

Engineering Calculus

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Abstract:

Calculus is a fundamental pillar of engineering mathematics, providing the essential tools to model, analyze, and solve real-world problems. This book, designed specifically for engineering students, aims to offer a comprehensive and intuitive introduction to the principles of calculus. Structured to enhance understanding and promote active learning, the book begins with differentiation, gradually building upon basic functions and advancing to more complex rules and techniques. Students will find a clear progression through chapters, covering topics such as the differentiation of parametric and implicit functions, logarithmic differentiation, and the application of differentiation to rates of change and curve analysis.

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This book is designed to not only teach the principles of calculus but also to highlight its practical significance and versatility in engineering. By working through the examples and exercises, students will develop a solid understanding of the material and gain confidence in applying calculus to solve engineering problems. It is my hope that this book serves as a valuable resource, helping students build a strong mathematical foundation that will support their academic and professional growth.

Keywords: Analysis of functions, Chain rule, Gradient of curve at a point, Parametric curve, Tangent line to a curve

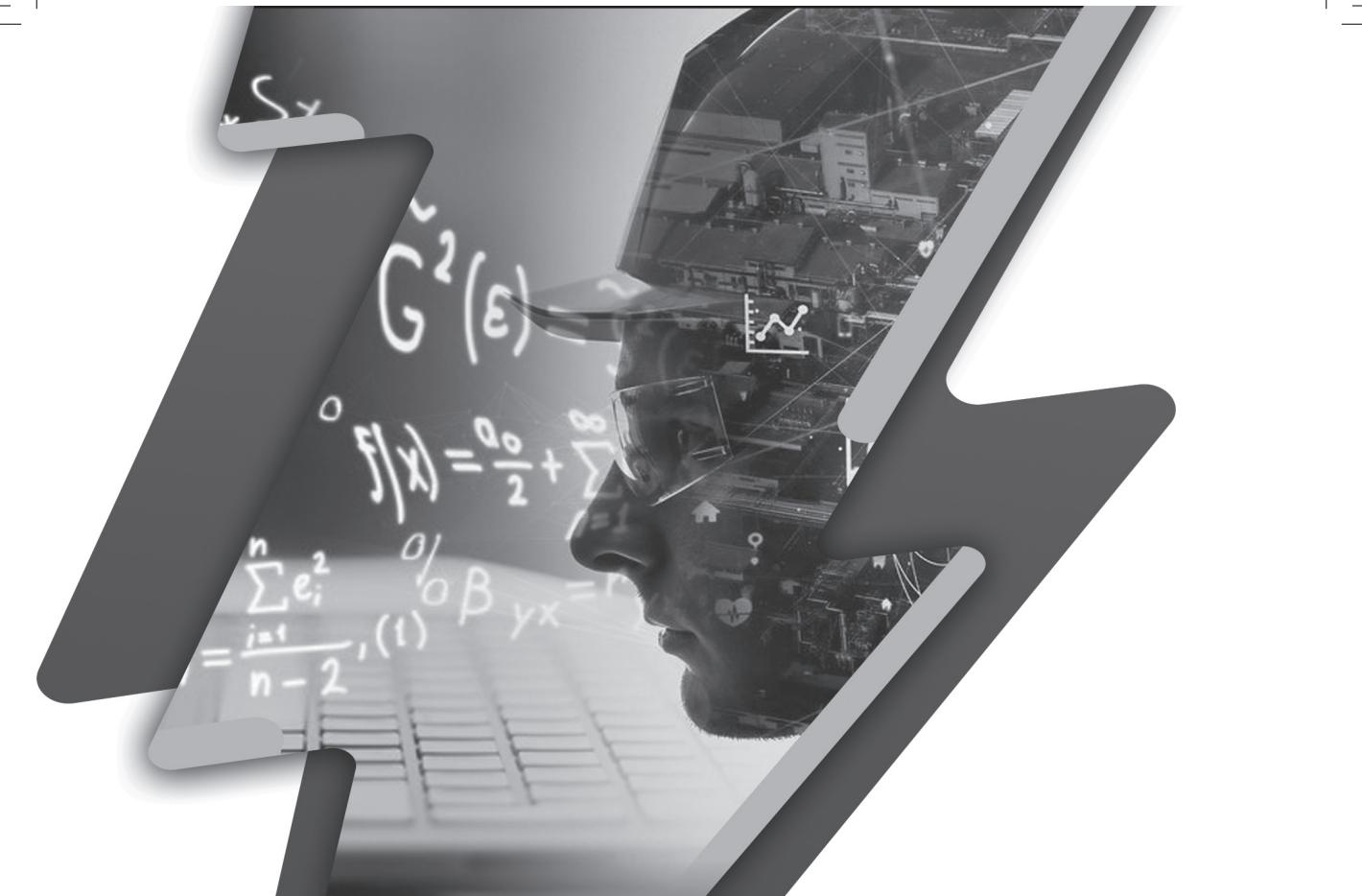


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Preface

Calculus is a fundamental pillar of engineering mathematics, providing the essential tools to model, analyze, and solve real-world problems. This book, designed specifically for engineering students, aims to offer a comprehensive and intuitive introduction to the principles of calculus. Structured to enhance understanding and promote active learning, the book begins with differentiation, gradually building upon basic functions and advancing to more complex rules and techniques. Students will find a clear progression through chapters, covering topics such as the differentiation of parametric and implicit functions, logarithmic differentiation, and the application of differentiation to rates of change and curve analysis.

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CHAPTER 1

Differentiation

1.1 DIFFERENTIATION OF COMMON FUNCTIONS

- A general rule for differentiating $y = ax^n$ emerges, where a and n are constants.
- The rule is: if $y = ax^n$ then $\frac{dy}{dx} = anx^{(n-1)}$
- For example, if $y = 4x^3$ then $a = 4$ and $n = 3$, and

$$\frac{dy}{dx} = anx^{(n-1)} = (4)(3)x^{(3-1)} = 12x^2$$

If $y = ax^n$ and $n = 0$ then $y = ax^{(0)}$ and

$$\frac{dy}{dx} = (a)(0)x^{(0-1)} = 0$$

- Figure 1 shows a graph of $y = \sin x$. The gradient is continually changing as the curve moves from 0 to A to B to C to D. The gradient, given by $\frac{dy}{dx}$, may be plotted in a corresponding position below $y = \sin x$.

CHAPTER 2

Applications of Differentiation

2.1 RATES OF CHANGE

2.1.1 Introduction

- Rates of change is a rate of how much one quantity change in relation to another quantity.
- If y is a function of x , for example, $y = 2x^2 + 5x$.
Then, $\frac{dy}{dx}$ is the rate of change of y with respect to x .
- In other words, rates of change is how much something changes over how much something else changes.

For example: Rate of change = $\frac{\text{Change in distance, } dx}{\text{Change in time, } dt}$

$$= \frac{dx}{dt}$$

- Rate of change can be positive (increases) or negative (decreases). When a quantity does not change over time, it is called zero rate of change.

Example 2.1

At time t , the volume of water in a leaking tank is $V \text{ cm}^3$, where

$$V = 75 - 10t - 2t^2.$$

Find the rate of water flows from the tank at $t = 5$ seconds.

CHAPTER 3

Integration

3.1 INTEGRATION AS ANTI-DERIVATIVE

- The reverse of differentiating is anti-differentiating and the result of anti-differentiating is called an anti-derivative.

THEOREM

If $F(x)$ is an anti-derivative of a function $f(x)$, then

$$\int f(x)dx = F(x) + C$$

The constant C is called the **constant of integration**.

This equation is read as “the anti-derivative of $f(x)$, with respect to x , is the set of functions $F(x) + C$.”

Example 3.1

Find an anti-derivative of $f(x) = 6x^3$.

Solution:

$$\int 6x^3 dx = \frac{6x^4}{6} + C$$

Simplify into:

$$= \frac{3}{2}x^4 + C \quad \text{Check: } \frac{dy}{dx} \left(\frac{3}{2}x^4 + C \right) = \frac{3}{2}(4x^3) = 6x^3$$

CHAPTER 4

Applications of Intergration

4.1 AREA BETWEEN TWO CURVES

- There are two cases, which are the area between two curves with respect to the x and y -axis.
- Case I

If f and g are continuous functions on an interval $[a, b]$, and if $f(x) \geq g(x)$ for all x in $[a, b]$ (**Figure 4.1**), then the area of the region bounded above by $y = f(x)$, below by $y = g(x)$, on the left by the line $x = a$, on the right by the line $x = b$ is

$$A = \int_a^b [f(x) - g(x)] dx$$

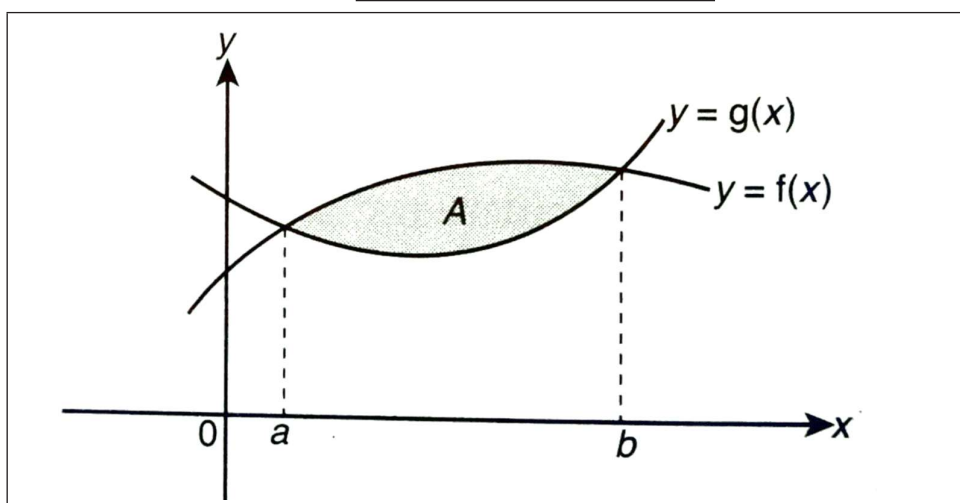


Figure 4.1: Area between two curves (upper and lower functions)

CHAPTER 5

First Order Differential Equation

5.1 FORMATION AND SOLUTION OF DIFFERENTIAL EQUATION

- A differential equation is an equation involving an unknown function and its derivatives.
- In general, differential equations often appear in mathematical models to predict future values of physical quantities based on present quantities and rates of change.
- Two (2) major types of differential equations are Ordinary Differential Equation (ODE) and Partial Differential Equation (PDE). ODE involves a function of a single variable and its derivatives while PDE involves a multi-variable function and its partial derivatives.
- **Ordinary Differential Equation**
A differential equation which contains only ordinary derivatives with at least one of the differential coefficients is called an ordinary differential equation.
- Examples of ordinary differential equations;

$$\frac{dy}{dx} - 3x = 2y$$

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} - x = 0$$

$$y''' + y'' + y' - 9y = e^x$$

$$x^2y'' - xy' + 9y = x$$

Synopsis

This book is designed specifically for engineering students, aims to offer a comprehensive and intuitive introduction to the principles of calculus. Structured to enhance understanding, the book begins with differentiation, gradually building upon basic functions and advancing to more complex rules and techniques. Students will find a clear progression through chapters, covering topics such as the differentiation of parametric and implicit functions, logarithmic differentiation, and the application of differentiation to rates of change and curve analysis.

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Dr. Noorwirdawati Ali is a Senior Lecturer in the Department of Civil Engineering, Faculty of Civil Engineering and Built Environment at Universiti Tun Hussein Onn Malaysia (UTHM). She earned her Bachelor's degree in Civil Engineering from Universiti Teknologi Malaysia in 2006 and Doctor of Philosophy in Civil Engineering from UTHM in 2014. Dr. Noorwirdawati teaches a range of subjects, including Reinforced Concrete Design, Structural Steel Design, Calculus and others. Her research focuses on repair and strengthening in material engineering, with a particular emphasis on structural applications besides research on construction materials. Dr. Noorwirdawati has authored over 60 academic publications, and her work has been widely cited, underscoring her contributions to the field. Through her teaching and research, Dr. Noorwirdawati continues to advance knowledge and foster innovation in engineering education and practice.



Ts. Dr. Sharifah Salwa Mohd Zuki is a Senior Lecturer in the Department of Civil Engineering at the Faculty of Civil Engineering and Built Environment, Universiti Tun Hussein Onn Malaysia (UTHM). She earned her Doctor of Philosophy in Structural Engineering from Universiti Sains Malaysia in 2017 and her Bachelor's degree in Civil Engineering from Universiti Teknologi Malaysia in 2009. Dr. Salwa's expertise lies in civil engineering, with a specialization in structural engineering. Her research interests encompass fire engineering, structural retrofitting, composite structures, and green building technology. In academia, Dr. Salwa has demonstrated a strong commitment to teaching and curriculum development. She has coordinated and taught courses such as Calculus, Mechanics of Materials, Statics and Dynamics, and Structural Design. She has also contributed to various university committees focused on academic quality assurance, accreditation, and program development.



Assoc. Prof. Ir Dr. Faisal Sheikh Khalid is a prominent Associate Professor at Universiti Tun Hussein Onn Malaysia (UTHM), renowned for his impactful contributions to research, academia, and engineering innovation. His work expertise lies in advanced materials, structure reinforced concrete, sustainable engineering and smart infrastructure systems, where he has consistently demonstrated excellence through groundbreaking studies and innovative solutions. His extensive portfolio includes numerous high-impact publications in internationally indexed journals, conference proceedings, and technical reports. His research outputs have not only expanded the knowledge base but also led to practical advancements that benefit industry and society. Dr. Faisal's achievements in the academic and professional arenas are underscored by his active involvement in research grants and collaborations, both locally and internationally. He has led and participated in funded projects that address critical issues in sustainable building materials development, structural and concrete design, contributing significantly to global research ecosystem. In addition to his academic endeavors, Dr. Faisal actively contributes to professional organizations and engineering communities, sharing his knowledge and expertise to foster sustainable growth in the field.



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