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A Comparative Study on Longitudinal Dynamics Stability Between Two Aircraft Models

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Abstract: The present work presents a comparative study on the longitudinal dynamic's stability behavior for two aircraft models, namely the Learjet 24 and the Cessna 182. The longitudinal flight dynamics behaviors are evaluated by introducing a disturbance to the elevator. This device uses a single doublet impulse as well as multiple doublet impulses. The governing equation of longitudinal flight motion, which was derived based on a small perturbation theory and a linearized process by dropping the second order and above to the disturbance quantities, allowed one to formulate the governing equation of flight motion in the form of an equation known as the longitudinal equation of flight motion. This equation describes the flight behavior of an aircraft and can be expressed in the disturbance quantity as translational velocity in the x-direction u, angle of attack α, and pitch angle θ. The implementation in the case of the Cessna 182 and the Learjet 24, where the Cessna 182 uses a single doublet impulse or a multiple doublet impulse, demonstrates that the aircraft response in these three variable states is better than that of the Learjet 24.

Keywords: Longitudinal stability, Cessna 182, Learjet 24, longitudinal, multiple, and single doublet impulse flight simulation.

1. Introduction

The aircraft is a flying vehicle that can fly freely in any direction and in any rotation. As a result, the aircraft has six degrees of freedom, and if it is assumed to be rigid, then the aircraft can fly to follow six paths freely. It can move forward, sideways, and down, and it can rotate about its axes with yaw, pitch, and roll. To describe the state of a system that has six degrees of freedom, one will involve six unknown quantities. To obtain these six unknowns, six simultaneous equations are needed for the equations of motion of an aircraft. Basically, it is hard to formulate the governing equation of flight motion. So, some sort of simplification needs to be introduced, such as the aircraft being considered a rigid body having a constant mass and a symmetrical shape between the left and right sides of the vertical plane. In addition, because the airplane is flying at a low altitude and a relatively low speed, it gives the impression that the earth is flat and can be considered an inertial reference frame. To formulate the governing equation of flight motion, one may require two reference frames: the inertial frame of reference and the fixed body reference frame. Using these two reference frames, one can derive the governing equation of flight motion, in terms of the force equation and momentum equation. By using a small perturbation theory and a linearization process, the governing equation of flight motion can be decoupled to become the governing equation for longitudinal flight motion and lateral-directional flight motion.

The present work focused on the longitudinal flight motion, with a view to understanding the dynamic stability if some sort of small disturbance is introduced to the motion. Here two type aircraft models are used, namely the Learjet

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24 and Cessna 182 aircraft, while the disturbance models will be applied are a Single Doublet Impulse and a Multiple Doublets Impulse.

2. Governing Equation of aircraft flight motion.

Basically, the governing equation of aircraft flight motion as rigid body is already well established. The governing equation of flight motion starts by introducing two reference frames, namely the body fixed axis and Earth reference frame which will act as its the inertial frame. Fig. 1 shows these two reference frames. On the body fixed axis all quantities related to the flight behavior such as linear and angular velocity, forces and moments are defined with the positive sign as indicated in Fig. 2.

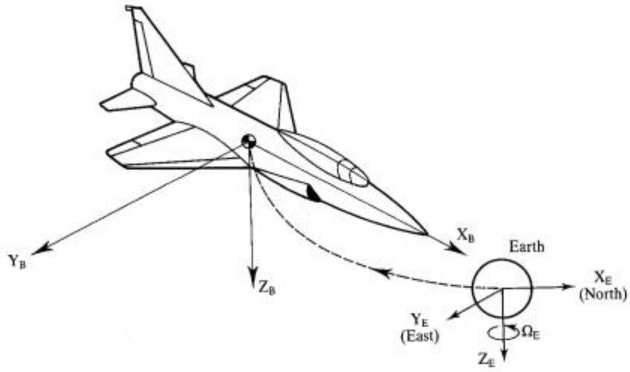

Fig. 1 - The fixed body axis and the earth reference frame [1]

On the body fixed axis all quantities related to the flight behavior such as linear and angular velocity, forces and moments are defined with the positive sign as indicated in Fig. 2.

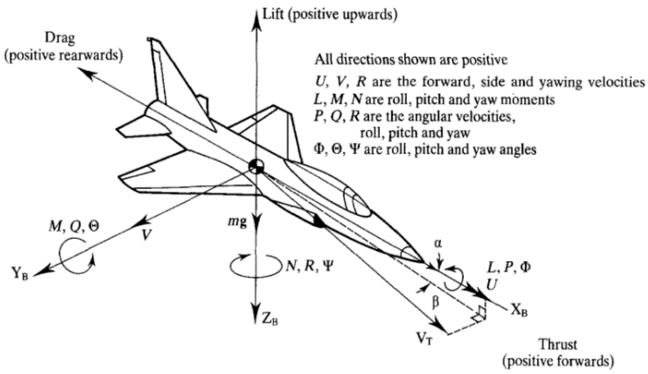

Fig. 2 - The definition of linear and angular velocity, forces and moment works on the aircraft [1]

The attitude aircraft which refer to the earth reference frame described by the Euler angle, this Euler angle consist of three angles namely, Bank (roll) angle Φ. Elevation (pitch) angle Θ and heading (yaw) angle Ψ. The aircraft position at any instant refer to the Earth reference system denoted as $[x_E, y_E, z_E]$. Fig. 3 show the definition of Euler in the context with the attitude of the aircraft.

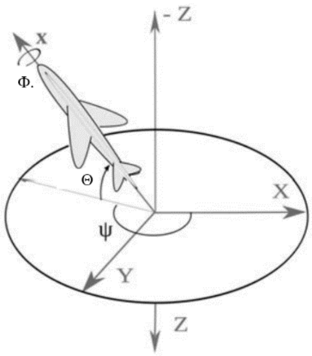

Fig. 3 - The Euler angle and the attitude of the aircraft [3]

Here one can notice that the flight behaviors of the aircraft can be adequately described by 12 variable states and stated in two groups given below [4]:

$$\boldsymbol{\nu} = \begin{bmatrix} U \\ V \\ W \\ P \\ Q \\ R \end{bmatrix} = \begin{bmatrix} \text{longitudinal (forward) velocity} \\ \text{lateral (transverse) velocity} \\ \text{vertical velocity} \\ \text{roll rate} \\ \text{pitch rate} \\ \text{yaw rate} \end{bmatrix} \quad (1)$$

$$\boldsymbol{\eta} = \begin{bmatrix} x_E \\ y_E \\ z_E \\ \Phi \\ \Theta \\ \Psi \end{bmatrix} = \begin{bmatrix} \text{earth-fixed x position} \\ \text{earth-fixed y position} \\ \text{earth-fixed z position} \\ \text{roll angle} \\ \text{pitch angle} \\ \text{yaw angle} \end{bmatrix} \quad (2)$$

The first group variable state $\boldsymbol{\nu}$ is related to the implementation of the Newton 2nd law of momentum conservation. While the second group related with the aircraft position and altitude. These 12 variable states represent the unknown quantities; hence 12 equations are needed to solve the unknown, Nelson [5] had provide the governing equation of flight motion which relate to these twelve variables states in the form

$$F_x - mg\sin\Theta = m\left(\dot{U} + QW - RV\right) \quad (3)$$

$$F_y + mg\cos\Theta\sin\Phi = m\left(\dot{V} + UR - PV\right) \quad (4)$$

$$F_z + mg\cos\Theta\cos\Phi = m\left(\dot{W} + PV - QU\right) \quad (5)$$

$$L = \dot{P}I_{XX} - \dot{R}I_{XZ} - PQI_{XZ} + RQ(I_{ZZ} - I_{YY}) \quad (6)$$

$$M = \dot{Q}I_{YY} + PR(I_{XX} - I_{ZZ}) + (P^2 - R^2)I_{XZ} \quad (7)$$

$$N = \dot{R}I_{ZZ} - \dot{P}I_{XZ} + PQ(I_{YY} - I_{XX}) + QRI_{XZ} \quad (8)$$

$$\begin{bmatrix} \dot{\Phi} \\ \dot{\Theta} \\ \dot{\Psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\Phi\tan\Theta & \cos\Phi\tan\Theta \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \sin\Phi\sec\Theta & \cos\Phi\sec\Theta \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} \dot{x_E} \\ \dot{y_E} \\ \dot{z_E} \end{bmatrix} = [A] \begin{bmatrix} U \\ V \\ W \end{bmatrix} \quad (10)$$

$$[A] = \begin{bmatrix} \cos\Psi\cos\Theta & -\sin\Psi\cos\Phi + \cos\Psi\sin\Theta\sin\Phi & \sin\Psi\sin\Phi + \cos\Psi\cos\Phi\sin\Theta \\ \sin\Psi\cos\Theta & \cos\Psi\cos\Phi + \sin\Phi\sin\Theta\sin\Psi & -\cos\Psi\sin\Phi + \sin\Theta\sin\Psi\cos\Phi \\ -\sin\Theta & \cos\Theta\sin\Phi & \cos\Theta\cos\Phi \end{bmatrix} \quad (11)$$

Eq. (1) to (11) represent the governing equation of general flight motion which they are a non-linear and coupled each to other. However, for practical application most of aircraft fly at specific flight condition namely at a steady state flight and perturbed flight. Steady state flight means that the linear and angular acceleration with respect to the body frame $[x, y, z]_b$ are zero. While perturbed flight condition describes that those flight condition, all motion variables of the aircraft experience a deviation from a set of original steady state values. As result at this flight condition the variable states, force and moments can be written as:

$$U = U_1 + u; \quad V = V_1 + v; \quad W = W_1 + w \quad (12)$$

$$P = P_1 + p; \quad Q = Q_1 + q; \quad R = R_1 + r \quad (13)$$

$$\Phi = \Phi_1 + \phi; \quad \Theta = \Theta_1 + \theta; \quad \Psi = \Psi_1 + \psi \tag{14}$$

$$F_{Ax} = F_{Ax_1} + f_{Ax}; \quad F_{Ay} = F_{Ay_1} + f_{Ay}; \quad F_{Az} = F_{Az_1} + f_{Az} \tag{15}$$

$$L_A = L_{A_1} + l_A; \quad M_A = M_{A_1} + m_A; \quad N_A = N_{A_1} + n_A \tag{16}$$

$$F_{Tx} = F_{Tx_1} + f_{Tx}; \quad F_{Ty} = F_{Ty_1} + f_{Ty}; \quad F_{Tz} = F_{Tz_1} + f_{Tz} \tag{17}$$

$$L_T = L_{T_1} + l_T; \quad M_T = M_{T_1} + m_T; \quad N_T = N_{T_1} + n_T \tag{18}$$

Above equation describe that the variable state composed two values, the values at the steady state condition denoted by subscript (1) and the perturbed quantity denoted by small later. Introducing Eq. (12-18) into the forces and moment equation and ignoring all term contain the second order and above of the perturbed quantities, one obtain the forces and moment equation in steady and perturbed flight condition as:

$$m(\dot{u} + Q_1 w + qW_1 - R_1 v - rV_1) = -mg\theta\cos\Theta_1 + f_x \tag{19}$$

$$m(\dot{v} + U_1 r + R_1 u - P_1 w - pW_1) = -mg\theta\sin\Phi_1\sin\Theta_1 + mg\phi\cos\Phi_1\cos\Theta_1 + f_y \tag{20}$$

$$m(\dot{w} + P_1 v + pV_1 - Q_1 u - U_1 q) = -mg\theta\cos\Phi_1\sin\Theta_1 - mg\phi\sin\Phi_1\cos\Theta_1 + f_z \tag{21}$$

$$\dot{p}I_{XX} - \dot{r}I_{XZ} - (P_1 q + Q_1 p)I_{XZ} + (R_1 q + Q_1 r)(I_{ZZ} - I_{YY}) = L \tag{22}$$

$$\dot{q}I_{YY} + (P_1 r + pR_1)(I_{XX} - I_{ZZ}) + \left(2{P_1}^2 p - 2{R_1}^2 r\right)I_{XZ} = M \tag{23}$$

$$\dot{r}I_{ZZ} - \dot{p}I_{XZ} + (P_1 q + pQ_1)(I_{YY} - I_{XX}) + (Q_1 r + R_1 q)I_{XZ} = N \tag{24}$$

While the equation of aircraft altitude, Eq. (9) becomes:

$$p = \dot{\Phi} - \dot{\Psi}_1\,\theta\cos\Theta_1 - \dot{\Psi}_1\sin\Theta_1 \tag{25}$$

$$q = -\dot{\Theta}_1\,\phi\cos\Phi_1 - \dot{\theta}\,\sin\Phi_1 - \dot{\Psi}_1\,\theta\sin\,\Phi_1\sin\Theta_1 + \dot{\Psi}_1\,\phi\,\cos\,\Phi_1\cos\Theta_1 + \dot{\Psi}_1\,\sin\,\Phi_1\cos\Theta_1 \tag{26}$$

$$r = -\dot{\Psi}_1\,\theta\cos\Phi_1\sin\Theta_1 - \dot{\Psi}_1\,\phi\sin\,\Phi_1\cos\Theta_1 - \dot{\theta}\,\sin\Phi_1 + \dot{\Psi}_1\cos\,\Phi_1\cos\Theta_1 - \dot{\Theta}_1\,\phi\cos\Phi_1 \tag{27}$$

In perturbed flight conditions, it is not necessary one has to focus as well to cover to the aircraft position with respect to the earth reference frame, since in the perturbed flight one just deals with a short time a flight dynamics behavior. If one imposing that the aircraft start from the flight condition as a steady-state rectilinear wing-level flight. One can impose the following initial condition:

- Angular velocities $P_1 = Q_1 = R_1 = 0$
- Euler angles $\Phi_1 = const.$, $\Theta_1 = const.$, $\Psi_1 = const.$;
- Lateral velocity $V_1 = 0$;
- Roll angle $\Phi_1 = 0$, $\sin\Phi_1 = 0$, $\cos\Phi_1 = 1$ (wing level).

The implementation above condition into Eq. (28) to (36) can be written as:

$$m(\dot{u} + qW_1) = -mg\theta\cos\Theta_1 + f_x \tag{28}$$

$$m(\dot{v} + U_1 r - pW_1) = mg\phi\cos\Theta_1 + f_y \tag{29}$$

$$m(\dot{w} - U_1 q) = -mg\theta\sin\Theta_1 + f_z \tag{30}$$

$$\dot{p}I_{XX} - \dot{r}I_{XZ} = L \tag{31}$$

$$\dot{q}I_{YY} = M \tag{32}$$

$$\dot{r}I_{ZZ} - \dot{p}I_{XZ} = N \tag{33}$$

$$p = \dot{\phi} - \dot{\psi}\sin\Theta_1 \tag{34}$$

$$q = \dot{\theta} \tag{35}$$

$$r = \dot{\psi}\cos\Theta_1 \tag{36}$$

Above set equation can be decoupling to become equation which deal with a longitudinal flight motion, the motion just related to the vertical plane and the lateral – directional flight motion. In these two types of flight motion, for the case of longitudinal flight motion becomes:

$$(\dot{u} + qW_1) = -mg\theta\cos\Theta_1 + f_x \tag{37}$$

$$m(\dot{w} - U_1 q) = -mg\theta\sin\Theta_1 + f_z \tag{38}$$

$$\dot{q}I_{YY} = M \tag{39}$$

$$q = \dot{\theta} \tag{40}$$

The present work just deals with longitudinal flight, so it would relate to the case of solving Eq. (37)-(40). This equation can be simplified by replacing the perturbed velocity component w in term of angle of attack α and the pitch rate q in term of perturbed elevation angle θ by using the following relation

$$q = \dot{\theta} \;\; \Rightarrow \;\; \dot{q} = \ddot{\theta} \tag{41}$$

$$w \approx V_{p1}\alpha \;\Rightarrow \dot{w} \approx V_{p1}\dot{\alpha} \tag{42}$$

In above equation, V_{p1} is the aircraft velocity presented in the stability reference frame. By using Eq. (41-42), the longitudinal Eq. (37-40) can be converted from four equation into three equation in term with the variable state velocity component in x- direction u, angle of attack, α and the elevation (pitch) angle θ, in the form as:

$$\dot{u} = -g\theta\cos\Theta_1 + \frac{1}{m}f_x \tag{43}$$

$$\left(V_{p1}\dot{w} - V_{p1}\dot{q}\right) = -g\theta\sin\Theta_1 + \frac{1}{m}f_z \tag{44}$$

$$\ddot{\theta}I_{YY} = M \tag{45}$$

If the force and moment are linearized in presented in non-dimensional aerodynamics coefficients, the complete form of the longitudinal equation of motion becomes:

$$\dot{u} = -g\cos\Theta_1\,\theta + \left(X_u + X_{T_u}\right)u + X_\alpha\alpha + X_{\delta_E}\delta_E \tag{46}$$

$$V_{P_1}\dot{\alpha} = -g\sin\Theta_1\,\theta + Z_u u + Z_\alpha\dot{\alpha} + \left(Z_q + V_{P_1}\right)\dot{\theta} + Z_{\delta_E}\delta_E \tag{47}$$

$$\ddot{\theta} = \left(M_u + M_{T_u}\right)u + \left(M_\alpha + M_{T_\alpha}\right)\alpha + M_{\dot{\alpha}}\dot{\alpha} + M_q\dot{\theta} + M_{\delta_E}\delta_E \tag{48}$$

Where the coefficients of the term which appear in above equations are defined as given in the Table. 1

Table 1 - The definition coefficient in the longitudinal flight motion [5]

Longitudinal Dimensional Stability and Control Derivatives	
$X_u = -\dfrac{q_1 S\left(C_{DU\beta} + C_{D1}\right)}{mU_1}\;\dfrac{ft}{sec^2}$	$X_{Tu} = -\dfrac{q_1 S(C_{Txu} + 2C_{Tx1})}{mU_1}\;\dfrac{1}{sec}$
$X_\alpha = -\dfrac{q_1 S(C_{D\alpha} - C_{L1})}{mU_1}\;\dfrac{ft}{sec^2}$	$X_{\delta e} = -\dfrac{q_1 S\, C_{D\delta E}}{m}\;\dfrac{ft}{sec^2}$

$Z_u = -\frac{q_1 S(C_{LU} + 2C_{L1})}{mU_1}\ \frac{ft}{sec^2}$	
$Z_{\dot{\alpha}} = -\frac{q_1 S(C_{L\dot{\alpha}})}{2mU_1}\ \frac{1}{sec}$	$Z_\alpha = -\frac{q_1 S(C_\alpha + C_{D1})}{2mU_1}\ \frac{ft}{sec^2}$
$Z_{\delta E} = -\frac{q_1 S(C_{L\delta E})}{m}\ \frac{1}{sec}$	$Z_q = -\frac{q_1 S\bar{c}\ (C_{Lq})}{2mU_1}\ \frac{ft}{sec}$
$M_u = \frac{q_1 S(C_{mU} + C_{m1})}{mI_{yy}}\ \frac{1}{ft\ sec}$	$M_{Tu} = \frac{q_1 S(C_{mTU} + C_{mT1})}{U_1 I_{yy}}\ \frac{1}{ft\ sec}$
$M_\alpha = \frac{q_1 S\bar{c}\ C_{m\alpha}}{I_{yy}}\ \frac{1}{sec^2}$	$M_{T\alpha} = \frac{q_1 S\bar{c}\ C_{mT\alpha}}{I_{yy}}\ \frac{1}{sec^2}$
$M_{\delta E} = \frac{q_1 S\bar{c}\ C_{m\delta E}}{I_{yy}}\ \frac{1}{sec^2}$	$M_q = \frac{q_1 S\bar{c}\ C_{m\delta E}}{I_{yy}} \frac{\bar{c}}{2U_1}\ \frac{1}{sec^2}$

3. The Solution of Longitudinal Flight Motion

Based on small perturbation theory and linearized implementation, the governing equation, which originally consisted of 12 equations, may be reduced to three equations for the scenario when the aircraft is flying in longitudinal motion. These equations are given in Eq. (46-48). If Laplace transforms are applied to that equation and assuming the zero initial condition for the disturbed quantities u, α and θ, the Laplace transform for the Eq. (46-48) becomes:

$$\left(s - \left(X_u + X_{T_u}\right)\right) u(s) - X_\alpha \alpha(s) + +g\cos\Theta_1\,\theta(s) = X_{\delta_E}\delta_E(s) \quad (49)$$

$$-Z_u u(s) + \left(s\left(V_{P_1} - Z_{\dot{\alpha}}\right) - Z_\alpha\right)\alpha(s) + \left(-s\left(Z_q + V_{P_1}\right) + g\sin\Theta_1\right)\theta(s) = Z_{\delta_E}\delta_E(s) \quad (50)$$

$$-\left(M_u + M_{T_u}\right)u(s) - \left(M_{\dot{\alpha}}s - \left(M_\alpha + M_{T_\alpha}\right)\right)\alpha(s) + s\left(s - M_q\right)\theta(s) = M_{\delta_E}\delta_E(s) \quad (51)$$

or in matrix notation can be written as:

$$\begin{bmatrix} (s - (X_u + X_{T_u})) & -X_\alpha & +g\cos\Theta_1 \\ -Z_u & (s(V_{P_1} - Z_{\dot{\alpha}}) - Z_\alpha) & (-s(Z_q + V_{P_1}) + g\sin\Theta_1) \\ -(M_u + M_{T_u}) & -(M_{\dot{\alpha}}s - (M_\alpha + M_{T_\alpha})) & s(s - M_q) \end{bmatrix} \begin{bmatrix} \frac{u(s)}{\delta_E(s)} \\ \frac{\alpha(s)}{\delta_E(s)} \\ \frac{\theta(s)}{\delta_E(s)} \end{bmatrix} = \begin{bmatrix} X_{\delta_E} \\ Z_{\delta_E} \\ M_{\delta_E} \end{bmatrix} \quad (52)$$

To solve Eq. (52), Cramer rule can be used to solve unknown quantities to yields:

$$\frac{u(s)}{\delta_E(s)} = \frac{\begin{vmatrix} X_{\delta_E} & -X_\alpha & +g\cos\Theta_1 \\ Z_{\delta_E} & (s(V_{P_1} - Z_{\dot{\alpha}}) - Z_\alpha) & (-s(Z_q + V_{P_1}) + g\sin\Theta_1) \\ M_{\delta_E} & -(M_{\dot{\alpha}}s - (M_\alpha + M_{T_\alpha})) & s(s - M_q) \end{vmatrix}}{\begin{vmatrix} (s - (X_u + X_{T_u})) & -X_\alpha & +g\cos\Theta_1 \\ -Z_u & (s(V_{P_1} - Z_{\dot{\alpha}}) - Z_\alpha) & (-s(Z_q + V_{P_1}) + g\sin\Theta_1) \\ -(M_u + M_{T_u}) & -(M_{\dot{\alpha}}s - (M_\alpha + M_{T_\alpha})) & s(s - M_q) \end{vmatrix}} = \frac{Num_u(s)}{\bar{D}_1(s)} \quad (53)$$

$$\frac{\alpha(s)}{\delta_E(s)} = \frac{\begin{vmatrix} (s - (X_u + X_{T_u})) & X_{\delta_E} & +g\cos\Theta_1 \\ -Z_u & Z_{\delta_E} & (-s(Z_q + V_{P_1}) + g\sin\Theta_1) \\ -(M_u + M_{T_u}) & M_{\delta_E} & s(s - M_q) \end{vmatrix}}{\begin{vmatrix} (s - (X_u + X_{T_u})) & -X_\alpha & +g\cos\Theta_1 \\ -Z_u & (s(V_{P_1} - Z_{\dot{\alpha}}) - Z_\alpha) & (-s(Z_q + V_{P_1}) + g\sin\Theta_1) \\ -(M_u + M_{T_u}) & -(M_{\dot{\alpha}}s - (M_\alpha + M_{T_\alpha})) & s(s - M_q) \end{vmatrix}} = \frac{Num_\alpha(s)}{\bar{D}_1(s)} \quad (54)$$

$$\frac{\theta(s)}{\delta_E(s)} = \frac{\begin{vmatrix} (s-(X_u+X_{T_u})) & -X_\alpha & X_{\delta_E} \\ -Z_u & (s(V_{P_1}-Z_{\dot\alpha})-Z_\alpha) & Z_{\delta_E} \\ -(M_u+M_{T_u}) & -(M_{\dot\alpha}s-(M_\alpha+M_{T_\alpha})) & M_{\delta_E} \end{vmatrix}}{\begin{vmatrix} (s-(X_u+X_{T_u})) & -X_\alpha & +g\cos\Theta_1 \\ -Z_u & (s(V_{P_1}-Z_{\dot\alpha})-Z_\alpha) & (-s(Z_q+V_{P_1})+g\sin\Theta_1) \\ -(M_u+M_{T_u}) & -(M_{\dot\alpha}s-(M_\alpha+M_{T_\alpha})) & s(s-M_q) \end{vmatrix}} = \frac{Num_\theta(s)}{\overline{D}_1(s)} \tag{55}$$

The expressions for the coefficients of the numerator polynomials ($Num_u(s)$, $Num_\alpha(s)$ and $Num_\theta(s)$) are given by:

$$\begin{aligned} &Num_u(s) = A_u s^3 + B_u s^2 + C_u s + D_u \\ &A_u = X_{\delta_E}(V_{P_1} - Z_{\dot\alpha}) \\ &B_u = -X_{\delta_E}\left[(V_{P_1}-Z_{\dot\alpha})M_q + M_{\dot\alpha} + (Z_q+V_{P_1})\right] + Z_{\delta_E}X_\alpha \\ &C_u = X_{\delta_E}[M_q Z_\alpha + M_{\dot\alpha} g\sin\Theta_1 - (M_\alpha+M_{T_\alpha})(Z_q+V_{P_1})] \\ &D_u = g\sin\Theta_1 X_{\delta_E}(M_u+M_{T_u}) + g\cos\Theta_1 Z_{\delta_E}(M_\alpha+M_{T_\alpha}) + M_{\delta_E}(g\cos\Theta_1 Z_\alpha - g\sin\Theta_1 X_\alpha) \end{aligned} \tag{56}$$

$$\begin{aligned} &Num_\alpha(s) = A_\alpha s^3 + B_\alpha s^2 + C_\alpha s + D_\alpha \\ &A_\alpha = Z_{\delta_E} \\ &B_\alpha = X_{\delta_E}Z_u - Z_{\delta_E}\left((X_u+X_{T_u}) + M_q\right) + M_{\delta_E}(Z_q+V_{P_1}) \\ &C_\alpha = X_{\delta_E}\left[(Z_q+V_{P_1})(M_u+M_{T_u}) - M_q Z_u\right] + Z_{\delta_E}M_q(X_u+X_{T_u}) - M_{\delta_E}\left[(Z_q+V_{P_1})(X_u+X_{T_u})\right] \\ &D_\alpha = -g\sin\Theta_1 X_{\delta_E}(M_u+M_{T_u}) + g\cos\Theta_1 Z_{\delta_E}(M_u+M_{T_u}) + M_{\delta_E}\left[g\sin\Theta_1 (X_u+X_{T_u}) - g\cos\Theta_1 Z_u\right] \end{aligned} \tag{57}$$

$$\begin{aligned} &Num_\theta(s) = A_\theta s^2 + B_\theta s + C_\theta \\ &A_\theta = Z_{\delta_E}M_{\dot\alpha} + M_{\delta_E}(V_{P_1}-Z_{\dot\alpha}) \\ &B_\theta = X_{\delta_E}\left[Z_u M_{\dot\alpha} + (V_{P_1}-Z_{\dot\alpha})(M_u+M_{T_u})\right] + Z_{\delta_E}\left[(M_\alpha+M_{T_\alpha}) - M_{\dot\alpha}(X_u+X_{T_u})\right] \\ &\qquad - M_{\delta_E}\left[(V_{P_1}-Z_{\dot\alpha})(X_u+X_{T_u}) - Z_\alpha\right] \\ &C_\theta = X_{\delta_E}\left[(M_\alpha+M_{T_\alpha})Z_u - (M_u+M_{T_u})Z_\alpha\right] - Z_{\delta_E}\left[(M_\alpha+M_{T_\alpha})(X_u+X_{T_u}) + X_\alpha(M_u+M_{T_u})\right] + \\ &M_{\delta_E}\left[(X_u+X_{T_u})Z_\alpha - Z_\alpha Z_u\right] \end{aligned} \tag{58}$$

While denominator $\overline{D}_1(s)$ is given as:

$$\begin{aligned} &\overline{D}_1(s) = A_1 s^4 + B_1 s^3 + C_1 s^2 + D_1 s + E_1 \\ &A_1 = (V_{P_1} - Z_{\dot\alpha}) \\ &B_1 = -(V_{P_1}-Z_{\dot\alpha})(X_u + X_{T_u} + M_q) - Z_\alpha - M_{\dot\alpha}(Z_q+V_{P_1}) \\ &C_1 = (X_u+X_{T_u})\left[M_q(V_{P_1}-Z_{\dot\alpha}) + Z_\alpha + M_{\dot\alpha}(Z_q+V_{P_1})\right] + M_q Z_\alpha - Z_u X_\alpha + M_{\dot\alpha} g\sin\Theta_1 - \\ &(M_\alpha+M_{T_\alpha})(Z_q+V_{P_1}) \\ &D_1 = g\sin\Theta_1\left[(M_\alpha+M_{T_\alpha}) - M_{\dot\alpha}(X_u+X_{T_u})\right] + g\cos\Theta_1\left[M_{\dot\alpha}Z_u + (M_u+M_{T_u})(V_{P_1}-Z_{\dot\alpha})\right] \\ &\qquad - X_\alpha(M_u+M_{T_u})(Z_q+V_{P_1}) + Z_u X_\alpha M_q + (X_u+X_{T_u})[(M_\alpha+M_{T_\alpha})(Z_q+V_{P_1}) - M_q Z_\alpha] \\ &E_1 = g\cos\Theta_1\left[Z_u(M_\alpha+M_{T_\alpha}) - Z_\alpha(M_u+M_{T_u})\right] + g\sin\Theta_1\left[(M_u+M_{T_u})X_\alpha - (X_u+X_{T_u})(M_\alpha+M_{T_\alpha})\right] \end{aligned} \tag{59}$$

Through Eq. (53) to Eq. (59), one can evaluate how the response of the aircraft in the speed u, the angle of attack α and the pitch angle θ, if the aircraft disturb in line with the elevator movement. In the presence of a sudden vertical flow, their effects can be simulated by using elevator deflection angle to follow a particular sequence. Here the flight behaviour of the aircraft in the longitudinal motion investigates by introducing the movement of the elevator deflection follow two type of disturbance models (a) a single doublet impulse and (b) a multiple doublet impulse.

In a single doublet impulse, as function of time the elevator deflected follows the sequence as given by function below:

$$\delta_{si}(t) = \begin{cases} 0 & 0 \le t \le 200 \\ -4^0 & 200 < t \le 205 \\ 0 & 205 < t \le 3200 \\ 4^0 & 3200 < t \le 3205 \\ 0 & 3205 \le t \le 12001 \end{cases} \tag{60}$$

While as a multiple doublet impulse, the elevator deflection will follow function as below

$$\delta_{mi}(t) = \begin{cases} 0 & 0 \le t \le 200 \\ -4^0 & 200 < t \le 215 \\ 0 & 215 < t \le 400 \\ 4 & 400 < t \le 415 \\ 0 & 415 < t \le 800 \\ -2 & 800 < t \le 815 \\ 0 & 815 < t \le 1000 \\ 2 & 1000 < t \le 1015 \\ 0 & 1015 < t \le 12001 \end{cases} \tag{61}$$

To solve longitudinal flight problem by using Eq. (53) to Eq. (59) need the geometry, mass, the aircraft inertia, the flight condition, and certain aerodynamic characteristics of the aircraft are needed. In the case of Learjet 24 and Cessna 182 aircraft, the required data for longitudinal flight dynamic analysis as given in the Table 2.

Table 2 Geometry, Mass, Inertia and Aerodynamics Data of the Learjet 24 and the Cessna 182 Aircraft.

Aircraft Model	**Learjet 24**	**Cessna 182**
Wing Surface (ft^2)	230.00	174.00
Mean Aerodynamic Chord (MAC) (ft)	7.000	4.90
Wingspan (ft)	34.00	36.00
Flight Condition		
Altitude (Ft)	40000.0	5000.0
Mach Number	0.7000	0.2010
True airspeed ($\frac{Ft}{Sec}$)	677.00	220.10
Dynamic pressure (lbs/ft^2)	134.60	49.60
Location of CG - % MAC	0.3200	0.2640
Steady-state angle of attack (deg)	1.50	0.0000
Aircraft's mass and inertia data		
Mass (lbs)	9000.00	2650.00
Moment of inertia x-axis (slug ft^2)	6000.00	948.00
Moment of inertia y-axis (slug ft^2)	17800.0	1346.0
Moment of inertia z-axis (slug ft^2)	25000.0	1967.0
Moment of inertia xz-axis (slug ft^2)	1400.00	0.00000
Aircrafts longitudinal steady state input data		
C_{L_1}	0.2800	0.3070
C_{D_1}	0.0279	0.0320
C_{m_1}	0.0000	0.0000
$C_{T_{X_1}}$	0.0279	0.0320
C_{mT_1}	0.0000	0.0000
C_{D_0}	0.0216	0.0270
C_{D_u}	0.1040	0.0000
C_{D_α}	0.2200	0.1210
C_{TX_u}	- 0.0700	- 0.0960
C_{L_0}	0.1300	0.3070
C_{L_u}	0.2800	0.0000
C_{L_α}	5.84	4.41
$C_{L_{\dot\alpha}}$	2.20	1.70
C_{L_q}	4.70	3.90
C_{m_0}	0.0500	0.0400
C_{m_u}	0.0700	0.0000
C_{m_α}	- 0.6400	- 0.6130
$C_{m_{\dot\alpha}}$	- 6.70	- 7.27
C_{m_q}	- 15.50	- 12.40
C_{mT_u}	- 0.0030	0.0000
C_{mT_α}	0.0000	0.0000
C_{L_β}	- 0.1000	- 0.0923
C_{L_p}	- 0.4500	- 0.4840
C_{L_r}	0.1400	0.0798
C_{Y_β}	- 0.7300	- 0.3930
C_{Y_p}	0.0000	-0.0750
C_{Y_r}	0.4000	0.2140

C_{n_β}	0.1240	0.0587
C_{nT_β}	0.0000	0.0000
C_{n_p}	- 0.0220	- 0.0278
C_{n_r}	- 0.2000	- 0.0937
Aircrafts control derivatives data		
$C_{D_{\delta_E}}/C_{D_{i_H}}$	0.0/0.0	0.0000
$C_{L_{\delta_E}}/C_{L_{i_H}}$	0.46/0.94	0.4300
$C_{m_{\delta_E}}/C_{m_{i_H}}$	-1.24/-2.5	-1.122
$C_{L_{\delta_A}}$	0.1780	0.2290
$C_{L_{\delta_R}}$	0.0210	0.0147
$C_{Y_{\delta_A}}$	0.0000	0.000
$C_{Y_{\delta_R}}$	0.1400	0.1870
$C_{n_{\delta_A}}$	- 0.0200	- 0.02160
$C_{n_{\delta_R}}$	-0.740	-0.0645

4. Result and Discussion

The current study compares the longitudinal flight characteristics of two aircraft. Those two aircraft are the Learjet 24 and the Cessna 182. As mentioned in the previous subchapter, there are two types of disturbance models used in this study: a single doublet impulse (Eq. (60)) and a multiple doublet impulse (Eq. (61). The Learjet 24 is designed to fly at a high subsonic velocity (M = 0.7), while the Cessna 182 flies at a low subsonic velocity (M=0.2). Using the same condition of single doublet input, namely that both aircraft's elevators were deflected at 4^0, up and down. Fig. 4-6 shows that the aircraft can return to its origin as the disturbance disappears. Because of the single doublet signal, the short period mode occurs in the immediate transient and lasts for a few seconds. The phugoid mode occurs shortly after the short period mode ends and often lasts quite a long time.

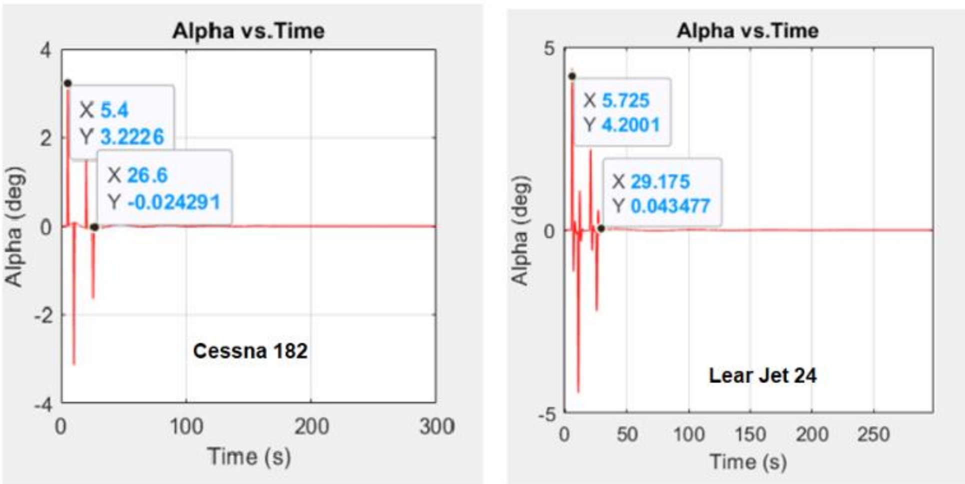

Fig. 4 - Comparison aircraft response the angle of attack between Cessna 182 and Learjet 24 in a single doublet impulse

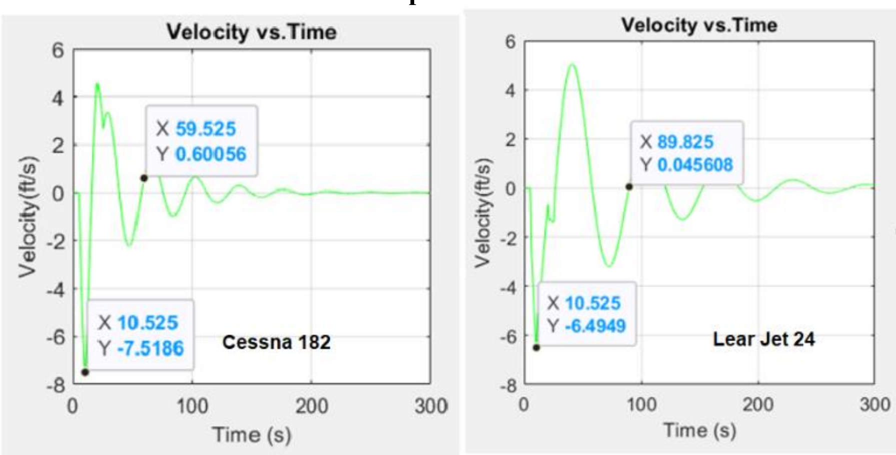

Fig. 5 - Comparison aircraft response the velocity in x-direction between Cessna 182 and Learjet 24. in a single doublet impulse

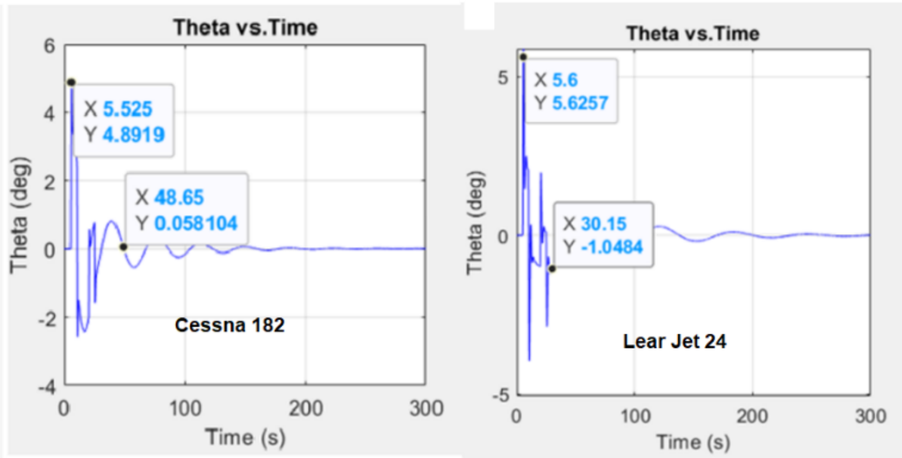

Fig. 6 - Comparison aircraft response the pitch angle in x- between Cessna 182 and Learjet 24 in a single doublet impulse.

While in the case of a disturbance model with a multiple doublet impulse, the comparison response between these two airplanes from the angle of attack response is shown in Fig. 7, while the velocity and pitch angle responses are shown in Fig. 8 and Fig. 9. Results in Fig. 7-9 show that the aircraft can return to its origin as the disturbance disappears, but it takes a longer time compared to the single doublet input disturbance.

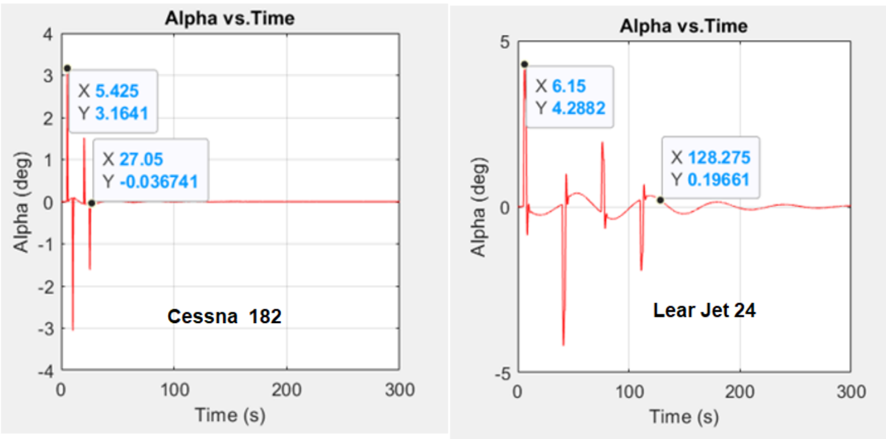

Fig. 7 - Comparison aircraft response the angle of attack α -between Cessna 182 and Learjet 24. Case a multiple doublet impulse.

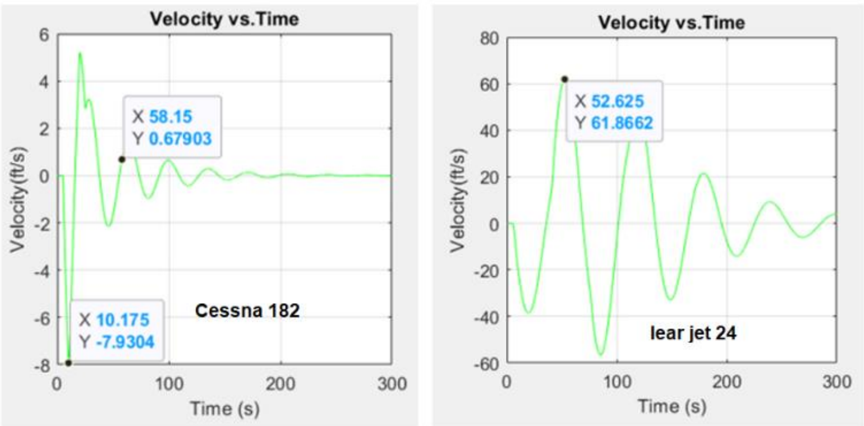

Fig. 8 - Comparison aircraft response the velocity in x-direction between Cessna 182 and Learjet 24. Case a multiple doublet impulse

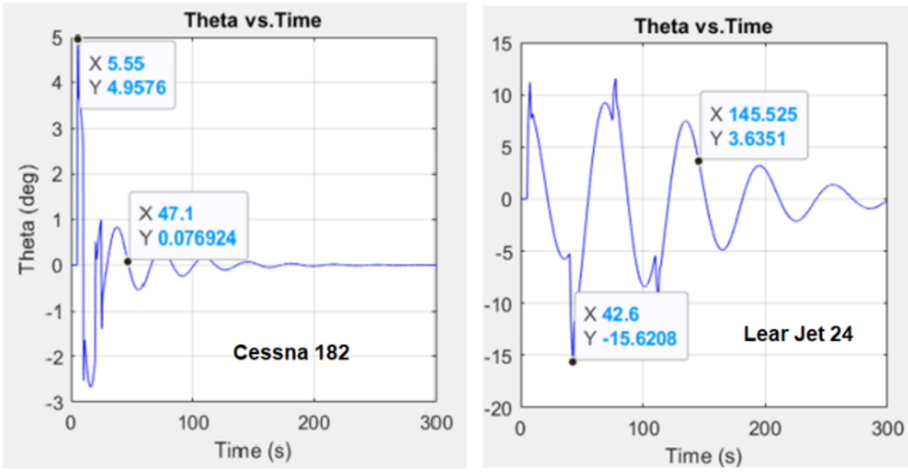

Fig. 9 - Comparison aircraft response pitch angle in x-direction between Cessna 182 and Learjet 24. Case a multiple doublet impulse

5. Conclusion and Future Work

Using the results from two model aircraft, one can identify that the longitudinal equation of flight motion can be solved by using the Laplace transform. The comparison shows how the velocity in the x-direction, the angle of attack, and the pitch angle change over time. Results in terms of velocity disturbance u in the x-axis direction, the angle of attack, and the pitch angle for the case of the Cessna 182 show that this airplane will be able to dump the oscillation as the disturbance is lifted. While for the Learjet 24, when a single doublet impulse is applied, the aircraft response is as good as the Cessna 182. As the disturbance is gone, the oscillation just occurred for a short time. However, when the multiple doublet impulse occurs, this airplane needs more time to make the disturbance disappear. Simulation results for the two different aircraft models are presented, and a comparative analysis between them is shown successfully.

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