# Introduction to Differential Equations for Engineering Students

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**Abstract:**

This book is written based on the teaching experience and the lecture notes of the authors at Universiti Tun Hussein Onn Malaysia, Malaysia and Telkom University, Indonesia, for more than 15 years. The chapters in this book are designed to cover the syllabus for the course of differential equations, which is one of the compulsary mathematics courses for most of the engineering programme in university. This book consists of six chapters, included Chapter 0: short Mathematical Revision, which is an introduction and a brief revision on basic mathematic that required for the fundamental differential equations. The contents start with the basic concept of ordinary differential equations (ODEs), which is the most immediately useful part of the calculus. The next chapters followed by the first and second order of ODEs with constant coefficient, Laplace transforms, Fourier series and ended with an introduction to partial differential equations.

**Keywords:** differential equations, engineering programme, calculus.

# Introduction to DIFFERENTIAL EQUATIONS for Engineering Students

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# PREFACE

In the name of Allah, the Almighty who give us the enlightenments, the truth, the knowledge and with regards to Prophet Muhammad for guiding us to the straight path. We thanks to Allah for giving guidance and strength to write this book.

This book is written based on the teaching experience and the lecture notes of the authors at Universiti Tun Hussein Onn Malaysia, Malaysia and Telkom University, Indonesia, for more than 15 years. The chapters in this book are designed to cover the syllabus for the course of differential equations, which is one of the compulsory mathematics courses for most of the engineering programme in university. This book consists of six chapters, included Chapter 0: Short Mathematical Revision, which is an introduction and a brief revision on basic mathematic that required for the fundamental differential equations. The contents start with the basic concept of ordinary differential equations (ODEs), which is the most immediately useful part of the calculus. The next chapters followed by the first and second order of ODEs with constant coefficient, Laplace transforms, Fourier series and ended with an introduction to partial differential equations.

Mathematician scholar said that confidence is half the battle in mathematics. Therefore, this book was written through the effort of "learning of example" so that the students have more confidence for self-learning. It is very encouraging to learn how to do something to be able to get it nearly right most times. Finally, we hope that this book will meet the requirements and the expectations of all the undergraduate engineering students. For further improvement, we shall highly appreciate and gratefully receive and an acknowledge every comment, suggestions for inclusion or exclusion of topics and error in this book from the readers.

Bukhari Manshoor
Azian Hariri
Hamidon Salleh

# *A Short Mathematical Revision*

## 0.1 Definition of Derivative

The derivative of the function $y = f(x)$, sometimes written as $dy/dx$ or $f'(x)$ or is defined as the slope of the tangent line to the curve $y = f(x)$ at the point $(x, y)$. This slope is obtained by a limit, and is defined as,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \tag{0.1}$$

### 0.1.1 Differentiating a Combination of Functions

**Sum or difference rule**

The derivative of the sum of $f(x)$ and $g(x)$ is $(f + g)' = f' + g'$.

Similarly, the derivative of difference is $(f - g)' = f' - g'$.

**Product rule**

The derivative of the product of $f(x)$ and $g(x)$ **is,**

$$(fg)' = f'g + fg'$$

and should be memorized as "*the derivative of the first times the second plus the first times the derivative of the second*."

**Quotient rule**

The derivative of the quotient of $f(x)$ and $g(x)$ is,

$$\left(\frac{f}{g}\right)' = \frac{(f'g - fg')}{g^2}$$

and should be memorized as "*the derivative of the top times the bottom minus the top times the derivative of the bottom over the bottom squared*."

**Chain rule**

The derivative of the composition of $f(x)$ and $g(x)$ is,

$$\left[f\left(g(x)\right)\right]' = f'\left(g(x)\right).g'(x)$$

and should be memorized as "*the derivative of the outside times the derivative of the inside*."

### 0.1.2 Differentiating Elementary Function

**Power rule**

The derivative of a power of x is given by,

$$\frac{d\left(x^p\right)}{dx} = px^{p-1}$$

**Trigonometric functions**

The derivatives of $\sin x$ and $\cos x$ are,

$$\left(\sin x\right)' = \cos x \quad \text{and} \quad \left(\cos x\right)' = -\sin x$$

We can say that **"***the derivative of sine is cosine,"* and *"the derivative of cosine is minus sine***."** For the second derivatives of both $\sin x$ and $\cos x$, its satisfy,

$$\left(\sin x\right)'' = -\sin x \quad \text{and} \quad \left(\cos x\right)'' = -\cos x$$

# *Ordinary Differential Equations – Basic Concepts*

## 1.1 Introduction

In our daily life, there are a number of problems that rise to different types of a differential equation. A differential equation is an equation for a function that relates the values of the function to the values of its derivatives. As a simple briefing, an ordinary differential equations (ODEs) is a differential equation for a function of a single variable, for example $x(t)$, while a partial differential equations (PDEs) is a differential equation for a function of several variables, such as $V(x, y, t)$. An ODEs contains ordinary derivatives and a PDEs contains partial derivatives. As we can see that the PDEs have several variables and typically PDE's are much harder to solve than ODE's. For the beginning, we will start with the understanding of basic concepts of ODE.

Solution of the engineering problem (basically in daily life problem that related to ODEs) begins with deriving the problem from physical or other problems (modelling), solving them by standard mathematical methods, and interpreting solutions and their graphs in terms of a given problem, as simplified in Figure 1.1.

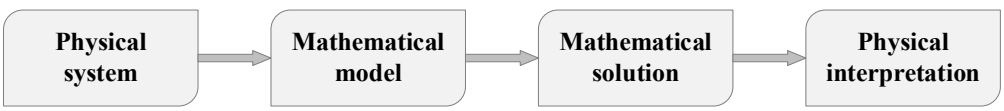


**Figure 1.1:** Modelling, solving and interpreting

Let consider some examples of the problem around that involves ordinary differential equations (ODEs). Classical case of the motion of an object that falls under the influence of gravity can be expressed in term of ODEs as,

$$\frac{d^2y}{dt^2} = g \tag{1.1}$$

where $y$ is a distance travelled by the object in time $t$ .

In heat transfer, the rate of cooling of some object is determined by Newton's Law of cooling. Newton's Law of cooling state that the rate of change of temperature (in time) is directly proportional to the difference between the temperature of the object and that of the surrounding temperature. The statement can be written in ODEs as,

$$\frac{dT}{dt} = -k\left(T - T_s\right)$$

where $T$ is the temperature of the body while $T_s$ is surrounding temperature at time $t$. The constant $k$ here known as a constant of proportionality, represents how fast the heat propagates.

For application in mechanical motion, we can see some examples that relate with spring system. For the free oscillations, a basic mechanical system consisting of a mass on an elastic spring, which moves left and right. The system can be written in ODEs as,

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0 \tag{1.3}$$

where $m,c$ and $k$ are the mass, damper coefficient and spring constant, respectively, and $x$ is the displacement of the body from its equilibrium position at time $t$.

Each examples of ODEs above, we need to determine a solution which is in the form of dependent variable expressed in terms of it independent variables. For the case of object that falls under the influence of gravity, the solution is in terms of $t$. $T$ in terms of $t$ for the heat transfer case and $x$ in terms of $t$ for the spring system case. The solution for each cases known as a general solution since the solution have some constants. To obtain the particular solution of the ODEs, we need to have an initial and boundary condition which is also be discuss briefly in this basic concept of ODEs.

# 1st Order Ordinary Differential Equations

## 2.1 Introduction

The general 1st order ordinary differential equation for the function $y = f(x)$ is given by

$$\frac{dy}{dx} = F(x, y) \tag{2.1}$$

where $f(x,y)$ can be any function of the independent variable $x$ and the dependent variable $y$. If $f(x,y) = -M(x,y) / N(x,y)$, this equation can be written as

$$M(x, y)dx + N(x, y)dy = 0 \tag{2.2}$$

Although it is not always possible to find an analytical solution of the differential equations for $y = f(x)$, it is always possible to determine a unique numerical solution given an initial value $f(x_0) = y_0$, and provided $f(x_0, y_0)$ is a well-behaved function. The differential equation (2.1) gives the slope $f(x_0, y_0)$ of the tangent line to the solution curve $y = y(x)$ at the point $(x_0, y_0)$. With a small step size $\Delta x = x_1 - x_0$ , the initial condition $(x_1 - x_0)$ can be marched forward to $(x_1, y_1)$ along the tangent line using Euler's method, as in Figure 2.1.

$$y_1 = y_0 + \Delta x f(x_0, y_0)$$

This solution $(x_1, y_1)$ then becomes the new initial condition and is marched forward to $(x_2, y_2)$ along a newly determined tangent line with slope given by $f(x_1, y_1)$. For small enough $\Delta x$, the numerical solution converges to the exact solution.

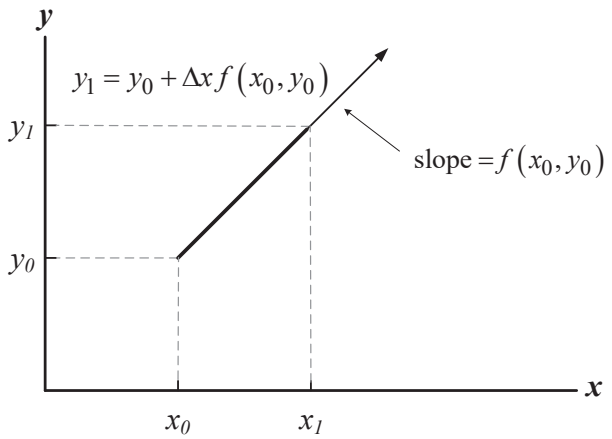


**Figure 2.1:** The differential equation $dy/dx = f(x, y)$, $y(x_0) = y_0$, is integrated to $x = x_1$ using the Euler method $y_1 = y_0 + \Delta x\, f(x_0, y_0)$, with $\Delta x = x_1 - x_0$

## 2.2 Methods of Solution

In this chapter, we are going to show how to determine a numerical solution of the 1st order ordinary differential equation, and then learn techniques for solving analytically some special forms of the differential equation, namely separable, homogenous (reduction to separable equations), linear and exact of 1st order equations. The four types of differential equations according to the form of $F(x,y)$ are brief as in Table 2.1.

**Table 2.1:** Differential equations according to the form of $F(x,y)$

| Types of differential equations | $F(x,y)$ |
|---|---|
| Separable | $F(x,y) = f(x)g(y)$ |
| Homogeneous | $F(x,y) = F\left(\frac{y}{x}\right)$ |
| Exact | $F(x,y) = -\frac{M(x,y)}{N(x,y)}$ |
| Linear | $F(x,y) = r(x) - p(x)y$ |

# $2^{nd}$ Order Ordinary Differential Equations

## 3.1 Introduction

From the previous chapter, the $1^{st}$ order ordinary differential equation has been discussed. The solution of the linear equation can be solved by using integrating factor. Generally, the integrating factor is written in the form of an exponential function. For this chapter, we centred only on linear ordinary differential equations of second order. In our discussions, we will look at into the classification of the equations, followed by the concepts of linear dependence and independence. The most important part that we will discussed is the solutions of linear homogeneous and non-homogeneous differential equations with constant coefficients. For the non-homogeneous linear differential equations, there are the most common technique that can be used to solve this kind of equation, which are; undetermined coefficients method and variation of parameters method. We also will look briefly on the Euler equation, which is a special linear differential equation with variable coefficients by using certain substitution.

## 3.2 Terminology and Classification

The most frequently applying differential equation in the field of science and engineering is the $2^{nd}$ order ODEs. The one-dimensional heat equation of heat transfer, the dynamics equation of dynamics and the circuit equations of electricity are all the same. Although these are different phenomena, by make some assumption, they are often described by a common form of $2^{nd}$ order ODEs. The $2^{nd}$ order ODEs can be classified into two form which is ***Linear*** and ***Nonlinear***. The $2^{nd}$ order ODEs is called as linear if it can be written as follow.

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = F(x) \tag{3.1}$$

and nonlinear if it cannot be written in this form. For the equation above, $p$ $(x)$, $q$ $(x)$ and $F$ $(x)$ is function in $x$ variable. Linear equation is called as ***Homogeneous*** if $F$ $(x)$ in RHS of equation (3.1) is zero, $F$ $(x) = 0$, while if $F$ $(x) \neq 0$, then equation is called ***Nonhomogeneous.***

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = F(x) \tag{3.2}$$

The function $p(x)$ and $q(x)$ in equation (3.1) and (3.2) are known as a coefficients of ODEs. If $p(x)$ and $q(x)$ replaced by constant coefficient we call the differential equation as linear ODEs with constant coefficient, as in equation (3.3).

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = F(x) \tag{3.3}$$

Table 3.1 shows the summary and examples of the terminology for combination of linear, homogeneous and nonhomogeneous of the 2nd ODEs.

**Table 3.1:** 2nd order Ordinary Differential Equations

| Types of 2nd ODEs | Equations | Constant |
|---|---|---|
| Linear homogeneous | $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + x^2y = 0$ | $p(x) = x$ , $q(x) = x^2$ |
| Linear homogeneous with constant coefficient | $2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 6y = 0$ | $a = 2$ , $b = 4$ , $c = 6$ |
| Linear non-homogeneous | $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + x^2y = \cos(x)$ | $p(x) = x$ , $q(x) = x^2$ |
| Linear non-homogeneous with constant coefficient | $2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 6y = \cos(x)$ | $a = 2$ , $b = 4$ , $c = 6$ |

Let look at some examples to give more overview regarding the terminology and classification discussed above.

# *Laplace Transforms*

## 4.1 Introduction

Laplace transform methods have a key role to play in the modern approach to the analysis and design of engineering systems. The stimulus for developing these methods was the pioneering work of the English electrical engineer Oliver Heaviside (1850 -1925) in developing a method for the systematic solution of ODE's with constant coefficients. Research on the problem continued for many years before it was eventually recognized that an integral transform developed by the French mathematician Pierre Simon de Laplace (1749 -1827) almost a century before provided a theoretical foundation for Heaviside's work. It was also recognized that the use of this integral transform provided a more systematic alternative for investigating differential equations than the method proposed by Heaviside. It is this alternative approach that is the basis of the **Laplace transform method.**

The Laplace transform is an example of a class called **integral transforms,** and it takes a function $f(t)$ of one variable (which we shall refer to as **time**) into a function $F(s)$ of another variable $s$ **(the complex frequency).** The attraction of the Laplace transform is that it transforms differential equations in the $t$ (time) domain into algebraic equations in the $s$ (frequency) domain. Solving differential equations in the $t$ domain therefore reduces to solving algebraic equations in the domain. Another advantage of using the Laplace transform for solving differential equations is that initial conditions play an essential role in the transformation process, so they are automatically incorporated into the solution. The Laplace transform is therefore an ideal tool for solving initial-value problems such as those occurring in the investigation of electrical circuits and mechanical vibrations.

The main idea of this method is to Laplace transform the constant-coefficient differential equation for $f(t)$ into a simpler algebraic equation for the Laplace-transformed function $F(s)$, solve this algebraic equation, and then transform $F(s)$ back into $F(t)$. The correct definition of the Laplace transform and the properties that this transform satisfies makes this solution method possible. Schematically, a system may be represented as in Figure 4.1.

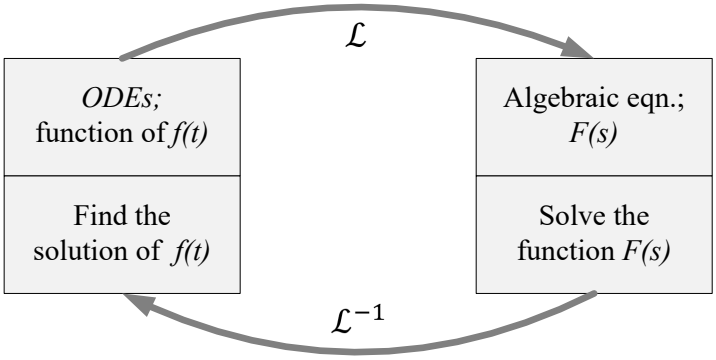


**Figure 4.1:** The Laplace transform operator

## 4.2 Definition and Notation

The Laplace transform of $f(t)$, denoted by $F(s) = \mathcal{L}\{f(t)\}$, is defined by the integral transform,

$$F(s) = \int_0^\infty e^{-st} f(t)\, dt \tag{4.1}$$

where is a complex variable and $e^{-st}$ is called the **kernel** of the transformation. It is usual to represent the Laplace transform of a function by the corresponding capital letter, so that we write,

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt \tag{4.2}$$

There are a few observations relating to the definition of equation (4.2).

a) The symbol $\mathcal{L}$ denotes the Laplace transform operator; when it operates on a function $f(t)$, it transforms it into a function $F(s)$ of the complex variable $s$. The operator transforms the function $f(t)$ in the $t$ domain (usually called the **time domain**) into the function $F(s)$ in the domain (usually called the **complex frequency domain**). This relationship is depicted graphically in Figure 4.1.

b) Because the upper limit in the integral is infinite, the domain of integration is infinite. Thus the integral is an example of an **improper integral**, that is,

# *Fourier Series*

## 5.1 Introduction

Many problems in engineering and science involved with a vibration and frequency oscillation, or in other speaking the problem are periodic. Some example of such problem are the current and voltage in an alternating current circuit, rotating parts of machines, vibrations of string, progression of waves and sound and others. These periodic functions can be analysed into their constituent components (fundamentals and harmonics) by a method called Fourier analysis. The Fourier analysis allows us to model phenomena which appear frequently in engineering problem that involved with periodic functions. Since an analysis problem that related to periodic functions may be complicated, the Fourier analysis come as an ingenious idea to represent the complicated function in term of simple periodic function namely cosines and sines series. This simplifying can be done by adding more and more trigonometric functions (cosines and sines series) together. The sum of these special trigonometric functions is called the Fourier Series.

Fourier series are the series of sine and cosine terms representing a general periodic function. The periodic function constitutes a very important tools in solving problems that involve ordinary and partial differential equations. Mostly, all periodic function which is occurred in an application with period *2L* can be represented by trigonometric series in terms of sines and cosines of the form,

$$f(x) = a_o + \sum_{n=1}^{\infty} \left\{ a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right\}$$

This series (known as *Fourier Series*) was a main work of *Analytic Theory of Heat* by Joseph Fourier (1768 – 1830), French physicist and mathematician who developed a theory of heat conduction in 1822. This will be discussed in terms of these series and their engineering use from a practical point of view. It contains material sufficient for further applications follow in the next chapter on partial differential equations.

### 5.1.1 Periodic Function and Trigonometric Series

A function of $f(x)$ is called periodic if $f(x)$ is defined for all real , and if there is some positive number, $p$ such that,

$$f(x+p)=f(x) \tag{5.1}$$

The graph of such function is obtained by periodic repetition of its graph in any interval of length $p$, as Figure 5.1.

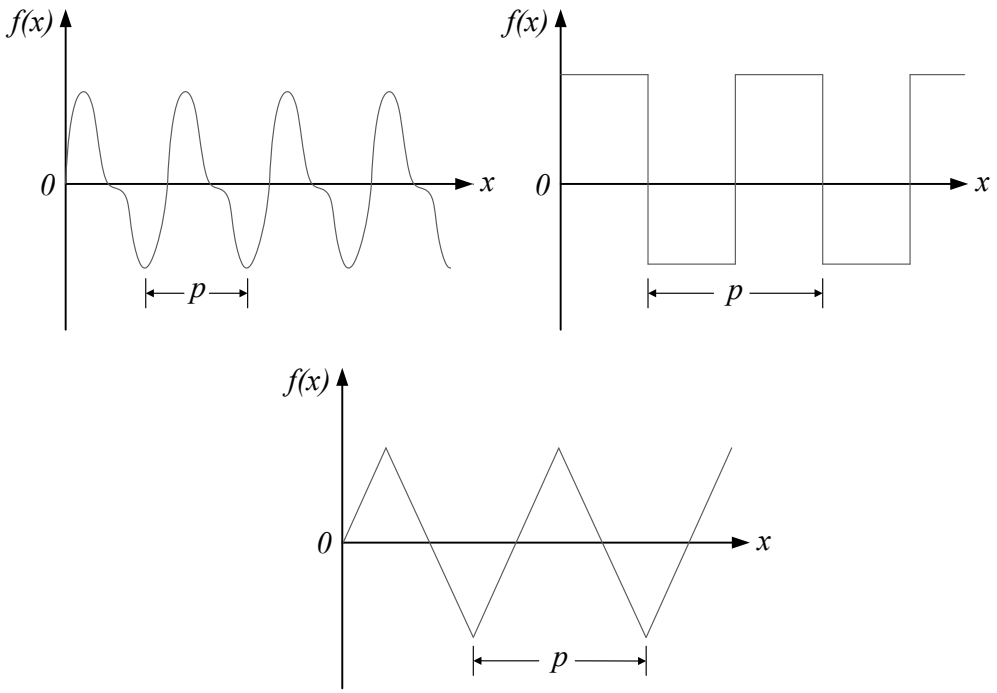


**Figure 5.1:** Periodic function

Most familiar periodic functions are the sine and cosine functions. From the definition of equation (5.1), the function $f$ = *constant* is also a periodic function for every positive . From the equation, we have $f(x+2p)=f[(x+p)+p]=f(x+p)=f(x),$ and for any integer , we can write the equation for the periodic function for all $x$ as,

$$f(x+np)=f(x) \tag{5.2}$$

Hence $2p$, $3p$, $4p$,... are also periods of $f(x)$. Furthermore, if $f(x)$ and $g(x)$ have period,

# *Partial Differential Equations*

## 6.1 Introduction

Partial differential equations (PDEs) is simply an equation that involves both a function and its partial derivatives. In this chapter, we are mainly concerned with techniques to find a solution to a given partial differential equation, and to ensure good properties to that solution. Most of the equations of interest arise from physics, and we will use $x$, $y$, $z$ as the usual spatial variables, and for the time variable. Various physical quantities will be measured by some function $u = u(x, y, z, t)$ which could depend on all three spatial variable and time, or some subset. The partial derivatives of $u$ will be denoted with the following notation.

$$U_x = \frac{\partial u}{\partial x}, \quad U_{xx} = \frac{\partial^2 u}{\partial x^2}, \quad U_t = \frac{\partial u}{\partial t}, \quad U_{xt} = \frac{\partial^2 u}{\partial x \partial t}$$

The partial differential equations arise from mathematical models of physical world. The modelling usually begins with an observation, then mathematical techniques are applied so that the resulting equations are transformed into convenient form of solution. Some typical partial differential equations that arise in physics are;

**Laplace's equation:**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

which is satisfied by the temperature $u = u(x, y, z)$ in a solid body that is in thermal equilibrium, or by the electrostatic potential $u = u(x, y, z)$ in a region without electric charges such as in analysis of electrostatic, gravitational, and velocity potentials in fluid mechanics.

**Heat equation:**

$$\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

which is satisfied by the temperature $u = u(x, y, z, t)$ of a physical object which conducts heat, where $k$ is a parameter depending on the conductivity of the object. This equation is sometimes called the diffusion equation since the diffusion or dissolved substances in solution is analogous to the flow of heat in solid. The equation also arises in the study of the flow of electricity in a long cable or transmission line, which is known a telegraph equation.

**Wave equation:**

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

which models the vibrations of a string in one-dimension $u = u(x, t)$, the vibrations of a thin membrane in two dimensions $u = u(x, y, t)$ or the pressure vibrations of an acoustic wave in air $u = u(x, y, z, t)$. The constant gives the speed of propagation for the vibrations.

The examples of partial differential equations mentioned above are some of the most important partial differential equations that we can find in engineering applications. All the three examples showed are the 'three-dimensional' which are contained all three components of spatial dimensions, $x$, $y$, $z$, and $t$ represents a time (except for Laplace's equation, describes the steady-state where the time, $t$ is dependent).

## 6.2 Basic Concepts

Partial differential equations (PDEs) is an equation for an unknown function of two or more independent variables that involves partial derivatives. It is an equation of the form for the given function $F$ and the unknown function $u$,

$$F\left(x, u, u_{x_1} \ldots, u_{x_n}, u_{x_1 x_2}, \ldots, u_{x_n x_n}, \ldots\right) = 0$$

where $x = (x_1, \ldots, x_n)$ and $x_{x1}, \ldots, x_{xn}$, $u_{x_1 x_2}, \ldots, u_{x_n x_n}, \ldots$ denotes partial derivatives of $u$, for some function, $F$, of several variables. Any equation containing at least one of the partial derivatives is called partial differential equations. Notation are usually used for

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# INDEX

# Introduction to DIFFERENTIAL EQUATIONS for Engineering Students

This book is written based on the teaching experience and the lecture notes of the authors at Universiti Tun Hussein Onn Malaysia, Malaysia and Telkom University, Indonesia, for more than 15 years. The chapters in this book are designed to cover the syllabus for the course of differential equations, which is one of the compulsary mathematics courses for most of the engineering programme in university. This book consists of six chapters, included Chapter 0: short Mathematical Revision, which is an introduction and a brief revision on basic mathematic that required for the fundamental differential equations. The contents start with the basic concept of ordinary differential equations (ODEs), which is the most immediately useful part of the calculus. The next chapters followed by the first and second order of ODEs with constant coefficient, Laplace transforms, Fourier series and ended with an introduction to partial differential equations.

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