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PAAT

http://penerbit.uthm.edu.my/ojs/index.php/paat

e-ISSN: 2821-2924

Progress in Aerospace and Aviation Technology

The Lateral and Directional Stability Behavior for Two Aircraft Models

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DOI: https://doi.org/10.30880/paat.2022.02.02.008
Received 15 November 2022; Accepted 19 December 2022; Available online 31 December 2022

Abstract: The current study looks at flight behavior in lateral and directional motion. The investigation involves setting up the aircraft under an equilibrium condition with a small disturbance applied to it. Plotting the side slip angle, roll angle, and yaw angle with respect to time in the presence of disturbances can be used to investigate flight behavior in lateral and directional motion. Here the disturbance can be simulated by the movement of the aileron or rudder, in which these two control surfaces can be designed to move in a single impulse or multiple impulse disturbance mode. These two disturbance modes are used on the Beechcraft 99 and Cessna T37 aircraft. Both impulse disturbance models are used for the aileron and the rudder. However, in the current work, the Beechcraft 99 receives a single impulse, whereas the Cessna T37 receives multiple impulses. The implementation of such disturbances found that the Beechcraft 99 represented the aircraft that would be able to go back to its initial condition in response to a single impulse disturbance mode while the Cessna T37 aircraft requires a little more time to return to its original yaw angle. Following the implementation of these two types of disturbances, it was discovered that each aircraft has the ability to return to its initial condition, but at varying times to reach its steady state solution.

Keywords: Lateral stability, directional stability, flight simulation

1. Introduction

Understanding an aircraft's flight behavior can be accomplished in two ways: (1) by investigating it over a long period of time, or (2) by investigating it in a short period of time, with an order of magnitude in just a few seconds [1]. The first approach is called "understanding aircraft behavior from the aircraft performance point of view." Through this approach, one can estimate the range, endurance, take-off distance, landing distance, etc. The second approach to understanding the flight behavior in a short time is called the aircraft's dynamics and stability point of view. This approach allows one to identify the aircraft response if some sort of disturbance is applied to it. The disturbance may appear naturally in the form of gust velocities or be due to deliberately deflecting control surfaces.

Basically, the flight dynamic stability of the aircraft can be split into two directions of flight, namely the longitudinal dynamic stability and the lateral and directional dynamic stability. The present work focuses on the lateral and directional dynamic stability of the aircraft as applied to the cases of two aircraft models, namely the Beechcraft 99 and the Cessna T37. The lateral and directional flight behavior can be investigated through the behaviors of the side slip angle β, roll angle Φ and yaw angle Ψ plotted with respect to time in the presence of disturbance. Such disturbances can be created

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through the movement of the aileron or rudder in single impulse mode or multiple impulse mode. The Beechcraft 99 aircraft is subjected to the implementation of single impulses for aileron and rudder, and it has been discovered that this aircraft is capable of returning the side slip angle (β), roll angle (Φ), and yaw angle (Ψ) to their initial conditions as soon as the disturbance is disappeared. These three variable states give slightly different responses when the multiple impulse is applied. With the implementation of this disturbance model on the Cessna T37, this aircraft needs more time to make its yaw angle return to its initial condition.

2. Governing Equation of Flight Motion

To formulate the governing equation of flight motion one needs at least two types of coordinate systems. The first coordinate system is the inertial frame reference F_E and the second one is the body fixed coordinate frame F_B . The first coordinate frame of reference is used to allow the implementation the Newton's second law of translational motion and the angular motion, While the second coordinate frame of reference is used to describe the motion on the airplane. Fig. 1 describes these two coordinates system is related.

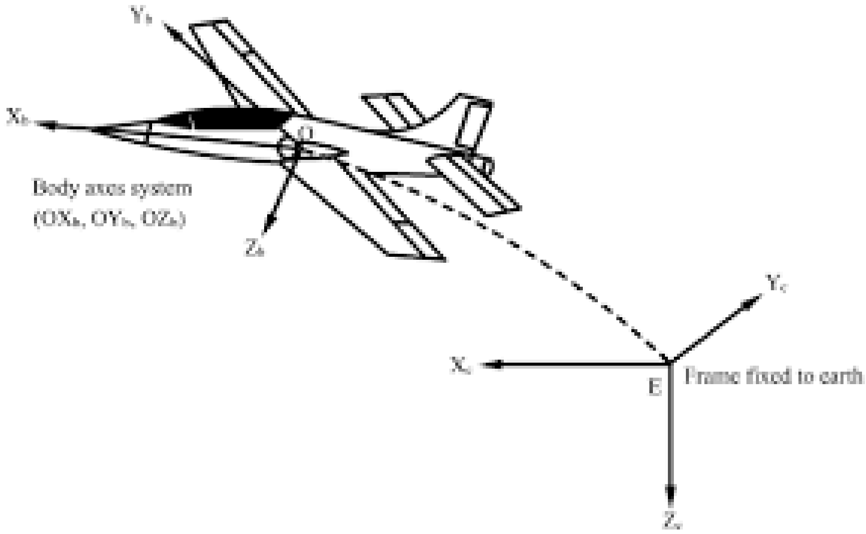

Fig. 1 - Earth fixed and body fixed co-ordinate systems [1]

In more precisely, the flight behavior presented in term of velocities, position and attitude can be described in 12 state variables as follows [2]:

$$\boldsymbol{v} = \begin{bmatrix} U \\ V \\ W \\ P \\ Q \\ R \end{bmatrix} = \begin{bmatrix} \text{longitudinal (forward)velocity} \\ \text{lateral (transverse) velocity} \\ \text{vertical} \\ \text{roll rate} \\ \text{pitch rate} \\ \text{yaw rate} \end{bmatrix} \tag{1}$$

$$\boldsymbol{\eta} = \begin{bmatrix} x_E \\ y_E \\ z_E \\ \Phi \\ \Theta \\ \Psi \end{bmatrix} = \begin{bmatrix} \text{earth-fixed x position} \\ \text{earth-fixed y position} \\ \text{earth-fixed z position} \\ \text{roll angle} \\ \text{pitch angle} \\ \text{yaw angle} \end{bmatrix} \tag{2}$$

While for the forces and moments are denoted in vector notation as $\boldsymbol{F^B} = [X \quad Y \quad Z]^T$ and $\boldsymbol{M^B} = [L \quad M \quad N]^T$ where X, Y, and Z are the longitudinal, transverse, and vertical forces, and L, M, and N are the roll, pitch, and yaw moments. The definition of notation vector velocity $\boldsymbol{\nu}$ **,** position and attitude $\boldsymbol{\eta}$ and forces and moment as shown in the Fig. 2.

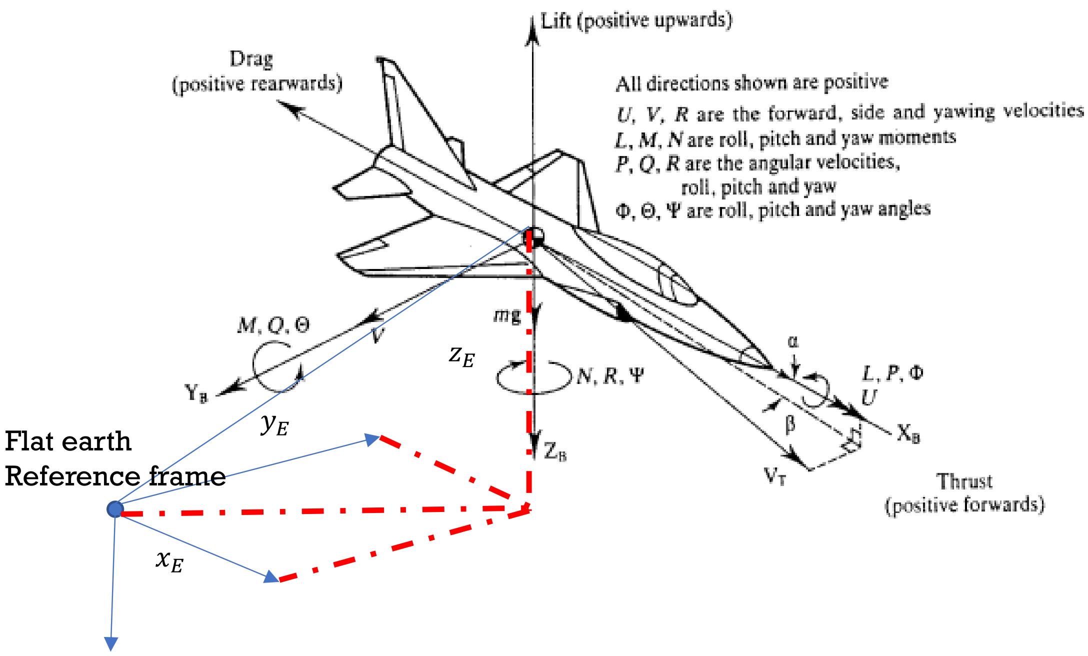

Fig. 2 - Definition notation velocity, position and forces [3]

The implementation the Newton second law of translational and angular motion generate the dynamic equation of flight motion as [2]:

$$X = m[\dot{U} + QW - RV + g \sin \Theta] \tag{3}$$

$$Y = m[\dot{V} + UR - WP - g \cos \Theta \sin \Phi] \tag{4}$$

$$Z = m[\dot{W} + VP - QU - g \cos \Theta \cos \Phi] \tag{5}$$

$$L = I_x \dot{P} - I_{xz}[\dot{R} + PQ] + (I_z - I_y) QR \tag{6}$$

$$M = I_y \dot{Q} + I_{xz}[P^2 - R^2] + (I_x - I_z) PR \tag{7}$$

$$N = I_z \dot{R} - I_{xz}[\dot{P} - QR] + (I_y - I_x) PQ \tag{8}$$

While the governing equation of flight motion related to the attitude and position of the aircraft with respect to the Earth fixed reference frame is called as kinematic equation:

$$\begin{bmatrix} \dot{x}_E \\ \dot{y}_E \\ \dot{z}_E \end{bmatrix} = [A] \begin{bmatrix} U \\ V \\ W \end{bmatrix} \tag{9}$$

where,

$$[A] = \begin{bmatrix} \cos\Psi\cos\Theta & -\sin\Psi\cos\Phi + \cos\Psi\sin\Theta\sin\Phi & \sin\Psi\sin\Phi + \cos\Psi\cos\Phi\sin\Theta \\ \sin\Psi\cos\Theta & \cos\Psi\cos\Phi + \sin\Phi\ \sin\Theta\sin\Psi & -\cos\Psi\sin\Phi + \sin\Theta\sin\Psi\cos\Phi \\ -\sin\Theta & \cos\Theta\sin\Phi & \cos\Theta\cos\Phi \end{bmatrix} \tag{10}$$

and,

$$\begin{bmatrix} \dot{\Phi} \\ \dot{\Theta} \\ \dot{\Psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\Phi\tan\Theta & \cos\Phi\ \tan\Theta \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \dfrac{\sin\Phi}{\cos\Theta} & \dfrac{\cos\Phi}{\cos\Theta} \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \tag{11}$$

The system equation that describes flight behavior can be solved using the Fourth Order Runge-Kutta Scheme, as demonstrated by Ozdemir and Kavsaoglu [4]. It is true that the system equation, which consists of 12 first-order differential equations, is non-linear and that the equations are coupled to each other. However, for a particular flight condition and if one is just interested in investigating the flight behavior due to a disturbance imposed on the equilibrium flight condition in a short time, it is not necessary to solve these twelve equations. Here one may just focus on the first six equations, namely the dynamic equations.

To include the presence of thrust, T, aircraft weight, *mg* and aerodynamic forces, the dynamics Eq. (3-8) can be written as [5]:

$$m\left[\dot{U} + QW - RV\right] = -mg\sin\Theta + F_{Ax} + T_{Ax} \quad (12)$$

$$m\left[\dot{V} + UR - WP\right] = mg\cos\Theta\sin\Phi + F_{Ax} + T_{Ax} \quad (13)$$

$$m\left[\dot{W} + VP - QU\right] = mg\cos\Theta\cos\Phi + F_{Az} + T_{Az} \quad (14)$$

$$I_x\dot{P} - I_{xz}\left[\dot{R} + PQ\right] + \left(I_z - I_y\right)QR = L_A + L_T \quad (15)$$

$$I_y\dot{Q} + I_{xz}[P^2 - R^2] + (I_x - I_z)PR = M_A + M_T \quad (16)$$

$$I_z\dot{R} - I_{xz}\left[\dot{P} - QR\right] + \left(I_y - I_x\right)PQ = N_A + N_T \quad (17)$$

In solving the above equation, one can introduce that the aircraft is in steady state condition [4]. It is meant that the linear and the angular accelerations with respect to the aircraft body frame *x*, *y*, and *z* are zero. Hence,

$$\boldsymbol{v} = \begin{bmatrix} U \\ V \\ W \\ P \\ Q \\ R \end{bmatrix} = \boldsymbol{const};\ \dot{\boldsymbol{v}} = \begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \\ \dot{P} \\ \dot{Q} \\ \dot{R} \end{bmatrix} = \mathbf{0}$$

In the steady state if the corresponding flight conditions are denoted by subscript 1, and in this flight condition, the flight is perturbed, in such condition the flight variables can be written as:

$$U = U_1 + u; \quad V = V_1 + v; \quad W = W_1 + w \quad (18)$$

$$P = P_1 + p; \quad Q = Q_1 + q; \quad R = R_1 + r \quad (19)$$

$$\Phi = \Phi_1 + \phi; \quad \Theta = \Theta_1 + \theta; \quad \Psi = \Psi_1 + \psi \quad (20)$$

$$F_{Ax} = F_{Ax_1} + f_{Ax}; \ \ F_{Ay} = F_{Ay_1} + f_{Ay}; \ \ F_{Az} = F_{Az_1} + f_{Az} \quad (21)$$

$$L_A = L_{A_1} + l_A; \ \ M_A = M_{A_1} + m_A; \ \ N_A = N_{A_1} + n_A \quad (22)$$

$$F_{Tx} = F_{Tx_1} + f_{Tx}; \ \ F_{Ty} = F_{Ty_1} + f_{Ty}; \ \ F_{Tz} = F_{Tz_1} + f_{Tz} \quad (23)$$

$$L_T = L_{T_1} + l_T; \ \ M_T = M_{T_1} + m_T; \ \ N_T = N_{T_1} + n_T \quad (24)$$

and imposing the conditions associated with steady-state flight conditions:

$$\dot{U} = \dot{U}_1 + \dot{u} = \dot{u}\,; \qquad \dot{V} = \dot{V}_1 + \dot{v} = \dot{v}; \quad (25)$$

$$\dot{W} = \dot{W}_1 + \dot{w} = \dot{w}\,; \quad \dot{P} = \dot{P}_1 + \dot{p} = \dot{p}; \quad (26)$$

$$\dot{Q} = \dot{Q}_1 + \dot{q} = \dot{q}\,; \qquad \dot{R} = \dot{R}_1 + \dot{r} = \dot{r}; \quad (27)$$

Introducing disturbance in steady flight by using the relationship as given by Eq. (18-27) and substitute into the dynamics equation as given by Eq. (12-17), yields:

$$m[\dot{u} + (Q_1 + q)(W_1 + w) - (R_1 + r)(V_1 + v)] = -mg\sin(\Theta_1 + \theta) + \left(F_{Ax_1} + f_{Ax}\right) + \left(F_{Tx_1} + f_{Tx}\right) \quad (28)$$

$$m[\dot{v} + (U_1 + u)(R_1 + r) - (P_1 + p)(W_1 + w)] = mg\cos(\Theta_1 + \theta)\sin(\Phi_1 + \phi) + \left(F_{Ay_1} + f_{Ay}\right) + \left(F_{Ty_1} + f_{Ty}\right) \quad (29)$$

$$m[\dot{w} + (P_1 + p)(V_1 + v) - (Q_1 + q)(U_1 + u)] = mg\cos(\Theta_1 + \theta)\cos(\Phi_1 + \phi) + \left(F_{Az_1} + f_{Az}\right) + \left(F_{Tz_1} + f_{Tz}\right) \quad (30)$$

$$I_x\,\dot{p} - I_{xz}\dot{r} - I_{xz}\,(P_1 + p)(Q_1 + q) + \left(I_z - I_y\right)(Q_1 + q)(R_1 + r) = (L_{A1} + l_A) + (L_{T1} + l_T) \quad (31)$$

$$I_x\,\dot{q} + I_{xz}[(P_1 + p)^2 - (R_1 + r)^2] + (I_x - I_z)(P_1 + p)(R_1 + r) = (M_{A1} + m_A) + (M_{T1} + m_T) \quad (32)$$

$$I_z\dot{r} - I_{xz}\dot{p} + I_{xz}(Q_1 + q)(R_1 + r) + \left(I_y - I_x\right)(P_1 + p)(Q_1 + q) = (N_{A1} + n_A) + (N_{T1} + n_T) \quad (33)$$

The kinematic equation which gives the relationship between Euler angle (Φ, Θ, Ψ) and rate of angular velocity (p, q, r) in the context of the presence perturbation from the steady state flight condition can be given as [5]:

$$p = \dot{\Phi} - \dot{\Psi}_1\,\theta\cos\Theta_1 - \dot{\Psi}_1\sin\Theta_1 \quad (34)$$

$$q = -\dot{\Theta}_1\,\phi\cos\Phi_1 - \dot{\theta}\,\sin\Phi_1 - \dot{\Psi}_1\,\theta\sin\Phi_1\sin\Theta_1 + \dot{\Psi}_1\,\phi\cos\Phi_1\cos\Theta_1 + \dot{\Psi}_1\sin\Phi_1\cos\Theta_1 \quad (35)$$

$$r = -\dot{\Psi}_1\,\theta\cos\Phi_1\sin\Theta_1 - \dot{\Psi}_1\,\phi\sin\Phi_1\cos\Theta_1 - \dot{\theta}\,\sin\Phi_1 + \dot{\Psi}_1\cos\Phi_1\cos\Theta_1 - \dot{\Theta}_1\,\phi\cos\Phi_1 \quad (36)$$

In steady-state rectilinear wing-level flight, one can imposed the following assumptions:

- The angular velocities $P_1 = Q_1 = R_1 = 0$;
- Euler angles $\Phi_1 = conts$; $\Theta_1 = const$; and $\Psi_1 = const$.
- Lateral velocity $V_1 = 0$;
- Roll angle $\Phi_1 = 0$, $\sin\Phi_1 = 0$, and $\cos\Phi_1 = 1$ (wing level).

The usage of the abovementioned condition would simplify Eq. (28-36) to become:

$$m[\dot{u} + qW_1] = -mg\theta\cos\Theta_1 + (f_{Ax} + f_{Tx}) \quad (37)$$

$$m[\dot{v} + rU_1 - pW_1] = mg\phi\cos\Theta_1 + \left(f_{Ay} + f_{Ty}\right) \quad (38)$$

$$m[\dot{w} - qU_1] = -mg\theta\cos\Theta_1 + (f_{Az} + f_{Tz}) \quad (39)$$

$$I_x\,\dot{p} - I_{xz}\dot{r} = l_A + l_T \quad (40)$$

$$I_x\,\dot{q} = m_A + m_T \quad (41)$$

$$I_z\dot{r} - I_{xz}\dot{p} = n_A + n_T \quad (42)$$

$$p = \dot{\phi} - \dot{\psi}\sin\Theta_1 \quad (43)$$

$$q = \dot{\theta} \quad (44)$$

$$r = \dot{\psi}\cos\Theta_1 \quad (45)$$

In the lateral – directional flight motion, one will deal with the motion of *x-z* plane which will move out of some *x-z* plane fixed in space. Translation in the y direction, roll about the x axis, and yaw about the z axis would all cause the *x-z* plane of the aircraft to move out of that arbitrarily fixed *x-z* plane in space [6]. The governing equation of flight motion consists of the *Y* force (Eq. (38)), *L* moment (Eq. (40)), and *N* moment equation (Eq. (45)). However, to make the number of unknown quantities the same as the number of equations, two additional equations from the kinematics formulation are needed. As result in the governing equation of the lateral and directional flight motion becomes:

$$m[\dot{v} + rU_1 - pW_1] = mg\ \phi \cos\Theta_1 + (f_{Ay} + f_{Ty}) \tag{46}$$

$$I_x\,\dot{p} - I_{xz}\dot{r} = l_A + l_T \tag{47}$$

$$I_z\dot{r} - I_{xz}\dot{p} = n_A + n_T \tag{48}$$

$$p = \dot{\phi} - \dot{\Psi}\sin\Theta_1 \tag{49}$$

$$r = \dot{\Psi}\cos\Theta_1 \tag{50}$$

If the forces and moments from the aerodynamics and restrict to the motion in the lateral dan directional flight direction only, the governing equation of flight motion Eq. (37) to (45) can be simplified to becomes:

$$\left(V_{P_1}\dot{\beta} + V_{P_1}\dot{\psi}\right) = g\phi + Y_\beta\beta + Y_{\dot{\phi}}\dot{\phi} + Y_{\dot{\psi}}\dot{\psi} + Y_{\delta_A}\delta_A + Y_{\delta_R}\delta_R \tag{51}$$

$$\ddot{\phi} - \frac{I_{XZ}}{I_{XX}}\ddot{\psi} = L_\beta\beta + L_{\dot{\phi}}\dot{\phi} + L_{\dot{\psi}}\dot{\psi} + L_{\delta_A}\delta_A + L_{\delta_R}\delta_R \tag{52}$$

$$\ddot{\psi} - \frac{I_{XZ}}{I_{ZZ}}\ddot{\phi} = N_\beta\beta + N_{\dot{\phi}}\dot{\phi} + N_{\dot{\psi}}\dot{\psi} + N_{\delta_A}\delta_A + N_{\delta_R}\delta_R \tag{53}$$

where,

$$Y_{\dot{\phi}} = Y_P,\quad Y_{\dot{\psi}} = Y_r,\ L_{\dot{\phi}} = L_P,\ L_{\dot{\psi}} = L_r,\quad N_{\dot{\phi}} = N_p,\ N_{\dot{\psi}} = N_r \tag{54}$$

Table 1 shows the definition of variables which appear in the Eq. (51-53).

Table 1 - The definition variables in the lateral – directional flight motion [5]

$Y_\beta = \frac{q_1 S C_{Y\beta}}{m}\ \frac{ft}{sec^2}$	$Y_p = \frac{q_1 S C_{Yp}}{m}\frac{b}{2U_1}\ \frac{ft}{sec^2}$
$Y_r = \frac{q_1 S C_{Yr}}{m}\ \frac{b}{2U_1}\frac{ft}{sec^2}$	$Y_{\delta A} = \frac{q_1 S C_{Y\delta A}}{m}\ \frac{ft}{sec^2}$
$Y_{\delta R} = \frac{q_1 S C_{Y\delta r}}{m}\ \frac{ft}{sec^2}$	
$L_\beta = \frac{q_1 S C_{\ell\beta}}{I_{xx}}\ b\ \frac{1}{sec^2}$	$L_p = \frac{q_1 S C_{\ell p}}{I_{xx}}\frac{b}{2U_1}\ \frac{1}{sec}$
$L_r = \frac{q_1 S C_{\ell r}}{I_{xx}}\ \frac{b}{2U_1}\ \frac{1}{sec}$	$L_{\delta A} = \frac{q_1 S C_{\ell\delta A}}{I_{xx}} b\ \frac{1}{sec^2}$
$L_{\delta R} = \frac{q_1 S C_{\ell\delta R}}{I_{xx}}\ b\ \frac{1}{sec^2}$	
$N_\beta = \frac{q_1 S C_{n\beta}}{I_{zz}}\ b\ \frac{1}{sec^2}$	$N_{TB} = \frac{q_1 S C_{nT\beta}}{I_{zz}}\ b\ \frac{1}{sec^2}$
$N_p = \frac{q_1 S C_{np}}{I_{zz}}\ \frac{b}{2U_1}\ \frac{1}{sec}$	$N_r = \frac{q_1 S C_{nr}}{I_{zz}}\ \frac{b}{2U_1}\ \frac{1}{sec}$
$L_{\delta A} = \frac{q_1 S C_{n\delta A}}{I_{zz}}\ b\ \frac{1}{sec^2}$	$L_{\delta R} = \frac{q_1 S C_{\ell\delta A}}{I_{xx}} b\ \frac{1}{sec^2}$

3. The Solution of Lateral and Directional Flight Motion

As the first order differential equation with zero initial condition, one can use a Laplace transform for solving a differential equation system. In Laplace transform, Eq. (51-53) becomes:

$$\begin{bmatrix} (sV_{P_1} - Y_\beta) & -(sY_p + \mathrm{g}) & s(V_{P_1} - Y_r) \\ -L_\beta & s(s - L_p) & -s(sI_1 + L_r) \\ -N_\beta & -s(sI_2 - N_p) & s(s - N_r) \end{bmatrix} \begin{Bmatrix} \frac{\beta(s)}{\delta(s)} \\ \frac{\phi(s)}{\delta(s)} \\ \frac{\psi(s)}{\delta(s)} \end{Bmatrix} = \begin{Bmatrix} Y_\delta \\ L_\delta \\ N_\delta \end{Bmatrix} \tag{55}$$

where $\delta(s)$ can be considered either $\delta_A(s)$ or $\delta_R(s)$. Here the the dimensional control derivatives $Y_\delta, L_\delta, N_\delta$ can be associated either to the ailerons or to the rudder. To solve Eq. (55) and so the unknown quantities $\left[\frac{\beta(s)}{\delta(s)}, \frac{\phi(s)}{\delta(s)}, \frac{\psi(s)}{\delta(s)}\right]$ can be defined, one can use the Cramer's rule. The results are:

$$\frac{\beta(s)}{\delta(s)} = \frac{\begin{vmatrix} Y_\delta & -(sY_p + \mathrm{g}\cos\Theta_1) & s(V_{P_1} - Y_r) \\ L_\delta & s(s - L_p) & -s(sI_1 + L_r) \\ N_\delta & -s(sI_2 - N_p) & s(s - N_r) \end{vmatrix}}{\begin{vmatrix} (sV_{P_1} - Y_\beta) & -(sY_p + \mathrm{g}\cos\Theta_1) & s(V_{P_1} - Y_r) \\ -L_\beta & s(s - L_p) & -s(sI_1 + L_r) \\ -N_\beta & -s(sI_2 - N_p) & s(s - N_r) \end{vmatrix}} = \frac{N_\beta^\delta(s)}{\overline{D}_2(s)} \tag{56}$$

$$\frac{\phi(s)}{\delta(s)} = \frac{\begin{vmatrix} (sV_{P_1} - Y_\beta) & Y_\delta & s(V_{P_1} - Y_r) \\ -L_\beta & L_\delta & -s(sI_1 + L_r) \\ -\left(N_\beta + N_{T_\beta}\right) & N_\delta & s(s - N_r) \end{vmatrix}}{\begin{vmatrix} (sV_{P_1} - Y_\beta) & -(sY_p + g\cos\Theta_1) & s(V_{P_1} - Y_r) \\ -L_\beta & s(s - L_p) & -s(sI_1 + L_r) \\ -N_\beta & -s(sI_2 - N_p) & s(s - N_r) \end{vmatrix}} = \frac{N_\phi^\delta(s)}{\overline{D}_2(s)} \tag{57}$$

$$\frac{\psi(s)}{\delta(s)} = \frac{\begin{vmatrix} (sV_{P_1} - Y_\beta) & -(sY_p + g\cos\Theta_1) & Y_\delta \\ -L_\beta & s(s - L_p) & L_\delta \\ -\left(N_\beta + N_{T_\beta}\right) & -s(sI_2 - N_p) & N_\delta \end{vmatrix}}{\begin{vmatrix} (sV_{P_1} - Y_\beta) & -(sY_p + g\cos\Theta_1) & s(V_{P_1} - Y_r) \\ -L_\beta & s(s - L_p) & -s(sI_1 + L_r) \\ -N_\beta & -s(sI_2 - N_p) & s(s - N_r) \end{vmatrix}} = \frac{N_\psi^\delta(s)}{\overline{D}_2(s)} \tag{58}$$

where:

$$\begin{aligned} N_\beta^\delta(s) &= s(A_\beta s^3 + B_\beta s^2 + C_\beta s + D_\beta \\ A_\beta &= Y_\delta(1 - I_1 I_2) \\ B_\beta &= -Y_\delta\left(L_p + N_r + I_1 N_p + I_2 L_r\right) + Y_p(L_\delta + I_1 N_\delta) + Y_r(L_\delta I_2 + N_\delta) - (V_{p_1}(L_\delta I_2 + N_\delta) \\ C_\beta &= Y_\delta\left(L_p N_r - N_p L_r\right) + Y_p(L_r N_\delta - N_r L_\delta) + g(L_\delta + I_1 N_\delta) + Y_r\left(L_\delta N_p - N_\delta L_p\right) - V_{p_1}\left(L_\delta N_p - N_\delta L_p\right) \\ D_\beta &= g(L_r N_\delta - N_r L_\delta) \end{aligned} \tag{59}$$

$$\begin{aligned} N_\phi^\delta(s) &= s\left(A_\phi s^2 + B_\phi s + C_\phi\right) \\ A_\beta &= V_{P_1}(L_\delta + I_1 N_\delta) \\ B_\phi &= V_{P_1}(L_r N_\delta - N_r L_\delta) + Y_\beta(L_\delta + I_1 N_\delta) + Y_\delta\left(L_\beta + I_1 N_\delta\right) \\ C_\phi &= -Y_\beta(L_r N_\delta - N_r L_\delta) + Y_\delta\left(L_r N_\beta - N_\beta L_r\right) + V_{P_1}\left(L_\delta N_\beta - L_\beta N_\delta\right) - Y_r\left(L_\delta N_\beta - L_\beta N_\delta\right) \end{aligned} \tag{60}$$

$$\begin{aligned} N_\psi^\delta(s) &= A_\psi s^3 + B_\psi s^2 + C_\psi s + D_\psi \\ A_\psi &= V_{P_1}(N_\delta + I_2 L_\delta) \\ B_\psi &= V_{P_1}(L_\delta N_P - N_\delta L_P) - Y_\beta(N_\delta + I_2 L_\delta) + Y_\delta\left(L_\beta I_2 + N_\beta\right) \end{aligned} \tag{61}$$

$$C_\psi = -Y_\beta(L_\delta N_P - N_\delta L_P) + Y_P(L_\delta N_\beta - L_\beta N_\delta) + Y_\delta(L_\beta N_P - L_P N_\delta)$$
$$D_\psi = g(L_\delta N_\beta - L_\beta N_\delta)$$

The denominator $\overline{D}_2(s)$ is given as:

$$\begin{aligned}
\overline{D}_2(s) &= s(A_2 s^4 + B_2 S^3 + C_2 s^2 + D_2 s + E_2)\\
A_2 &= V_{P_1}(1 - I_1 I_2)\\
B_2 &= -Y_\beta(1 - I_1 I_2) - V_{P_1}(L_p + N_r + I_1 N_p + I_2 L_r)\\
C_2 &= V_{P_1}(L_P N_r - N_P L_r) + Y_\beta(L_p + N_r + I_1 N_p + I_2 L_r) - Y_p(L_\beta + I_1 N_\beta) + V_{P_1}(L_\beta I_2 + N_\beta)\\
&\quad - Y_r(L_\beta I_2 - N_\beta)\\
D_2 &= -Y_\beta(L_p N_r - N_p L_r) + Y_p(L_\beta N_r - L_r N_\beta) - g(L_\beta + I_1 N_\beta) + V_{P_1}(L_\beta N_p - L_p N_\beta)\\
&\quad - Y_r(L_\beta N_p - L_p N_\beta)\\
E_2 &= g(L_\beta N_r - L_r N_\beta)
\end{aligned} \tag{62}$$

Using Eq. (59-62), one can define how the side slip angle β, roll angle Φ and yaw angle Ψ will behave with respect to time for a given a disturbance $\delta(s)$.

4. Disturbance Models

To simulate the behavior of the airplane from the point view of lateral and directional flight motion by using Eq. (56-62), here the single impulse $\delta_{si}(t)$, and the multiple impulse $\delta_{mi}(t)$ are defined respectively as [5]:

$$\delta_{si}(t) = \begin{cases} 0 & 0 \le t \le 200 \\ -a & 200 < t \le 205 \\ 0 & 205 < t \le 3200 \\ b & 3200 < t \le 3205 \\ 0 & 3205 \le t \le 12001 \end{cases} \tag{63}$$

$$\delta_{mi}(t) = \begin{cases} 0 & 0 \le t \le 200 \\ -a & 200 < t \le 215 \\ 0 & 215 < t \le 400 \\ b & 400 < t \le 415 \\ 0 & 415 < t \le 800 \\ -d & 800 < t \le 815 \\ 0 & 815 < t \le 1000 \\ e & 1000 < t \le 1015 \\ 0 & 1015 < t \le 12001 \end{cases} \tag{64}$$

The amplitude of the disturbance can be prescribed the same or different values. In the case of a single impulse, the present work set the value of $a = b = 4$, while for the case multiple impulse use $a = b = 5$ while $c = d = 2$.

5. Discussion and Results

The current study compares the lateral and directional flight characteristics of two aircraft. Those two aircraft are the Beechcraft 99 and the Cessna T37. The Beechcraft Model 99 is a civilian aircraft produced by US aircraft manufacturer Beechcraft. This aircraft was designed as a commuter twin-engine aircraft with 15 to 17 passenger seats. The Cessna T37 aircraft is also manufactured by a US aircraft manufacturer named Cessna Aircraft Company, whose base is in Wichita, Kansas, in the U.S. state of Kansas. In the context of lateral-directional dynamic stability analysis, the required data for these two aircraft is shown in Table 2.

Table 2 - The Aircraft Data for Beechcraft 99 and Cessna T37 aircraft [5,6,7,8]

Aircraft Model		
Wing geometry data	Beechcraft 99	Cessna T37
Wing area S (ft^2)	280	182
Wing mean aerodynamic chord (ft)	6.5	5.47
Wingspan (ft)	46	33.8
Flight Condition		
Altitude (ft)	5000	30000

Mach number	0.310	0.549
True airspeed $\left(\frac{ft}{sec}\right)$	340	456
Dynamic pressure $\left(\frac{lb}{ft^3}\right)$	118.3	92.7
Location c.g (%Mac)	0.16	0.27
Steady state angle of attack (deg)	0	2
Mass Properties		
Mass (lb)	7000	6360
Moment of inertia x-axis I_{xx} ($slug\ ft^2$)	10085	7985
Moment of inertia y-axis I_{yy} ($slug\ ft^2$)	15148	3326
Moment of inertia z-axis I_{zz} ($slug\ ft^2$)	23046	11183
Product of inertia xz, I_{xn} ($slug\ ft^2$)	1600	0
Stability derivatives		
$C_{\ell\beta}$	-0.13	-0.0944
$C_{\ell p}$	-0.50	-0.442
$C_{\ell r}$	0.14	0.0926
$C_{Y\beta}$	-0.59	-0.346
C_{Yp}	-0.19	0.0827
C_{Yr}	0.39	0.3
$C_{n\beta}$	0.08	0.1106
$C_{nT\beta}$	0	0
C_{np}	0.019	-0.0243
C_{nr}	-0.197	-0.139
Control derivatives		
$C_{\ell\delta A}$	0.156	0.181
$C_{\ell\delta R}$	0.0109	0.015
$C_{Y\delta A}$	0	0
$C_{Y\delta R}$	0.145	0.2
$C_{n\delta A}$	-0.0012	-0.0254
$C_{n\delta R}$	-0.0772	-0.0365

5.1 Case the Beechcraft 99 Aircraft

If the single impulse as defined by Eq. (56) is applied to the aircraft, the output response of the aircraft to such a disturbance is shown in Fig. 3. In the dynamic simulation, the deflection of the aileron is set between -8° (down) and +5° (upward). Fig. 3 shows that the aircraft is able to return to its origin as the disturbance disappears. When this single impulse function is used to generate a disturbance for the rudder with a similar deflection angle, ($+5^0$), the aircraft responds as shown in Fig. 4. Using the above results, one can identify the sideslip effect, $\beta(t)$ and the yaw angle, $\Psi(t)$ is not the same as the roll angle, $\phi(t)$. The relative airflow that causes the rudder to deflect $-4°$ increases the angle of yaw $\Psi(t)$ by two times. As the disturbance fades, the sideslip angle $\beta(t)$ and angle roll, $\phi(t)$ are damped towards zero.

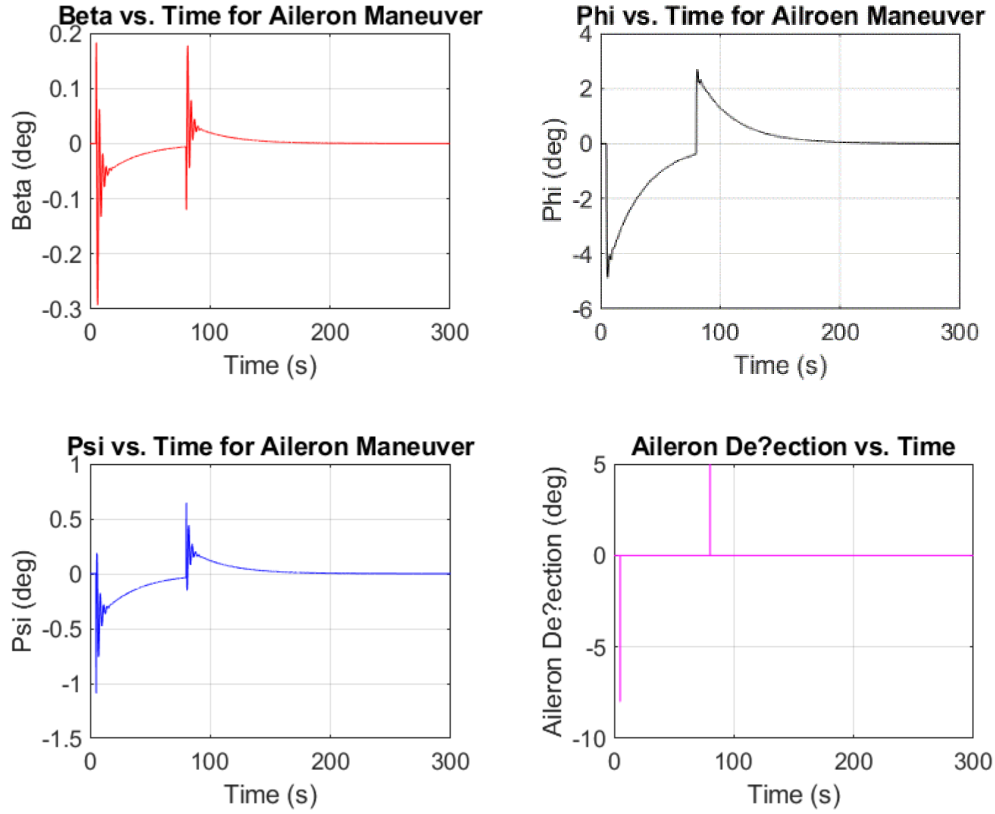

Fig. 3 - The aircraft response in a side slip angle $\beta(t)$, roll angle $\Phi(t)$ and the yaw angle $\Psi(t)$ due to a single impulse aileron disturbance model on the Beechcraft 99 aircraft.

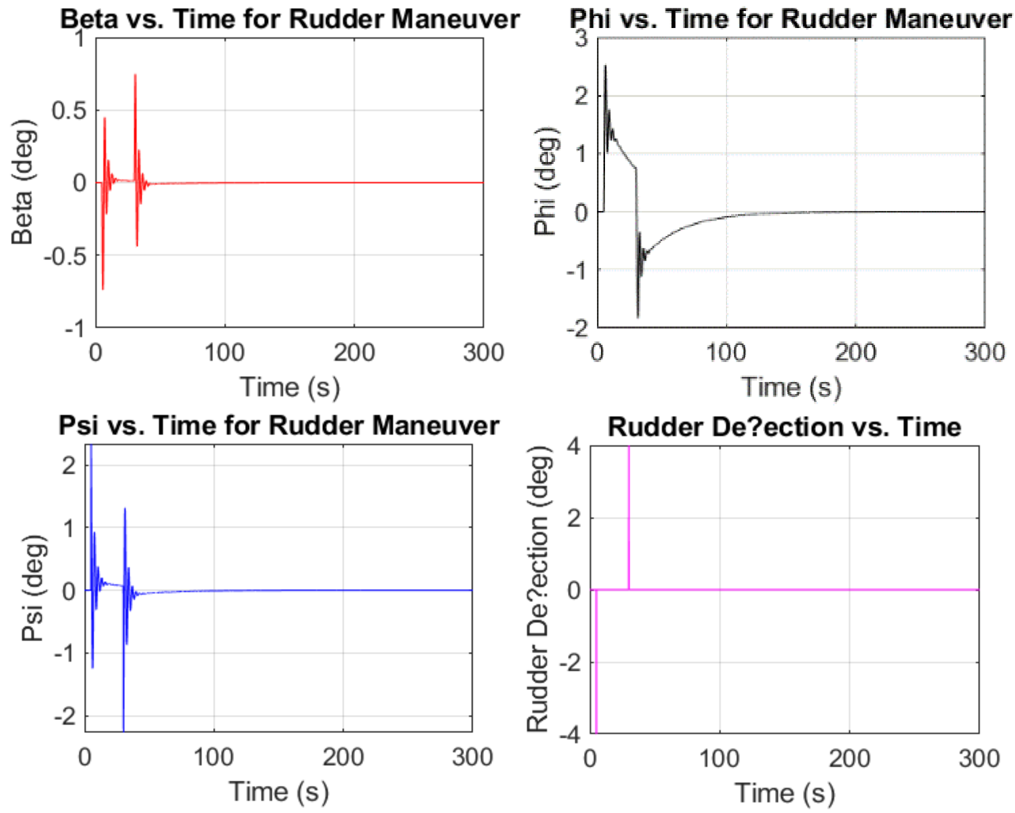

Fig. 4 - The aircraft response in a side slip angle $\beta(t)$, roll angle $\Phi(t)$ and the yaw angle $\Psi(t)$ due to a single impulse rudder disturbance model on the Beechcraft 99 aircraft.

5.2 Case the Cessna T37

For the case of the Cessna T37, the disturbance model in use is the multiple impulse as given by Eq. (57). This disturbance model uses the magnitudes $a = b = 5^0$ and $c = d = 2^0$. If multiple impulses are applied to simulate the presence of disturbance due to aileron, the aircraft response is shown in Fig. 6. Fig. 5 shows that side slip angle $\beta(t)$ peak at amplitude -0.3º and 0.3º which is similar to the peak of the disturbance. It is discovered that the side slip can return to its original state of 0º after 110 seconds. Considering the roll angle Φ vs. time: the roll angle reaches its peak value at 0.8º and $-1.3°$ before decreasing to 0º completely after 200 sec. While in view of yaw angle behavior, the yaw angle reaches their peak value at -5.5º and 0.5º. The yaw angle cannot go back to its original value.

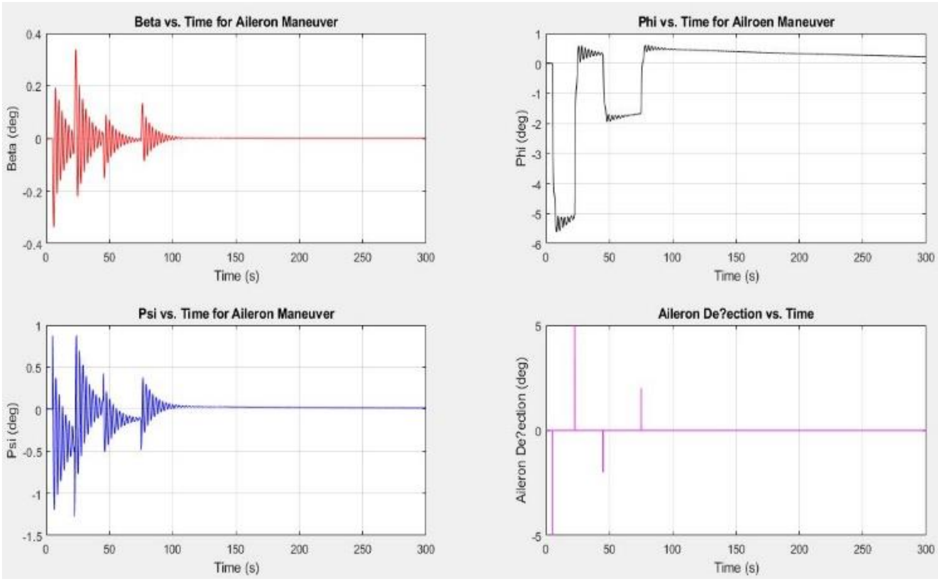

Fig. 5 - The Cessna T37's response to the multiple impulse aileron disturbance model

The same disturbance model as given by Eq. (57) is applied to the rudder. The aircraft responses for the side slip angle β(t), roll angle $\Phi(t)$, and yaw angle $\Psi(t)$, are shown in Fig. 6. This figure shows the rudder deflected upward (-4) and downward (+4)., The side slip angle β reaches the peak angle at -1 deg. When the second disturbance excited this angle, it was still oscillating; as a result, the oscillation vanished after 140 seconds. In similar appearances, the first disturbance still makes the roll angle oscillate at the time the second disturbance is introduced. In the presence of a second disturbance, the roll angle achieves its convergent solution at 300 sec.

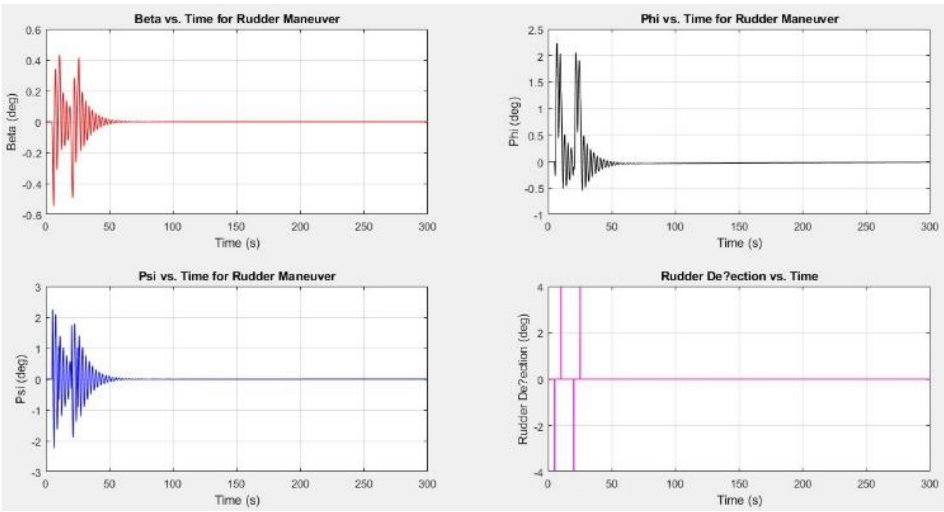

Fig. 6 - The Cessna T37's response to the multiple impulse of rudder disturbance model

6 Conclusion and Future Work

Using the results from two model aircraft, one can identify that the lateral–directional equation of flight motion can be solved by using the Laplace transform. Here one can obtain how the side slip angle, β, the roll angle, Φ and the yaw angle, Ψ change with respect to time. In the case of a single impulse applied to the aileron and rudder, the Beechcraft 99 aircraft were able to return to their original condition as the disturbance lifted. However, in the case of multiple impulses applied to the Cessna T37, there is a slight difference in response between aileron and rudder. In the case where a rudder acts as a source of disturbance, the three variable states (β, Φ, Ψ) of the Cessna T37 are plotted with respect to time, showing that the tendency goes to zero immediately as the disturbance disappears. However, when aileron is in use, the yaw angle response needs more time to go back to its initial condition. It is true that the present work treats aileron and rudder separately, therefore, an investigation of the aircraft response with aileron and rudder as a combined source of disturbance may need to be carried out. This is the proposed work for the future work.

Acknowledgement

This research was supported by Universiti Tun Hussein Malaysia (UTHM) through Tier 1 (vot Q112).

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