NUMERICAL METHODS with CASIO FX-570EX CLASSWIZ CALCULATOR

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Abstract: Motivated by our previous books "How to Use Calculator Casio fx-570 MS in Numerical Methods", "Numerical Methods Using Casio fx-570 ES Calculator" and "Numerical Methods Using Casio fx-570ES PLUS Calculator" which was published in 2006, 2011 and 2017 respectively as well as our "Numerical Method (1.1 Edition)" module which was published in 2005, we have come up with a subsequent edition entitled "Numerical Methods With Casio fx-570EX CLASSWIZ Calculator". Solving numerical methods using normal calculators can be a dreary task since it involves tedious iteration which may easily lead to careless mistakes. CALC memory in Casio fx-570MS calculator provides 9 variables (A, B, C, D, E, F, X, Y and M), allowing users to input different values for a function or several functions. Once the function has been input, it can be recalled for calculation. This recall ability helps to repeat the tabulation of a sequence of function values which is required in Numerical Methods. As a result, the numerical calculations become much faster and calculation errors are reduced. However, Casio fx-570MS calculator does not have the natural display function (display mathematical expressions like square roots and fractions).

Keywords: Approximate, mathematical, analytically, numerical

NUMERICAL METHODS

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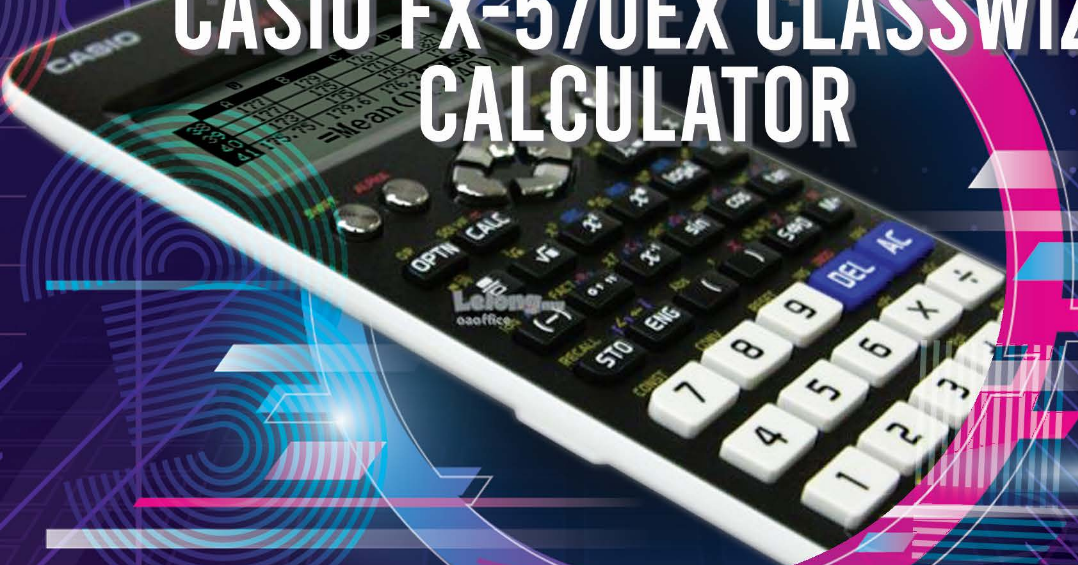

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2022

DEDICATIONS

To Teoh Eng Soon, Teoh Sin Yee and Teoh Jia Wei

To Kuan Kai Chin, Phang Yi Yang and Phang Yi Ler

To Chen Hui Ping, Phang Yi Jia and Phang Yi Ren

And our parents, families, friends and students.

Perpustakaan Negara Malaysia Cataloguing-in-Publication Data

Tay, Kim Gaik
NUMERICAL METHODS with CASIO FX-570EX CLASSWIZ CALCULATOR / TAY KIM GAIK, PHANG CHANG, PHANG PIAU.
ISBN 978-967-2817-78-9
1. Calculators.
2. Mathematical instruments.
3. Numerical calculations.
4. Government publications--Malaysia.
I. Phang, Chang. II. Phang, Piau.
III. Title.
510.284

Published by:
Penerbit UTHM
Universiti Tun Hussein Onn Malaysia
86400 Parit Raja,
Batu Pahat, Johor
Tel: 07-453 8529/8698
Fax: 07-453 6145

Website: http://penerbit.uthm.edu.my
E-mail: pt@uthm.edu.my
http://e-bookstore.uthm.edu.my

Penerbit UTHM is a member of
Majlis Penerbitan Ilmiah Malaysia
(MAPIM)

Printed by:
PERNIAGAAN TEKMAKRO
No. 10, Jalan Perdagangan 16,
Taman Universiti, 81300 Skudai,
Johor Bahru
No. Tel: 012-7021008

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PREFACE

Motivated by our previous books "**How to Use Calculator Casio fx-570 MS in Numerical Methods**", "**Numerical Methods Using Casio fx-570 ES Calculator**" and "**Numerical Methods Using Casio fx-570ES PLUS Calculator**" which was published in 2006, 2011 and 2017 respectively as well as our "**Numerical Method (1.1 Edition)**" module which was published in 2005, we have come up with a subsequent edition entitled "**Numerical Methods With Casio fx-570EX CLASSWIZ Calculator**".

Solving numerical methods using normal calculators can be a dreary task since it involves tedious iteration which may easily lead to careless mistakes. CALC memory in Casio *fx*-570MS calculator provides 9 variables (A, B, C, D, E, F, X, Y and M), allowing users to input different values for a function or several functions. Once the function has been input, it can be recalled for calculation. This recall ability helps to repeat the tabulation of a sequence of function values which is required in Numerical Methods. As a result, the numerical calculations become much faster and calculation errors are reduced. However, Casio fx-570MS calculator does not have the natural display function (display mathematical expressions like square roots and fractions).

The Casio fx-570 ES calculator has a natural display, but it can only handle 7 variables (A, B, C, D, X, Y, M). Hence, the Thomas algorithm cannot be implemented efficiently. This is why the Thomas algorithm was not shown in the second previous book but will be demonstrated in this book.

Similar to the Casio fx 570 MS and Casio Fx-570ES Plus calculators, the Casio fx-570EX CLASSWIZ calculator can handle 9 variables (A, B, C, D, E, F, X, Y, M) and has a natural display. The Casio fx-570EX CLASSWIZ calculator has a higher dimension of matrix, higher degree of polynomial, solve 4 simultaneous linear equations, spreadsheet mode and plotting ability. Besides, the Casio fx-570EX CLASSWIZ calculator is inexpensive and most students already own one since it is also required for other subjects. This is why we prefer this new calculator model in teaching and learning numerical methods for examination purposes.

Each numerical method in the previous two books is solved by one example which focuses more on step-by-step methods on how to implement numerical methods using Casio fx-570 MS and Casio fx-570 ES button graphics with their immediate calculator screens. However, in this book, each numerical method is solved using at least two examples that focus more on numerical methods. The first example still shows a step-by-step guide on implementing numerical methods using Casio fx-570EX CLASSWIZ button graphics similar to the Casio fx-570 ES calculator, but its calculator screen is not provided as in the previous two books. Besides, in the first example, an addition to step-by-step instructions and its calculator screen using spreadsheet mode for specific numerical methods is included too. The second example is solved numerically without a step-by-step guide using the Casio fx-570EX CLASSWIZ calculator due to the thickness of the book.

We need to emphasize that all the calculators including Casio fx-570EX CLASSWIZ are portable and relatively lower price compared to mathematical software. Hence, these calculators always are a better choice for the numerical method's teaching and learning process in the classroom and the examination hall.

This textbook was written based on the Engineering Mathematics IV syllabus of University Tun Hussein Onn Malaysia (UTHM). It serves as a guide for UTHM engineering students studying Numerical Methods

The book consists of nine chapters: Introduction to Numerical Methods and Errors, Nonlinear Equation, System of Linear Equations, Interpolation, Numerical Differentiation, Numerical Integration EigenValues, Ordinary Differential Equations and Partial Differential Equations.

Each chapter is presented without many formula derivations. This is because engineering students are only required to apply the concept without knowing the derivation of the formula. Comparison of different methods for the same topic and error analysis for each method are included too.

A summary of formulae is provided at the end of each chapter for easy reference. Exercises are provided at the end of the summary followed by model answers so that readers can enhance their understanding and problem-solving skills.

We hope this book will benefit lecturers and students. We are also grateful to Casio Japan for providing unique Casio fx-570EX CLASSWIZ calculator font for the production of this book. Finally, we would like to thank everyone who has helped us directly or indirectly in the process of writing this book. Thanks for using this book.

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ACKNOWLEDGEMENT

We wish to express our highest appreciation to Casio Japan for providing us the special Casio fx-570EX CLASSWIZ calculator font to type the keystroke of the Casio fx-570EX CLASSWIZ calculator and the emulator of the Casio fx-570EX CLASSWIZ calculator for capture the Casio fx-570EX CLASSWIZ calculator screen in preparing this book. Without these two things, the publication of this book would be more difficult.

On top of that, we are grateful to Mr. Cheong Tau Han from Universiti Teknologi MARA (UiTM) for introducing the Casio fx-570 EX CLASSWIZ calculator to be used in Numerical Methods and made the publication of this book becomes a reality.

Thanks to Mr. Chuah Wee Heng, who carefully read, learnt and notified us of the typo errors or mistakes for the previous third version of the book entitled "**Numerical Methods Using Casio fx-570ES PLUS Calculator".** Without his kind help, the same errors or mistakes might have existed in this new version of the book.

Besides, we would like to express our deepest gratitude to Publisher UTHM, especially Madam Zuraidah Johari who helped in the publication process of this book. We thank all lecturers or students who directly or indirectly pointed out the mistakes of previous books.

Lastly, we would like to use this golden opportunity to thank for the support given by the Faculty of Electrical and Electronic Engineering and Faculty of Applied Science and Technology, Universiti Tun Hussein Onn Malaysia (UTHM) and Faculty of Computer Science and Information Technology in Universiti Malaysia Sarawak (UNIMAS).

CHAPTER 1

INTRODUCTION TO NUMERICAL METHODS AND ERRORS

1.0 INTRODUCTION

In science and engineering, there are three different ways or approaches in problem-solving, which includes:

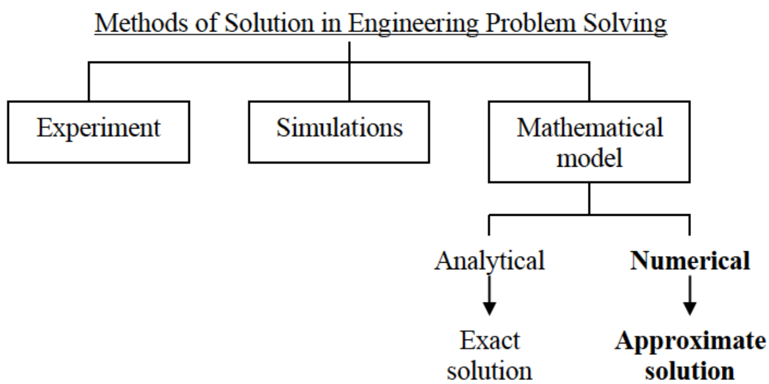

Figure 1.1: Methods of solution in engineering problem solving

As we know, in science and engineering, we set up **experiments** to obtain information or data, test hypothesis, make comparisons between theory and practice. Although the outcome is expressive, experiments usually involve a lot of equipment and facilities which are expensive.

Thus, scientists and engineers are also solving the problems by using **simulations**. Simulations can be defined as a model or representation of a course of events in science, engineering and business, especially by a computer calculation to study the effects of possible future changes or decisions. Graphical solutions are generally used to characterize the behavior of systems in simulations.

Besides, solutions can also be obtained by **mathematical modeling**. A mathematical model can be defined as a formulation or equation that expresses the essential features of a physical system or process in mathematical terms. Solutions are derived for some problems **analytically**, and the *exact* solutions are obtained. However, analytical solutions can be derived for only a limited class of problems, including those that can be approximated with linear models and those with simple geometry and low dimensionality.

CHAPTER 2
NONLINEAR EQUATIONS

2.0 INTRODUCTION

A linear equation is a polynomial equation of the first degree in the form of

$$f(x) = ax + b. \quad (2.1)$$

The graph of the linear equation is a straight line.

Any equation

$$f(x) = 0 \quad (2.2)$$

which is not in the form of (2.1) is called a nonlinear equation. From a graphical point of view, a nonlinear equation does not form a straight line. Example 2.1 shows some examples of nonlinear equations.

EXAMPLE 2.1

$x - \cos x = 0$, $3x^2 + 2x + 1 = 0$, and $4x^3 + 3x^2 + 2x + 1 = 0$

To find the roots of a nonlinear equation, one has to find the value of $x = x^*$ that satisfies $f(x^*) = 0$. From a graphical point of view, the value of x^* can be obtained from the graph $y = f(x)$ intercepting x-axis. By using the Intermediate Value Theorem, as given in Section 2.1, we can roughly locate the interval of the root.

In this chapter, four numerical methods for finding the root of nonlinear equations will be discussed. They are the bisection method, secant method, Newton-Raphson's method and fixed-point iteration method.

2.1 INTERMEDIATE VALUE THEOREM

If $f(x)$ is a continuous real-valued function in the interval $[a,b]$ and $f(a)f(b) < 0$, then it has at least one root in the interval.

CHAPTER 3
SYSTEM OF LINEAR EQUATIONS

3.0 INTRODUCTION

An instance of a system of linear equations is given as follows:

$$\begin{array}{rcrcrcl} x_1 & + & 2x_2 & - & x_3 & = & 1 \\ -3x_1 & + & x_2 & + & 2x_3 & = & 2 \\ 2x_1 & - & 5x_2 & + & x_3 & = & 3 \end{array} \qquad (3.1)$$

The above system of linear equations (3.1) can be solved by finding the values of unknowns x_1, x_2 and x_3 which simultaneously satisfy all three equations.

In general, a system with *m* linear equations and *n* unknowns (x_i, for $i = 1, \cdots, n$) can be written as

$$\begin{array}{lcccccl} a_{12}x_2 & + & \cdots & + & a_{1n}x_n & = & b_1 \\ a_{22}x_2 & + & \cdots & + & a_{2n}x_n & = & b_2 \\ & & & & & & \\ a_{m2}x_2 & + & \cdots & + & a_{mn}x_n & = & b_m \end{array} \qquad (3.2)$$

or in matrix form

$$\mathbf{Ax} = \mathbf{b}$$

where matrix $\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$ (3.3)

CHAPTER 4
INTERPOLATIONS

4.0 INTRODUCTION

In real life, we may have a certain range of paired (x, y) data set and need to estimate the corresponding value of y for an intermediate value of x which is not in the data set. To do so, we need the interpolations which provide techniques to approximate the function $f(x)$ at intermediate points x within the range of a given discrete data set.

More specifically, if we have several data points, $(x_0, f(x_0))$, $(x_1, f(x_1))$, $\ldots$, $(x_n, f(x_n))$, where $a \le x_0 < x_1 < \cdots < x_n \le b$ is usually available (see Figure 4.1(a)), we need a method to approximate the function $f(x)$ and use it to approximate the value of $f(x_k)$ at certain x_k over the entire closed interval $[a,b]$. For this purpose, a polynomial $P(x)$ of degree n, denoted by $P_n(x)$ will be constructed that passes through all $n+1$ points, (see Figure 4.1(b)) such that it satisfies the condition, $P(x_i) = f(x_i)$, $\forall i = 0, 1, 2, 3, \ldots, n$.

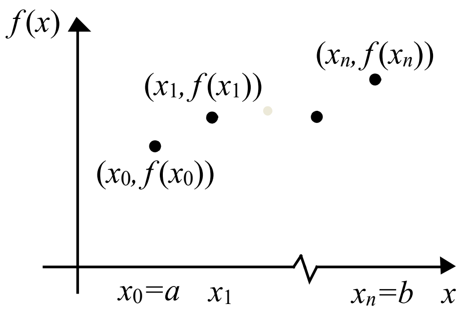

Figure 4.1 (a)

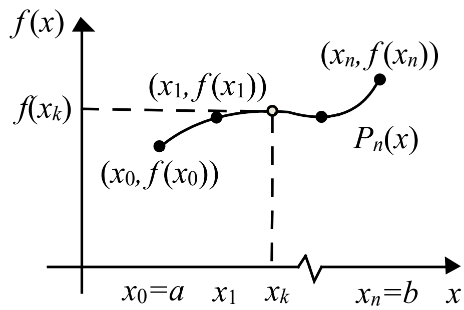

Figure 4.1 (b)

This chapter will discuss two types of interpolation, namely, **Lagrange interpolating** and **Newton's divided-difference**. All these methods are included in the polynomial interpolation method. Another category of interpolation is *piecewise polynomial interpolation* which includes a natural cubic spline and clamped cubic spline

Note that if we have $n+1$ data points, the polynomials obtained by the interpolating process have n degree, i.e. $P_n(x)$. Besides, we could only use the polynomial $P_n(x)$ obtained through the interpolation process to approximate the value of $f(x_k)$ at particular x_k in the interval of $x_0 < x_k < x_n$. For $x_k < x_0$ or $x_k > x_n$, we need to use *extrapolation* and it is beyond the scope of this chapter.

CHAPTER 5
NUMERICAL DIFFERENTIATION

5.0 INTRODUCTION

Differentiation is defined as the rate of change. Many physical situations involve the rate of change. For example, velocity is the rate of change of distance with respect to time while the acceleration is the rate of change of velocity with respect to time. The rate of change of a graph $y = f(x)$ against x, is the gradient of the curve y. This is equivalent to finding the slope of the tangent line to the function $y = f(x)$ at a point.

In cases where a function is not easily differentiated analytically or only a set of data is given, numerical differentiation can be used. In numerical differentiation, we will learn how to approximate the first derivative by using 2-point backward, 2-point forward, 3-point central and 5-point formulas whereas second derivative can be approximated from 3-point central and 5-point formulas for a tabulated data with the increment of h.

5.1 DIFFERENCE FORMULA

The difference formula for a function of a single variable can be developed using Taylor series.

5.1.1 First Derivative using Taylor's Series

Taylor series expansion for $f(x+h)$ can be written as

$$f(x+h) = f(x) + hf'(x) + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \ldots + \frac{f^{(n)}(x)}{n!}h^n \tag{5.1}$$

where h is the step size in the x space, i.e. $h = \Delta x$.

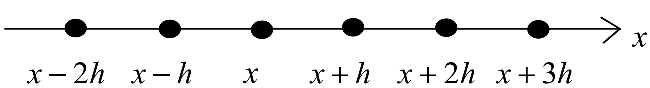

Figure 5.1: Illustration of discretization with step size h of x space

Solving Eq. (5.1) for $f'(x)$ up to the second derivatives gives **2-point forward difference** formula

$$f'(x) = \frac{f(x+h) - f(x)}{h} - h\frac{f''(x)}{2} + \ldots \tag{5.2}$$

CHAPTER 6
NUMERICAL INTEGRATION

6.0 INTRODUCTION

Integration can be defined as antidifferentiation. It has wide applications. For example, it can be used to find the area or volume below a curve or bounded by several curves. If acceleration is given, we can find velocity and distance by integration. Numerical integration is needed when direct integration is impossible (An example of integral that cannot be integrated analytically is $\int_0^3 e^{x^2} dx$) or integrates a tabulated data rather than integrates a known function. This chapter deals with definite integrals only. Several numerical methods can be used to approximate integrals such as a trapezoidal rule, Simpson's $\frac{1}{3}$ rule, Simpson's $\frac{3}{8}$ rule and Gauss-Quadrature.

6.1 TRAPEZOIDAL RULE

If $f(x)$ is a continuous function in the interval $[a, b]$, then the definite integral $\int_a^b f(x)\,dx$ is the area under the curve $y = f(x)$ with boundaries x-axis, $x = a$ and $x = b$ as shown in Figure 6.1 below:

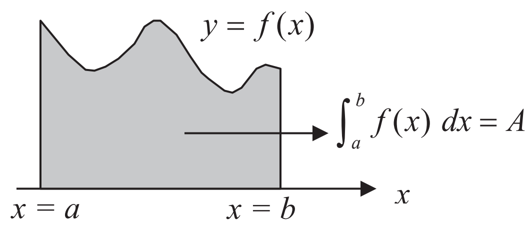

Figure 6.1: Approximating area under the curve $y = f(x)$

At first approximation, $\int_a^b f(x)\,dx$ can be approximated by an area of a trapezoid, A_1, with $n = 1$ segment.

$$A_1 = \int_a^b f(x)\,dx \approx \frac{(b-a)}{2}(f(a) + f(b)). \tag{6.1}$$

CHAPTER 7
EIGENVALUES

7.0 INTRODUCTION

Let **A** be an $n \times n$ matrix. A number (scalar) λ is called an **eigenvalue** of **A** if there exists a nonzero vector called **eigenvector** $\mathbf{v}$ such that

$$\mathbf{Av} = \lambda \mathbf{v}. \tag{7.1}$$

Eigenvalues and eigenvectors are widely used in various applications. For example, they can be used to determine the stability of a finite-difference scheme to solve a partial differential equation and find the solution for the system of differential equations.

Rewriting Eq. (7.1) as

$$\mathbf{Av} = \lambda \mathbf{Iv}, \tag{7.2}$$

where **I** is an identity matrix, yields

$$(\mathbf{A} - \lambda \mathbf{I})\mathbf{v} = 0 \tag{7.3}$$

The linear homogeneous system (7.3) has a nontrivial solution $\mathbf{v} \neq 0$ if and only if the matrix $\mathbf{A} - \lambda \mathbf{I}$ is singular that is $|\mathbf{A} - \lambda \mathbf{I}| = 0$. Solving $|\mathbf{A} - \lambda \mathbf{I}| = 0$ will produce polynomial $P_n(\lambda)$ with order n in terms of λ. The roots of the polynomial $P_n(\lambda)$ will yield n eigenvalues of matrix **A**. Polynomial $P_n(\lambda)$ is called **characteristic polynomial** and the equation $P_n(\lambda) = 0$ is called a **characteristic equation**. Substituting each eigenvalue into Eq. (7.3) will lead to its corresponding eigenvector.

Historical Notes
'*Eigen*' is a German word which means 'characteristic'.

CHAPTER 8
ORDINARY-DIFFERENTIAL EQUATIONS (ODES)

8.0 INTRODUCTION

Consider a function of $y = f(x)$, where x and y are the independent variable and dependent variable, respectively. Differentiate the function $y = f(x)$ with respect to the independent variable x, one time, two times and so on, we obtain its ordinary derivatives, $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots\right)$. An equation that consists of the function $y = f(x)$, and its ordinary derivatives is called ordinary differential equations (ODEs). A few examples are as follows:

$$2\frac{dy}{dx} + e^x y = \sin x$$

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = \tan x$$

ODEs arise in many physical phenomena such as engineering, population modeling, etc.

The solution of an ordinary differential equation is a function, $y = f(x)$ satisfying the ordinary differential equation.

8.1 INITIAL-VALUE PROBLEM (IVP)

Consider an IVP of the first-order differential equation

$$y' = f(x, y), \quad y(x_0) = y_0, \quad x_0 \le x \le x_n. \tag{8.1}$$

By using numerical methods, the interval of x is divided into n subintervals with width h such that $x_i = x_0 + ih$. We need to solve for y from $i = 1$ to n. Notice that, by using numerical methods, we only obtain discrete values of y. But by solving the differential equation analytically, we obtain a continuous function of $y(x)$. Five numerical methods, i.e. the Euler, Taylor, Heun, Midpoint and Fourth-order Runge-Kutta method will be discussed.

CHAPTER 9
PARTIAL DIFFERENTIAL EQUATIONS (PDEs)

9.0 INTRODUCTION

A partial differential equation (PDE) is an equation involving functions of several variables and their partial derivatives with respect to those variables. Let us focus on second-order linear PDEs with two variables x and y which can be written as

$$A\frac{\partial^2 u(x,y)}{\partial x^2}+B\frac{\partial^2 u(x,y)}{\partial x\partial y}+C\frac{\partial^2 u(x,y)}{\partial y^2}+D\frac{\partial u(x,y)}{\partial x}+E\frac{\partial u(x,y)}{\partial y} \quad (9.1)$$
$$+Fu(x,y)=G,$$

where A, B, C, D, E, F and G are constants or functions of x and y only. Table 9.1 below shows the classification of such PDEs.

Table 9.1: Classification of PDEs

B^2-4AC	Category	Example	Equations
$=0$	Parabolic	Heat equation	$\frac{\partial^2 u(x,t)}{\partial x^2}-\frac{1}{c^2}\frac{\partial u(x,t)}{\partial t}=0$
>0	Hyperbolic	Wave equation	$\frac{\partial^2 u(x,t)}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2}=0$
<0	Elliptic	Laplace's equation	$\frac{\partial^2 u(x,y)}{\partial x^2}+\frac{\partial^2 u(x,y)}{\partial y^2}=0$
		Poisson's equation	$\frac{\partial^2 u(x,y)}{\partial x^2}+\frac{\partial^2 u(x,y)}{\partial y^2}=f(x,y)$

PDEs can be solved by many numerical methods such as finite-difference method, finite-element method, wavelet, etc. In this chapter, we are going to discuss the solutions of PDEs using finite-difference method, a classical method which is the easiest method compared to other methods mentioned above.

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