CHAPTER 6

COST-EFFECTIVE COMMUNICATION NETWORK DESIGN USING PRIM'S AND KRUSKAL'S ALGORITHMS

*Muhammad Haziq Sufyan bin Suhaimi, Mohamad Nasrullah bin Delwes Penaroya, Muhammad Haziq Fikri bin Mohd Hasnul Redza, Mohamad Haqif Amri bin Roslan, Jamilah binti Mohd Ghazali**

Centre for Diploma Studies, Universiti Tun Hussein Onn Malaysia, Pagoh Higher Education Hub, 84600 Pagoh, Johor, MALAYSIA.
*Corresponding Email: jamilahg@uthm.edu.my

6.0 INTRODUCTION

Constructing a Minimum Spanning Tree (MST) using Prim's and Kruskal's algorithms provides an effective solution for minimizing the total cost of connecting multiple houses. A telecommunication company seeks to interconnect nine houses at the lowest possible cost. Applying graph-based optimization techniques determines the most efficient and cost-effective way to establish the network [1].

Designing an efficient and cost-effective communication network is a fundamental problem in network optimization. A well-structured network can significantly reduce infrastructure costs while ensuring full connectivity [2]. The problem of network design has been extensively studied in operations research, where various algorithms, including Prim's and Kruskal's, have been employed to develop optimal solutions. The MST is one of the most widely used methods for optimizing communication networks, transportation systems, and electrical grids [3].

In this study, the problem is modelled as a weighted undirected graph, where houses are represented as nodes and communication cables as edges. The weight of each edge corresponds to the installation cost in Ringgit Malaysia (RM). The objective is to construct an MST that connects all houses with the minimum total cost. Graph-based optimization approaches, particularly MST algorithms, are widely applied in designing telecommunication networks, ensuring cost efficiency while maintaining complete connectivity.

A telecommunications company requires an optimal plan to connect nine houses while minimizing total installation costs. By applying Prim's and Kruskal's algorithms, an MST can be constructed, providing an optimal and cost-efficient communication network design. The effectiveness of both algorithms is analyzed, comparing their performance in terms of computational efficiency and network cost reduction. The findings of this study will contribute to improving telecommunication infrastructure planning by providing a systematic approach to network design based on graph theory principles.

6.1 METHODOLOGY

The problem under consideration is modeled as a weighted undirected graph, a mathematical structure frequently used in network optimization. In this representation, individual houses serve as nodes, while the communication cables connecting them are depicted as edges. Each edge is assigned a weight corresponding to the installation cost in RM ensuring a cost-based approach to network design. A well-structured communication network is essential to minimize infrastructure costs while maintaining full connectivity. By leveraging graph theory principles, particularly MST algorithms, the most efficient and cost-effective network configuration can be identified. MST techniques are widely utilized in telecommunications, transportation, and electrical grid systems to reduce costs while ensuring uninterrupted connectivity.

For this study, Graph Online [4], a digital graph visualization tool, was used to construct and analyze the network model, as illustrated in Figure 6.1. This tool facilitates the representation of different connectivity scenarios, allowing for an optimized selection of edges that collectively form an MST with the minimum total installation cost. By applying Prim's and Kruskal's algorithms, the optimal network structure is determined, ensuring efficient resource allocation and long-term cost savings. Table 6.1 lists these edges and their corresponding weights.

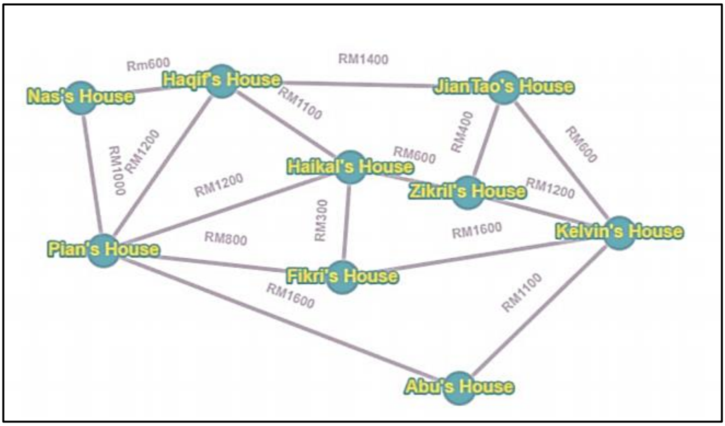

Figure 6.1: An undirected weighted graph representing installation cost in RM

Table 6.1: Edges and Weights in the Graph Representation

No.	Edge	Cost (RM)
1	Haikal's House - Fikri's House	300
2	Haikal's House - Zikril's House	600
3	Zikril's House - JianTao's House	400
4	Fikri's House - Pian's House	800
5	JianTao's House - Kelvin's House	600
6	Haikal's House - Haqif's House	1100
7	Haqif's House - Nas's House	600
8	Kelvin's House - Abu's House	1100

6.1.1 Prim's Algorithm

Prim's algorithm is a greedy algorithm used to construct a MST by incrementally adding edges with the lowest possible cost while ensuring that all nodes remain connected without forming cycles [5]. The algorithm begins with an arbitrary starting node and progressively selects the shortest available edge that links a visited node to an unvisited node. This process continues until all nodes are included in the MST, resulting in an optimized network with minimal overall cost. By using a priority queue, the algorithm efficiently determines the most cost-effective edges, making it particularly useful for dense graphs where a large number of connections exist between nodes. In the context of this study, Prim's algorithm is applied to determine the optimal way to interconnect houses within the network while minimizing installation costs.

Table 6.2: Calculation using Prim's Algorithm

Iteration	Edges	Visited Nodes	Unvisited Nodes	Cost (RM)
1	Haikal's House - Fikri's House	Haikal, Fikri	Others	300
2	Fikri's House - Pian's House	Haikal, Fikri, Pian	Others	800
3	Haikal's House - Zikril's House	Haikal, Fikri, Pian, Zikril	Others	600
4	Zikril's House - JianTao's House	Haikal, Fikri, Pian, Zikril, JianTao	Others	400

5	JianTao's House - Kelvin's House	Haikal, Fikri, Pian, Zikril, JianTao, Kelvin	Others	600
6	Haikal's House - Haqif's House	All Houses	-	1100
Total				RM 5500

The process begins with the smallest edge, ensuring minimal cost expansion while maintaining connectivity. The final MST efficiently interconnects all houses with a total cost of RM 5500.

6.1.2 Kruskal's Algorithm

Kruskal's algorithm begins by sorting all edges based on their weight and then iteratively adding the smallest edge to the MST, provided that it does not form a cycle. This process continues until all nodes are connected, ensuring that the resulting tree maintains the minimum possible total edge weight. Unlike Prim's algorithm, which grows the MST incrementally from a starting node, Kruskal's algorithm considers the entire edge set and follows a global approach, making it particularly effective for sparse graphs with fewer edges [6]. The cycle detection mechanism, typically implemented using the Union-Find data structure, ensures that only valid edges are included in the MST. In the context of this study, Kruskal's algorithm is applied to determine the most cost-efficient way to interconnect houses within a telecommunication network while minimizing overall installation costs.

Table 6.3: Calculation using Kruskal's Algorithm

Iteration	Edges	Visited Nodes	Unvisited Nodes	Cost (RM)
1	Haikal's House - Fikri's House	Haikal, Fikri	Others	300
2	Zikril's House - JianTao's House	Haikal, Fikri, Zikril, JianTao	Others	400
3	Nas's House - Haqif's House	All Houses	-	600
Total				RM 5500

The application of both Prim's and Kruskal's algorithms resulted in the construction of a MST with an overall cost of RM 5500. By systematically selecting edges to ensure full connectivity while minimizing total installation expenses, both algorithms demonstrate their practical applicability in real-world network planning and infrastructure optimization.

6.2 CONCLUSION

Both Prim's and Kruskal's algorithms constructed the MST with a total cost of RM 5500. This result demonstrates the effectiveness of both algorithms in solving the MST problem for communication network design. This highlights their applicability to real-world problems like minimizing infrastructure costs while ensuring full connectivity.

6.3 REFERENCES

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