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Solving Nanofluid Flow Over a Permeable Stretching and Shrinking Surfaces using Shooting Technique

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Abstract: The analysis of nanofluid flow over permeable stretching and shrinking surfaces using the shooting technique with Runge-Kutta-Felhberg in Maple software is carried out. The governing equations of nanofluid on boundary layer flow is solved from the partial differential equation and transformed into a system of ordinary differential equations by using an appropriate similarity transformation. The nonlinear ordinary differential equations are considered for different types of nanoparticles, which are copper and silver. The problem is numerically handled to investigate the influence of the nanofluid solid volume fraction or nanoparticle volume fraction parameter. The results showed how the governing parameters affected the skin friction coefficient, local Nusselt number, temperatures, and velocity profiles. It is found that the nanoparticle volume fraction has a substantial impact on fluid flow and heat transfer characteristics.

Keywords: Nanofluid, Stretching/Shrinking Surfaces, Shooting Technique, Runge-Kutta-Felhberg, Heat Transfer.

1. Introduction

Nanofluid is an intriguing topic in nanoscience. As defined in [1] it is the suspension of nanometer-sized particles ranging in size from 1 nm to 100 nm in a common fluid. Nanofluids are expected to have superior heat transfer characteristics due to the presence of nanoparticles that increase thermal conductivity. The scientists added a single type of nanosized particle to the fluid to form a compound called nanofluid, which was first introduced by [2], to improve the deficiency. According to research, nanoparticles have a high potential because they improve the heat transfer rate and thermal conductivity of the fluid. An examination of the heat transfers and the flow properties for two-dimensional oblique

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stagnation point flow of viscous, incompressible nanofluid flow over a curved stretching/shrinking surface in [3]. The nanofluid mixture is made by suspending aluminium oxide nanoparticles in water as a base fluid.

The impact of a nonlinearly permeable stretched sheet and a porous material on a three-dimensional magnetohydrodynamic fluid stream across it in terms of heat production or absorption that is dependent on space and temperature investigated in [4]. In the presence of a uniform magnetic field, an electrically conducting fluid is considered. The boundary layer theory and a low magnetic Reynolds number are used to solve the problem. A system of ordinary differential equations replaces the governing partial differential equations and are numerically solved by using the RKF-45 technique. The recent scientific investigation of heat transfer of nanofluid flow over a shrinking surface in the presence of thermal radiation is described in [5]. The energy equations indicate the effect of heat radiation. The governing nonlinear semi-differential equations for momentum and energy are converted into dimensionless ordinary differential equations together with the convective boundary conditions on the temperature profile using the similarity variables. After applying the transformations, the emerging parameters were calculated analytically to obtain an accurate solution to the two properties. Due to the decreasing surface, which also influences the temperature profile, a dual-nature solution is formed. The results for velocity, temperature, skin friction, Nusselt number, and current line were observed.

In the present study, the problem of nanofluid flow over a permeable stretching or shrinking surface investigated by [6] is solved using the shooting technique with Runge-Kutta-Felhberg (RKF45) in Maple software with different types of nanoparticles, which are copper (Cu) and silver (Ag). Runge Kutta Fehlberg (RKF45) is a fourth-order approach with a fifth-order error estimator. The error in the solution may be calculated and controlled by employing the higher-order embedded technique, which allows for the automated determination of adaptive step size. RKF45 method which can provide even more accurate solutions than bvp4c solver. Even though the RKF45 method is quite complex in its iterations, it encompasses an analysis of four slopes to approximate the point desired. The governing equation is a three-dimensional partial differential equation, where it will be reduced to ordinary differential equations by applying similarity transformations technique. The results of present study are compared to those obtained by [6] for several values of skin friction coefficient when other parameters are kept constant.

2. Methodology

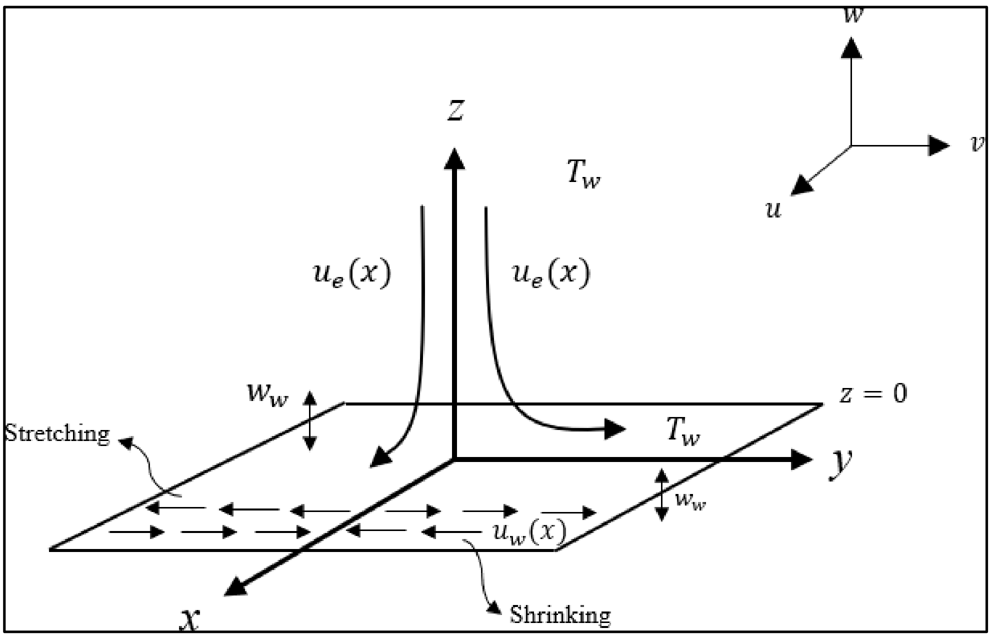

Figure 1: Physical model and coordinate system for stretching/shrinking surfaces

Based on Figure 1, the physical models and coordinate system for the three-dimensionalboundary layer flow and heat transfer near the stagnation point over a permeable stretching and shrinking surface in a water-based nanofluid comprising copper (Cu) and silver (Ag) are considered. Under the speculations and by applying the nanofluid model equations published in [6], the governing boundary layer equations of the nanofluid flow for the continuity, momentum, and energy equations may be expressed as follows, respectively:

$$\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}}+\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{y}}+\frac{\partial \boldsymbol{w}}{\partial \boldsymbol{z}}=\mathbf{0}, \qquad \boldsymbol{Eq}.1$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=u_e\frac{\partial u_e}{\partial x}+v_{nf}\frac{\partial^2 u}{\partial z^2}, \qquad Eq.2$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\mu_{nf}\frac{\partial^2 T}{\partial z^2}, \qquad Eq.3$$

subject to the boundary conditions:

$$u=u_w(x)=ax, \quad v=0, \quad w=w_w, \quad T=T_w \ \ at \ \ z=0,$$
$$u=u_e(x)=cx, \quad T=T_\infty \ \ as \ \ z\to\infty, \qquad Eq.4$$

Table 1: Fluid and nanoparticles thermophysical properties [7]

Physical properties	Fluid phase (water)	Cu	Ag
C_p(J/kg K)	4179	385	235
ρ(kg/m^3)	997.1	8933	10500
k(kg/m^3	0.613	400	429

where $u_w(x)$ is the sheet stretching/shrinking velocity, $u_e(x)$ is the velocity of the nanofluid's external flow with c is a constant, $c>0$ for a stretching surface and $c>0$ for a shrinking surface. The velocity of mass flux is $w_w>0$ for injection and $w_w<0$ for suction. Further, μ_{nf} and α_{nf} are the effective viscosity of the nanofluid and the thermal diffusivity of the nanofluid respectively which are defined as follows:

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}, \alpha_{nf}=\frac{k_{nf}}{(\rho C_p)}, \frac{k_{nf}}{k_{nf}}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)},$$
$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s, \rho_{nf}=(1-\phi)\rho_f+\phi\rho_s, \qquad Eq.5$$

where $\phi, \rho_{nf}, (\rho C_p)_{nf}, k_{nf}, \rho_f, \rho_s, \mu_f, k_f, k_s, (\rho C_p)_f, (\rho C_p)_s$ are the nanoparticle volume fraction, effective density of the nanofluid, heat capacity of the nanofluid, effective thermal conductivity of the nanofluid, reference density of the fluid fraction, reference velocity of the solid fraction, viscosity of the fluid fraction, thermal conductivity of the fluid, thermal conductivity of the solid, heat capacity of the fluid and the heat capacity of the solid respectively. By referring to [8] and [9], the similarity solutions are depicted in terms of the following variables:

$$u=cxf'(\eta), v=c(m-1)yf'(\eta), w=-\sqrt{cv}mf(\eta), \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \eta=\sqrt{\frac{c}{v}}z \qquad Eq.6$$

Here, we have $m = 1$ when the sheet shrinks in one direction and $m = 2$ when the sheet shrinks asymmetrically. *Eq. 1* is automatically satisfied. Then, by substituting $Eq.$ 6 into $Eq.$ 2 and $Eq.$ 3, the partial differential equations are generated to the ordinary differential equations:

$$\kappa_1 f''' + mff'' + 1 - f'^2 = 0, \qquad Eq. 7$$

$$\kappa_2 \theta' + Prmf\theta' = 0. \qquad Eq. 8$$

The boundary conditions $Eq.$ 4 become:

$$f(0) = s, f'(0) = \lambda, \theta(0) = 1, f'(\eta) \to 1, \theta(\eta) \to 0 \ as\ \eta \to \infty, \qquad Eq. 9$$

where $Pr = v_f/\alpha_f$ is the Prandtl number,

$s = -w_w/(cv)^{1/2}$ is the suction parameter $(s > 0)$, $\lambda = \alpha/c$ is the stretching $(\lambda > 0)$ or shrinking $(\lambda < 0)$ parameter, and κ_1 and κ_2 are two constants relate to the nanofluid's properties that can be presented as follow:

$$\kappa_1 = \frac{1}{(1-\phi)^{2.5}(\frac{1-\phi+\phi\rho_s}{\rho_f})}, \kappa_2 = \frac{k_{nf}/k_f}{(1-\phi)+(\frac{\phi(\rho C_p)_s}{(\rho C_p)_f})}. \qquad Eq. 10$$

The skin friction coefficient C_f and the local Nusselt number Nu are the two physical quantities:

$$Re_x^{1/2} C_f = \frac{1}{(1-\phi)^{2.5}} f''(0), Re_x^{-1/2} Nu_x = -\frac{k_{nf}}{k_f}\theta'(0), \qquad Eq. 11$$

where the dimensionless parameter of local Reynolds number is, $Re_x = u_e(x)x/v$.

3. Results and Discussion

The mathematical formulation of the nonlinear ordinary differential equations $Eq.$ 7 and $Eq.$ 8 along with the boundary conditions $Eq.$ 9 were obtained using shooting technique with Runge-Kutta-Felhberg (RKF45) in Maple software for different values of the nanoparticle volume fraction ϕ. The relative thickness of the momentum and thermal boundary layers are governed by the Prandtl number in the heat transfer problem. When Pr is small, it means that the heat diffuses quickly compared to the velocity. The Prandtl number of the base fluid is fixed at Pr = 6.7850; usually, the Prandtl number is assumed to be around 6.9 for water and Table 1 shows the thermophysical characteristics of the fluids and nanoparticles used in this investigation.

Table 2: Values of $f''(0)$ for $\phi = 0$ for Cu nanoparticles with different m and $s = 0$

	Hafidzuddin [6]		Present Results	
λ	$m = 1$	$m = 2$	$m = 1$	$m = 2$
1	0.0000	0.0000	0.0000	0.0000
0	1.232588	1.311938	1.232588	1.311938
-1	1.32882	0.30360	1.32882	0.30360

Table 2 is displayed to validate the results of the current method with the results obtained by [5] for various values of stretching/shrinking parameter (regular Newtonian fluid). The comparison in Table 2 are found to be in very good agreement, so we are certain that the results of the current study are acceptable.

According to Figure 2, the effect of silver nanoparticle volume fractions on the velocity profile of the nanofluid flow is smaller compared to copper. Based on Figure 3, we can observed that the heat transfer rate at the surface for copper is lower than sliver nanoparticles. Figures 2 and 3 show that the velocity and temperature profiles satisfied the far field boundary conditions asymptotically.

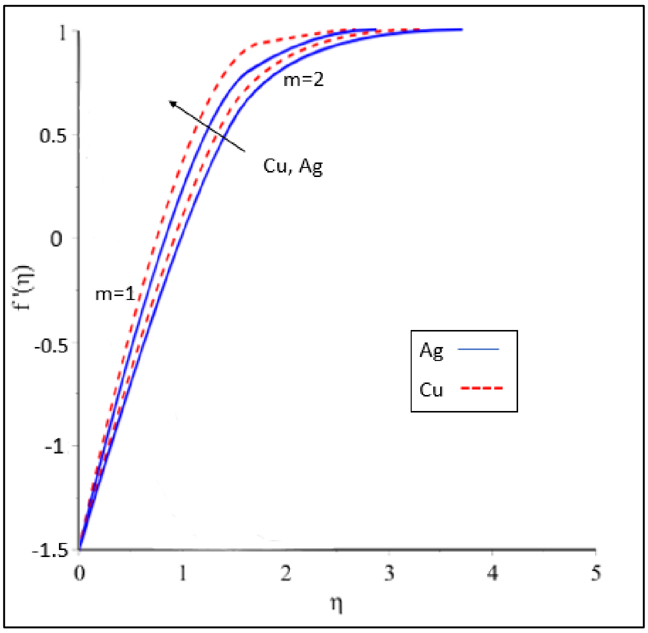

Figure 2: Velocity profiles $f''(\eta)$ for various of nanoparticles when $m = 1, 2, \lambda = -1.5, s = 0$ and $\phi = 0.1$

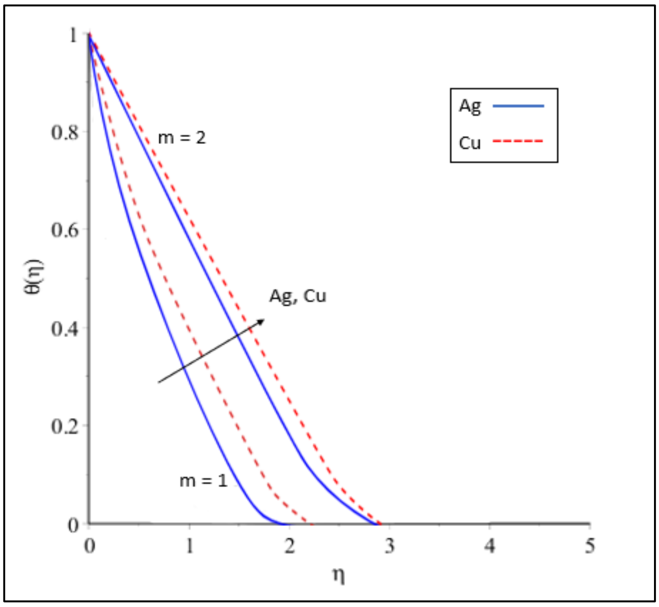

Figure 3: Temperature profiles $\theta(\eta)$ for various of nanoparticles when $m = 1, 2, \lambda = -1.5, s = 0$ and $\phi = 0.1$

4. Conclusion

In this study, the consequences of the nanofluid's solid volume fraction or nanoparticle volume fraction parameter on heat transfer in a nanofluid across a permeable stretching and shrinking sheet has been investigated. To validate the numerical values obtained, the outcomes of this research have been compared with the existing study in [6], and the results show a high degree of agreement. The nonlinear ordinary differential equations for two types of nanoparticles, copper and silver in the base fluid of water have been solved numerically with Prandtl number is fixed at 6.7850. For various values of the governing parameters, the skin friction coefficient, local Nusselt number, and velocity and temperature profiles are displayed and analyzed. The nanoparticle volume fraction is found to has a substantial impact on fluid flow and heat transfer characteristics.

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