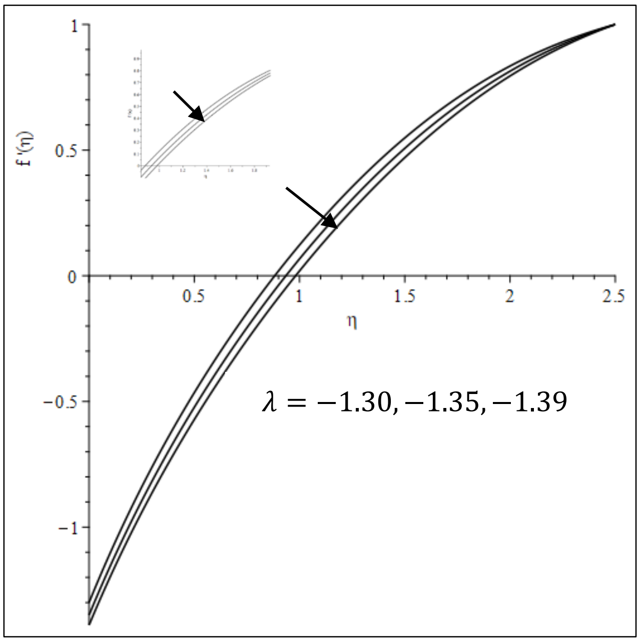

Figure 2: The velocity profiles for different values of λ when $s = 1, \beta = 0.1$ and $M = 0.2$

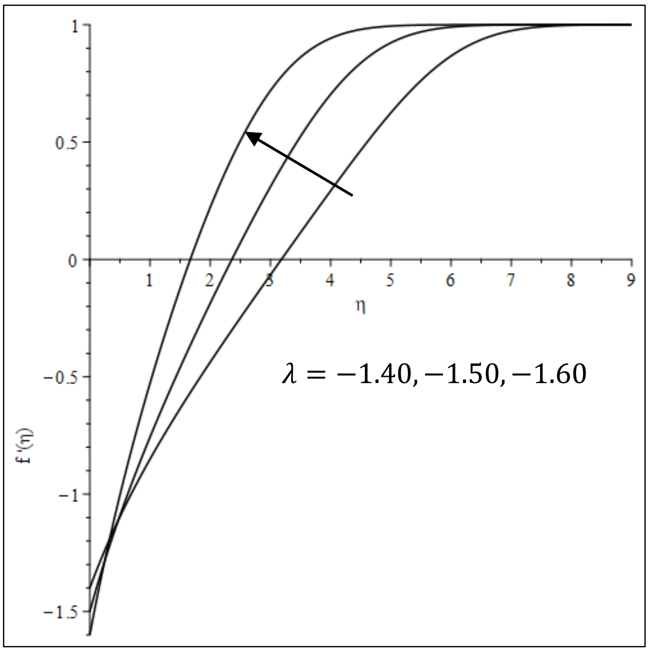

Figure 3: The velocity profiles for different values of λ when $s = 1, \beta = 0.33$ and $M = 0.2$

3.2 The effect on temperature profiles

Figures below show temperature profiles for different values of λ when $s = 1, M = 0.2$ and $\mathrm{Pr} = 1$ with $\beta = 0.1$ and $\beta = 0.33$ as in Figure 4 and Figure 5 respectively. Based on these figures, the infinity boundary conditions ($f \to 1$ and $\theta \to 0$ as $\eta \to \infty$) are satisfied asymptotically which support the validity of the numerical results obtained. Figure 4 shows the temperature profile increases as the values of λ decrease. This is because of the viscous dissipation tends to enhance temperature [10], while Figure 5 shows the boundary layer thickness is high and the temperature profile decreases as the values of λ increase.

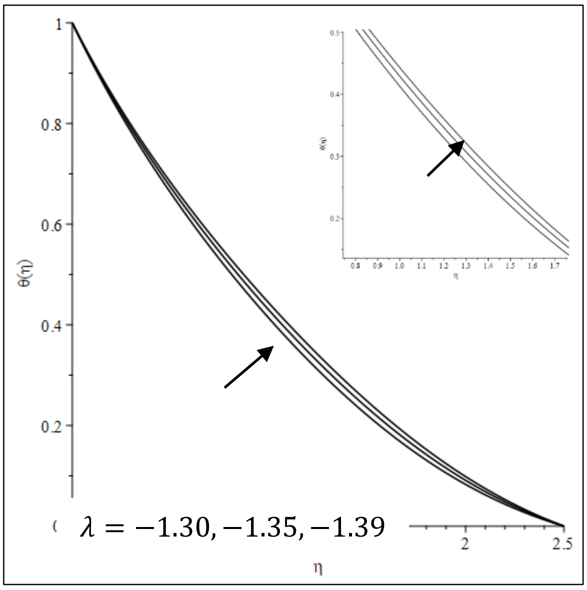

Figure 4: The temperature profiles for different values of λ when $s = 1, \beta = 0.1$, $M = 0.2$ and $\mathrm{Pr} = 1$

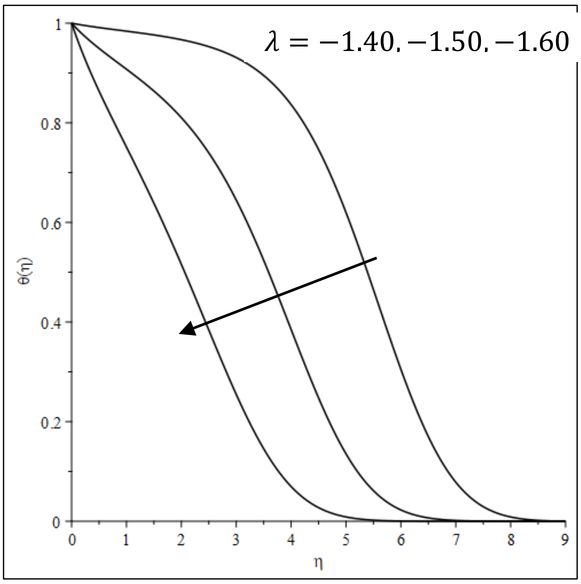

Figure 5: The temperature profiles for different values of λ when $s = 1, \beta = 0.33$, $M = 0.2$ and $\mathrm{Pr} = 1$

4. Conclusion

In conclusion, this study examined the steady two-dimensional MHD boundary layer flow and heat transfer of an incompressible and electrically conducting fluid over a shrinking wedge. The Runge-Kutta-Fehlberg (RFK45) in Maple software has been used to calculate the numerical findings. The shrinking strength and wedge angle are both important factors in determining whether a solution exists for the shrinking situation. As the wedge's angle rises, the range of (for which the solution exists) expands. For the effects of all the parameters involved, discussions on the skin friction coefficient and the local Nusselt number have been conducted.

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