Journal of Sustainable and Manufacturing in Transportation Vol. 3 No. 2 (2023) 38-55

JSMT

http://publisher.uthm.edu.my/ojs/index.php/jsmt

e-ISSN : 2821-2835

Journal of Sustainable and Manufacturing in Transportation

Commercial Aircraft Selection Decision Support Model Using Fuzzy Combinative Multiple Criteria Decision Making Analysis

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DOI: https://doi.org/10.30880/jsmt.2023.03.02.005
Received 29 September 2023; Accepted 14 January 2024; Available online 31 December 2023

Abstract: Multidimensional decision-making issues invariably entail a profusion of factors and dimensions necessitating consideration. In such intricate environments, decision-making can be notably challenging. To navigate this complexity, the realm of multiple criteria decision-making approaches is indispensable, helping in unraveling convoluted predicaments. This paper introduces a novel approach fuzzy combinative multiple criteria decision-making analysis — crafted to address fuzzy decision analysis challenges. At the heart of this method lies the utilization of the Euclidean distance as the primary measure and the rectilinear distance as the secondary measure, both calibrated in accordance with the negative-ideal solution. Determining the desirability of alternatives rests on these distances. Further augmenting the evaluation, the process involves combining the Euclidean and rectilinear distances through fuzzy combinative multiple criteria decision-making analysis. The efficacy and stability of the fuzzy decision analysis are underscored through a numerical example, illustrating the proposed method's procedural nuances. A comparative analysis comparing both Euclidean distance and rectilinear distance with the weighted sum model and the weighted product model is also conducted. This scrutiny underscores the efficiency of the proposed method while affirming the reliability of its outcomes.

Keywords: Euclidean distance, rectilinear distance, commercial aircraft selection, weighted sum model, weighted product model, fuzzy combinative multiple criteria decision-making analysis

1. Introduction

Commercial aircraft selection is a pivotal process guided by multifaceted factors that significantly influence the performance and efficiency of these intricate flying machines. Achieving optimal efficiency hinges on the meticulous design of aircraft that minimizes drag across all operational configurations. This necessitates the creation of airframes devoid of irregularities, characterized by smooth surfaces, flush-fitted doors, and precisely aligned control surfaces.

The process of selecting commercial aircraft for the aviation industry embodies a critical facet of aircraft design and performance. Commercial aircraft selection problem aims to provide comprehensive insights into the intricate strategy underpinning aircraft selection since the dawn of the aviation era, thereby ensuring that contemporary aircraft perpetually adhere to the highest benchmarks of safety and operational efficiency. The evolving landscape of air transportation, catalyzed by the deregulation, liberalization, and privatization trends, prompted airlines to embrace novel strategies in response. Among these strategies, selecting a commercial aircraft using multidimensional decision-making support model emerged as a prominent analytical paradigm. Competitive airlines in aviation sector, employing a point-to-point flight network structure and emphasizing cost-effectiveness, strive to cater to price-conscious passengers while maximizing profitability.

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In the contemporary aviation sector, characterized by intense competition, the challenge lies in providing aircraft models of high quality while simultaneously offering competitive pricing to end customers. Notably, aviation management plays a pivotal role in the broader realm of supply chain management. A pivotal strategic decision for airlines is fleet planning, especially commercial aircraft selection, which significantly impacts long-term viability and competitive success. In this context, aviation management delves into the complex arena of aircraft selection for fleet planning, exploring proper aircraft models. Given the multifaceted considerations—ranging from technical attributes and economic factors to passenger preferences and strategic alignment—this decision-making landscape necessitates a multiple criteria approach [1-4].

Commercial aircraft selection poses a complex, multi-criteria problem involving both qualitative and quantitative factors. Commercial aircraft selection criteria depend on industry characteristics, corporate strategies, and several variables specific to individual companies. In particular, the fact that management strategies, corporate culture and competitive positioning differ among institutions emphasizes the importance of expert evaluations in determining the relevant criteria. Traditionally, commercial aircraft selection methods have leaned towards multidimensional decisions. The challenge in commercial aircraft selection lies in identifying the optimal choice amidst diverse alternatives, with considerations spanning service quality, costs, and risks. Efficiently integrating expert assessments to facilitate sound decision-making requires a systematic approach [5-10].

The Analytical Hierarchy Process (AHP), the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), the Preference Analysis for Reference Ideal Solution (PARIS), and the Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE), and Fuzzy Multiple Criteria Decision Making (FMCDM) emerge as prevalent multiple-criteria decision-making (MCDM) methods [11]. This study introduces a novel approach, fuzzy combinative multiple criteria decision-making analysis (FC-MCDM), designed to address the intricate nature of aircraft selection decisions. To operationalize the FC-MCDM method, a multiple-criteria decision making framework is adopted, enriched with the application of Fuzzy sets to account for decision makers' perceptions. This method leverages two primary distance metrics—Euclidean and Rectilinear—calculated based on the negative-ideal solution. By considering a multitude of factors, the proposed method provides a robust framework to navigate the complex decision-making environment. The FC-MCDM method employs Euclidean and Rectilinear distances from the negative-ideal solution to gauge an alternative's overall performance. Notably, it relies on Euclidean distance as the primary assessment measure, with rectilinear distance used for comparisons. The parameter λ controlling the closeness of combinative Euclidean and Rectilinear distances is affording flexibility to the method.

The proposed FC-MCDM methodology is substantiated through practical application, wherein twenty commercial aircraft alternatives are evaluated using six decision criteria. This study contributes to the burgeoning body of research on commercial aircraft selection decision support models by introducing a holistic and robust approach that aligns with the unique challenges and objectives of airlines. In contrast to existing MCDM methods like the Weighted Sum Model (WSM) and the Weighted Product Model (WPM), the FMCDM method is, a distinctive in its comprehensive approach, seeks to provide a holistic solution to multiple-criteria decision-making challenges inherent in commercial aircraft selection. Through a detailed presentation of the FC-MCDM method, accompanied by a numerical example and comparative analysis, this research contributes to advancing the understanding and application of MCDM techniques in the aviation sector. The subsequent sections of this paper delineate the fuzzy sets theory, and FC-MCDM methodology in Section 2, the application of the proposed FC-MCDM model is presented in Section 3, and finally, research findings, and conclusions are presented in Section 4.

2. Methodology

This section delves into the realm of fuzzy sets, proposing corresponding linguistic variable values for fuzzy numbers. It also introduces the classical combinative Multiple Criteria Decision Making Analysis (MCDM) and Entropy Weight Method for determining criteria weights in a complex challenging MCDM problem [12-28].

2.1 Definition of Fuzzy Sets

A fuzzy set is a mathematical concept used to represent uncertainty and vagueness in set membership. It is defined as a generalization of a classical (crisp) set where elements have partial degrees of membership ranging between 0 (not a member) and 1 (full membership). Formally, a fuzzy set A in a universe of discourse X is defined by its membership function $\mu_A(x)$, which assigns a degree of membership $\mu_A(x)$ to each element x in X. Mathematically, a fuzzy set A in X is defined as [29-31]:

$$A = \{(x, \mu_A(x)) \mid x \in X, 0 \leq \mu_A(x) \leq 1\} \tag{1}$$

Where A represents the fuzzy set, x is an element from the universe of discourse X, and $\mu_A(x)$ is the membership function that assigns a degree of membership to x, where $0 \leq \mu_A(x) \leq 1$.

Basic Operations on Fuzzy Sets:
Union (OR):

For two fuzzy sets *A* and *B* with membership functions $\mu_A(x)$ and $\mu_B(x)$, the union of *A* and *B* denoted as $A \cup B$ is defined as:

$$\mu(A \cup B)(x) = \max(\mu_A(x), \mu_B(x)) \text{ for all } x \text{ in } X. \tag{2}$$

Intersection (AND):

For two fuzzy sets *A* and *B* with membership functions $\mu_A(x)$ and $\mu_B(x)$, the intersection of *A* and *B* denoted as $A \cap B$ is defined as:

$$\mu(A \cap B)(x) = \min(\mu_A(x), \mu_B(x)) \text{ for all x in X.} \tag{3}$$

Extended Operations on Fuzzy Sets:

In addition to the basic operations, fuzzy sets can be operated upon using extended operations, which provide more flexibility in handling fuzzy information.

Complement (NOT):

For a fuzzy set *A* with membership function $\mu_A(x)$, the complement of *A* denoted as A^C is defined as:

$$\mu_{A^C}(x) = 1 - \mu_A(x) \text{ for all } x \text{ in } X. \tag{4}$$

Fuzzy Addition (Algebraic Sum):

For two fuzzy sets *A* and *B* with membership functions $\mu_A(x)$ and $\mu_B(x)$, the fuzzy addition (*A* + *B*) is defined as:

$$\mu(A + B)(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) * \mu_B(x) \text{ for all } x \text{ in } X. \tag{5}$$

Fuzzy Scaling (Multiplication):

For a fuzzy set *A* with membership function $\mu_A(x)$ and a scalar *α*, the fuzzy scaling (*αA*) is defined as:

$$\mu(\alpha A)(x) = \alpha * \mu_A(x) \text{ for all } x \text{ in } X. \tag{6}$$

In fuzzy set theory, the inclusion and equality of fuzzy sets are important concepts for comparing and characterizing fuzzy sets.

Inclusion of Fuzzy Sets:

Fuzzy set *A* is said to be included in fuzzy set *B* if and only if the membership function of fuzzy set *A* is less than or equal to the membership function of fuzzy set *B* for all elements in the universal set *X*. This is denoted as A ⊆ B.

Equality of Fuzzy Sets

Fuzzy set *A* is said to be equal to fuzzy set *B* if and only if the membership functions of fuzzy sets *A* and *B* are equal for all elements in the universal set *X*. This is denoted as $A = B$.

Relationships between Inclusion and Equality. The inclusion and equality of fuzzy sets are related in the following ways:

1. Reflexivity: Every fuzzy set is included in itself.

   $A \subseteq A$ for all fuzzy sets A (7)

   This property states that every fuzzy set is a subset of itself. This is because the membership function of a fuzzy set is always less than or equal to itself for all elements in the universal set *X*.

2. Transitivity: If fuzzy set *A* is included in fuzzy set *B* and fuzzy set *B* is included in fuzzy set *C*, then fuzzy set *A* is included in fuzzy set *C*.

   If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$ for all fuzzy sets *A*, *B*, and *C* (8)

   This property states that if fuzzy set *A* is a subset of fuzzy set *B* and fuzzy set *B* is a subset of fuzzy set *C*, then fuzzy set *A* is a subset of fuzzy set *C*. This is because the membership function of fuzzy set *A* is always less than or equal to the membership function of fuzzy set *B* for all elements in the universal set *X*, and the membership function of fuzzy set *B* is always less than or equal to the membership function of fuzzy set *C* for all elements in the universal set *X*. Therefore, the membership function of fuzzy set *A* is always less than or equal to the membership function of fuzzy set *C* for all elements in the universal set *X*.

3. Antisymmetry: If fuzzy set *A* is included in fuzzy set *B* and fuzzy set *B* is included in fuzzy set *A*, then fuzzy set *A* is equal to fuzzy set *B*.

   If $A \subseteq B$ and $B \subseteq A$, then $A = B$ for all fuzzy sets *A* and *B* (9)

   This property states that if fuzzy set *A* is a subset of fuzzy set *B* and fuzzy set *B* is a subset of fuzzy set *A*, then fuzzy set *A* is equal to fuzzy set *B*. This is because the membership function of fuzzy set *A* is always less than or equal to the membership function of fuzzy set *B* for all elements in the universal set *X*, and the membership function of fuzzy set *B* is always less than or equal to the membership function of fuzzy set *A* for all elements in the universal set *X*. Therefore, the membership functions of fuzzy sets *A* and *B* must be equal for all elements in the universal set *X*.

4. Inclusiveness: If fuzzy set *A* is included in fuzzy set *B*, then fuzzy set *A* is a subset of fuzzy set *B*.

   If $A \subseteq B$, then $A \subseteq B$ for all fuzzy sets *A* and *B* (10)

   This property states that if fuzzy set *A* is a subset of fuzzy set *B*, then fuzzy set *A* is strictly included in fuzzy set *B*. This is because the membership function of fuzzy set *A* is always less than or equal to the membership function of fuzzy set *B* for all elements in the universal set *X*, and there must be at least one element in the universal set *X* for which the membership function of fuzzy set *A* is strictly less than the membership function of fuzzy set *B*.

5. Completeness: If fuzzy set *A* is equal to fuzzy set *B*, then the inclusion of fuzzy set *A* in fuzzy set *B* is complete.

   If $A = B$, then $A \subseteq B$ for all fuzzy sets *A* and *B* (11)

   This property states that if fuzzy set *A* is equal to fuzzy set *B*, then the inclusion of fuzzy set *A* in fuzzy set *B* is complete. This is because the membership functions of fuzzy sets *A* and *B* are equal for all elements in the universal set *X*, and therefore the membership function of fuzzy set *A* is always less than or equal to the membership function of fuzzy set *B* for all elements in the universal set *X*.

These extended operations broaden the scope for conducting more intricate operations and transformations on fuzzy sets, rendering them well-suited for modeling and managing uncertainty across a range of domains. These domains encompass not only fuzzy logic, fuzzy control, and fuzzy decision-making but also extend to the realm of intuitionistic fuzzy sets and their fuzzy MCDM applications in engineering and management.

2.2 Linguistic Variable Values

The relationships between criteria and alternatives are determined based on the linguistic variable values in Table 1. These linguistic variable values are represented by fuzzy numbers, which range from 0.0 (Extremely Unimportant/Extremely Low) to 1.0 (Extremely Important/Extremely High). The fuzzy numbers are used to represent the subjective judgments of the decision-makers about the relative importance of each criterion.

Table 1 - Correspondence of linguistic variable values

Linguistic variable value (L_i)	Fuzzy number $\mu_A(x)$

Extremely Important (EI) / Extremely High (EH)	1.0
Very Important (VI) / Very High (VH)	0.9
Intermediate	0,8
Important (I) / High (H)	0.7
Intermediate	0.6
Fair (F) / Medium (M)	0.5
Intermediate	0,4
Unimportant (U) / Low (L)	0.3
Intermediate	0.2
Very Unimportant (VU) / Very Low (VL)	0.1
Extremely Unimportant (EU) / Extremely Low (EL)	0.0

(Author's own work based on fuzzy sets [29])

The evaluation values in the initial decision matrix are based on the linguistic variable values scale in Table 1. For example, if a criterion is considered to be "Very Important" for a particular alternative, then the evaluation value for that criterion and alternative would be 0.9.

2.3 Combinative Multiple Criteria Decision Making Analysis

This section introduces a novel approach called combinative multiple criteria decision making analysis (CMCDM) to address the complexities of multidimensional problems. The CMCDM method enhances decision-making by combining the Euclidean distance and rectilinear distance into a single ranking metric. This combination, optimized through an objective criterion, provides decision-makers with unique advantages, such as evaluating confidence intervals for the relative importance of alternatives and reducing the estimated variance of ranking results. The CMCDM method offers a more comprehensive and reliable approach to ranking alternatives compared to traditional methods. The sequential steps specific to the proposed CMCDM methodology are as follows:

Step 1. Building the decision matrix.

$$X = [x_{ij}]_{mxn} \tag{12}$$

where m indicates the alternatives and n indicates the criteria. x_{ij} indicates the performance index value of the alternative i according to the criterion j.

Step 2. Normalizing the decision matrix.

$$r_{ij} = \begin{cases} \dfrac{x_{ij}}{\max\limits_i x_{ij}} if\ j \in \Omega_b \\ \dfrac{\min\limits_i x_{ij}}{x_{ij}} if\ j \in \Omega_c \end{cases} \tag{13}$$

where r_{ij} indicates the normalized value of the alternative i according to the criterion j. Also Ω_b and Ω_c represent the sets of benefit and cost criteria, respectively.

Step 3. A $m \times n$ matrix $(v_{ij})_{mxn}$ is built with the index elements of the weighted normalized decision matrix:

$$v_{ij} = \omega_j r_{ij} \tag{14}$$

where $\omega_j (0 < \omega_j < 1)$ denotes the weight of jth criterion, and $\sum_{j=1}^{n} \omega_j = 1$.

Step 4. Let $v_j = \min_{i=1 \ldots m} v_{ij}$, the vector $v = (v_j)_{j=1 \ldots n}$ is the negative ideal solution (NIS).

Step 5. Distances between each alternative i and a negative ideal solution (NIS) are calculated.

- Euclidean Distance. The Euclidean distance between an alternative i and NIS is as follows:

$$E_i = \sqrt{\sum_{j=1}^{n}\left(v_j - v_{ij}\right)^2} \tag{15}$$

- Rectilinear distance. The distance between two points is the sum of the absolute values of the difference of their respective Cartesian coordinates. The rectilinear distance between an alternative *i* and the NIS is the follows:

$$R_i = \sum_{j=1}^{n} | v_j - v_{ij} | \tag{16}$$

Final scores of alternative *i* for MCDM method are between 0 and 1. Alternatives are, then, ranked by descending order of final scores.

Step 6. The MCDM method effectively merges the significance of Euclidean and rectilinear distances by introducing a parameter, denoted as λ, which falls within the range of 0 to 1. This parameter plays a pivotal role in determining the overall relative distance importance by harmonizing the weighted relative importance of both Euclidean and Rectilinear distances. The value of λ serves as the governing factor for accentuating either the Euclidean or Rectilinear relative distance importance, with a setting of 1 placing a strong emphasis on the Euclidean distance and a setting of 0 emphasizing the rectilinear distance.

$$S_i = \lambda E_i + (1-\lambda)R_i, i = 1,2,...,n \tag{17}$$

where composite distance S_i metric combines Euclidean distance E_i and rectilinear distance R_i using a parameter λ. This parameter λ controls the balance between the two distance metrics in the composite distance, where a value of 0 would give full weight to the rectilinear distance, and a value of 1 would give full weight to the Euclidean distance, and values between 0 and 1 represent a weighted combination of the two. This composite distance metric artfully amalgamates the Euclidean and Rectilinear distances, ultimately yielding heightened levels of accuracy and precision in various applications.

Final scores of alternative *i* for MCDM method are between 0 and 1. Alternatives are, then, ranked by descending order of final scores.

Step 7. Weighted Sum Model (WSM). Operation refers to an additive aggregation in where criteria values are linearly combined using weights. Emphasis is on criteria values that are scaled and combined additively.

$$\Phi_i = \sum_{j=1}^{n} \omega_j r_{ij} \tag{18}$$

Final scores of alternative *i* for WSM method are between 0 and 1. Alternatives are, then, ranked by descending order of final scores.

Step 8. Weighted Product Model (WPM). Operation refers to a multiplicative aggregation, where criteria values are raised to the power of their weights and then multiplied. Emphasis is on criteria values that are multiplied together, giving greater influence on the highest values.

$$\Psi_i = \prod_{j=1}^{n} r_{ij}^{\omega_j} \tag{19}$$

Final scores of alternative *i* for WSM method are between 0 and 1. Alternatives are, then, ranked by descending order of final scores.

Step 9. The MCDM method is a unique combination of weighted sum model (WSM) and weighted product model (WPM).

$$\Theta_i = \lambda \Phi_i + (1-\lambda)\Psi_i, i = 1,2,...,n \tag{20}$$

where composite analysis Θ_i combines the WSM (Φ_i) method and the WPM (Ψ_i) method using a parameter λ. This parameter λ controls the balance between the two multidimensional models in the composite analysis, where a value of 0 would give full weight to the WPM method, and a value of 1 would give full weight to the WSM method, and values between 0 and 1 represent a weighted combination of the two. MCDM method is transformed into WPM when the value of λ is 0, and WSM method when it is 1. This composite analysis combines WSM and WPM methods, providing a high level of accuracy and precision in a variety of applications.

2.4 Entropy Weight Method

The entropy weight method is recommended as an approach within classical multiple criteria decision making (MCDM) methods to determine the objective weights assigned to criteria. Entropy is a fundamental concept in information theory, serving as a quantification of uncertainty. In the realm of decision-making, the Entropy method is a valuable technique employed to determine the weight coefficients of various criteria. It enables the calculation of criteria weights based on available data, effectively removing the need for subjective personal judgments and biases from decision-makers. Consequently, this approach fosters objectivity in the decision-making process. The Entropy method typically consists of five essential steps [32]:

Step 1. Building the decision matrix.

$$X = [x_{ij}]_{mxn} \tag{21}$$

where *m* indicates the alternatives and *n* indicates the criteria. x_{ij} indicates the performance index value of the alternative *i* according to the criterion *j*.

Step 2. Normalizing the decision matrix.

$$p_{ij} = \frac{x_{ij}}{\sum_{i=i}^{m} x_{ij}} \tag{22}$$

where p_{ij} indicates the normalized value of the alternative *i* according to the criterion *j*.

Step 3. Computing the entropy measure.

$$E_j = -\frac{1}{\ln(m)} \sum_{j=i}^{n} p_{ij} \ln p_{ij} \tag{23}$$

where E_j indicates the entropy value of the criterion *j*.

Step 4. Defining the objective weight.

$$\omega_j = \frac{1 - E_j}{\sum_{j=i}^{n} (1 - E_j)} \tag{24}$$

where $Div_j = 1 - E_j$ indicates the divergence of entropy value of the criterion *j*. ω_j indicates the objective criteria weight value of the criterion *j*.

3. Application

This section explores the application of Fuzzy Combinative Multiple Criteria Decision Making Analysis (FC-MCDM) as a Decision Support Model for the Commercial Aircraft Selection problem. The results and implications of the FC-MCDM model are presented and discussed, demonstrating its applicability and effectiveness in the context of Multiple Criteria Decision Making (MCDM) [33-46].

3.1 Multiple Criteria Decision-Making (MCDM) Analysis

In the realm of decision-making theory, a multiple criteria decision-making (MCDM) analysis problem is characterized by a defined set of alternative options $A_i = \{A_1, ..., A_i\}$ $(i > 2)$ for which the optimal choice must be determined. This decision-making process is guided by a predefined set of evaluation criteria $C_j = \{C_1, ..., C_j\}$ $(j > 1)$, each with its own associated score. The score assigned to each alternative is based on its performance in accordance with a specific criterion. Each of these criteria is assigned a normalized weight $\sum_{j=1}^{J}\omega_j = 1$, denoted as $\omega_j \in [0,1]$, with values ranging between 0 and 1, signifying their respective importance [11].

In an MCDM problem, all criteria C_j and all available alternatives A_i, along with their corresponding quantitative score values X_{ij}, are taken into consideration. The weighting factor (importance) assigned to each criterion C_j, often referred to as ω_j, is an essential component of the decision-making process. These normalized weighting factors collectively constitute a set, denoted as $\omega_j = \{\omega_1,...,\omega_j\}$ [11].

Depending on the specific context of the MCDM problem at hand, the assigned scores can be interpreted either as costs to minimize or as benefits to maximize. The matrix that captures these scores and is often termed the "score matrix" can alternatively be referred to as the "benefit matrix" or "payoff matrix" in existing literature. In essence, the fundamental objective of a classical MCDM problem is to identify and select the most suitable alternative, given the set of alternatives (A_i), the associated criteria (C_j), and the corresponding weighting factors (ω_j) assigned to these criteria.

3.2 Commercial Aircraft Selection Criteria

The multiple criteria decision-making (MCDM) methods can be effectively applied to determine the most suitable commercial aircraft selection. Therefore, the suggested FC-MCDM approach is used to evaluate and select the best commercial aircraft among several alternatives. To evaluate the commercial aircraft alternatives, a set of criteria is determined based on a comprehensive literature review and expert opinions. In the realm of civil aviation applications for commercial aircraft, various performance criteria are considered:

Payload capacity (C1): Refers to the weight of equipment, excluding avionics, fuel, and necessary systems for ensuring a safe takeoff, flight, and landing. The payload varies depending on the flight requirements.

Maximum speed (C2): The velocity of a commercial aircraft depends on its engine power. Different operations may demand high, low, or average speeds.

Maximum endurance (C3): Indicates the longest duration a commercial aircraft can operate safely in the air, considering the fuel capacity from the moment it takes off until landing.

Maximum altitude (C4): The "service ceiling" defines the maximum altitude that commercial airplanes are permitted to reach during flight, universally capped at 42,000 feet. This altitude limit is adhered to by most commercial aircraft due to its recognized role in optimizing operational efficiency.

Maximum range (C5): The farthest distance a commercial aircraft can be controlled by a pilot after taking off from its airport, considering the fuel capacity and payload, while ensuring a safe return.

Price (C6): The cost associated with acquiring a commercial aircraft, including ground equipment, contributes to its overall evaluation.

These defined criteria are essential in the meticulous evaluation and selection of commercial aircraft for various aviation applications. By considering these performance criteria, a comprehensive assessment can be conducted to evaluate commercial aircraft for their intended purposes in a rigorous manner. The initial decision matrix distinguishes between the optimization type (benefit or cost) for each attribute. The decision criteria used to evaluate alternative commercial aircraft options are categorized into two types: benefit criteria (C1-C5) and cost criteria (C6).

3.3 Commercial Aircraft Selection Problem

In this section, the applicability of decision-making approach FC-MCDM method has been demonstrated through a case involving an airline company operating in the aviation sector and its commercial aircraft selection problem. The objective of this company is to procure a commercial aircraft that aligns with the company's goals. In this regard, the responsibility for researching and analyzing commercial aircraft procurement falls on evaluation committee within the company. By following the steps within the fuzzy MCDM framework, a systematic approach is applied to effectively evaluate and compare the potential commercial aircraft options based on the identified decision criteria.

Step 1. The decision matrix is established.

The initial decision matrix $X = [x_{ij}]_{mxn}$ for the alternatives (A_i), the decision criteria (C_j), and the criteria weights (ω_j) is constructed. This matrix also specifies the type of optimization (benefit or cost) of each criterion.

Step 2. The decision matrix is normalized.

$$r_{ij} = \begin{cases} \{\mu_A(x), & \Omega_b \\ 1-\mu_A(x), & \Omega_c \end{cases} \quad (25)$$

where Ω_b denotes benefit type criteria, and Ω_c denotes cost type criteria,

Step 3. The criteria weights are calculated. The importance weights ω_j of decision criteria C_j are assessed by the decision makers.

Step 4. Weighted normalized matrix is computed.

$$v_{ij} = \omega_j r_{ij} \quad (26)$$

Step 5. Negative ideal solution (NIS) is computed.

$$v_j = \min_{i=1...m} v_{ij} \quad (27)$$

Step 6. Euclidean distance (E_i) between an alternative *i* and NIS (v_j) is computed.

$$E_i = \sqrt{\sum_{j=1}^{n}\left(v_j - v_{ij}\right)^2} \quad (28)$$

Step 7. Rectilinear distance (R_i) between an alternative *i* and NIS (v_j) is computed.

$$R_i = \sum_{j=1}^{n} | v_j - v_{ij} | \quad (29)$$

Step 8. FC-MCDM evaluation is conducted.

$$S_i = \lambda E_i + (1-\lambda) R_i, i = 1,2,...,n \quad (30)$$

Step 9. Weighted sum model (WSM) is conducted.

$$\Phi_i = \sum_{j=1}^{n} \omega_j r_{ij} \quad (31)$$

Step 10. Weighted Product Model (WPM)

$$\Psi_i = \prod_{j=1}^{n} r_{ij}^{\omega_j} \quad (32)$$

Step 11. FC-MCDM evaluation is conducted.

$$\Theta_i = \lambda \Phi_i + (1-\lambda)\Psi_i, i = 1,2,...,n \quad (33)$$

Step 12. The alternatives are ranked according to their score function (E_i , R_i , S_i , Φ_i , Ψ_i , Θ_i) values in decreasing order. The bigger score function value of alternatives A_i corresponds to the best MCDM solution A^*, that is $A^* = A_i$.

Step 13. The Spearman's Rank Correlation Coefficient (*ρ*) between the two ranks of each observation is calculated as follows:

$$\sigma = 1 - \frac{6\sum d_i^2}{n(n^2-1)} \quad (34)$$

where σ is Spearman's rank correlation coefficient, d_i is difference between the two ranks of each observation, and *n* is number of observations.

3.4 Determining the Weights of Criteria

In the fuzzy Multiple Criteria Decision-Making (MCDM) approach, the determination of criteria weights is accomplished using the fuzzy weight method, which replaces the entropy method.

Let *n* be the number of decision makers. *m* be the number of criteria. X_{ij} be the evaluation of decision maker *i* for criterion *j*, where $i = 1, 2, ..., m$, and $j = 1, 2, ..., n$. C_j be the aggregated value for criterion *j*. ω_j be the normalized weight for criterion *j*. The algorithm consists of the following steps:

Step 1. Convert linguistic variable values (L_i) to fuzzy numbers ($\mu_A(x)$) based on Table 1.

Step 2. Calculate the aggregated value C_j for each criterion using the geometric mean:

$$C_j = \left(\prod_{i=1}^{n} X_{ij}\right)^{\frac{1}{n}} \tag{35}$$

Step 3. Normalize the aggregated values C_j to obtain the weights of criteria ω_j using the formula:

$$\omega_j = \frac{C_j}{\sum_{j=i}^{n} C_j} \tag{36}$$

where $\sum_{j=1}^{n} \omega_j = 1$.

The weights of criteria calculated using the linguistic variable values (L_i) given in Table 1 and the fuzzy numbers ($\mu_A(x)$) are provided in Table 2 for three decision makers (DMs).

Table 2 – Weights of Criteria

	C1	C2	C3	C4	C5	C6	
DM1	0.9	0.3	0.7	0.5	0.7	0.5	
DM2	0.7	0.7	0.3	0.7	0.9	0.3	
DM3	0.5	0.5	0.7	0.7	0.9	0.5	
							Sum
C_j	0.680	0.472	0.528	0.626	0.828	0.422	3.555
ω_j	0.191	0.133	0.148	0.176	0.233	0.119	1

The evaluation grades of the three decision makers for the aircraft alternatives are given in Table3.

Table 3 - Decision makers' evaluation grades of aircraft alternatives

	C1	C2	C3	C4	C5	C6
A1	0.9	0.3	0.7	0.3	0.1	0.5
A2	0.7	0.5	0.3	0.7	0.9	0.3
A3	0.3	0.5	0.7	0.7	0.9	0.1
A4	0.9	0.3	0.5	0.3	0.7	0.5
A5	0.1	0.7	0.5	0.7	0.3	0.7
A6	0.9	0.7	0.3	0.9	0.1	0.5
A7	0.3	0.5	0.5	0.3	0.7	0.3
A8	0.7	0.3	0.7	0.5	0.5	0.1
A9	0.3	0.7	0.7	0.3	0.7	0.5
A10	0.1	0.9	0.9	0.7	0.3	0.7
A11	0.5	0.3	0.1	0.5	0.7	0.9
A12	0.9	0.1	0.7	0.5	0.3	0.7
A13	0.3	0.9	0.7	0.3	0.1	0.3
A14	0.1	0.3	0.7	0.9	0.3	0.9
A15	0.5	0.3	0.5	0.1	0.7	0.1

A16	0.3	0.1	0.3	0.7	0,9	0.5
A17	0.9	0.3	0.1	0.5	0,3	0.3
A18	0.1	0.9	0.3	0.3	0.1	0.1
A19	0.7	0.7	0.7	0.5	0.3	0.5
A20	0.5	0.3	0.9	0.1	0.9	0.7

The normalized decision matrix is given in Table 4. The cost criterion C_6 was converted to the benefit criterion.

Table 4 – Normalized Decision Matrix

	C1	C2	C3	C4	C5	C6
A1	0.9	0.3	0.7	0.3	0.1	0.5
A2	0.7	0.5	0.3	0.7	0.9	0.7
A3	0.3	0.5	0.7	0.7	0.9	0.9
A4	0.9	0.3	0.5	0.3	0.7	0.5
A5	0.1	0.7	0.5	0.7	0.3	0.3
A6	0.9	0.7	0.3	0.9	0.1	0.5
A7	0.3	0.5	0.5	0.3	0.7	0.7
A8	0.7	0.3	0.7	0.5	0.5	0.9
A9	0.3	0.7	0.7	0.3	0.7	0.5
A10	0.1	0.9	0.9	0.7	0.3	0.3
A11	0.5	0.3	0.1	0.5	0.7	0.1
A12	0.9	0.1	0.7	0.5	0.3	0.3
A13	0.3	0.9	0.7	0.3	0.1	0.7
A14	0.1	0.3	0.7	0.9	0.3	0.1
A15	0.5	0.3	0.5	0.1	0.7	0.9
A16	0.3	0.1	0.3	0.7	0.9	0.5
A17	0.9	0.3	0.1	0.5	0.3	0.7
A18	0.1	0.9	0.3	0.3	0.1	0.9
A19	0.7	0.7	0.7	0.5	0.3	0.5
A20	0.5	0.3	0.9	0.1	0.9	0.3

The weighted normalized decision matrix is calculated as given in Table 5.

Table 5 – Weighted Normalized Decision Matrix

	C1	C2	C3	C4	C5	C6
A1	0.172	0.040	0.104	0.053	0.023	0.060
A2	0.134	0.067	0.044	0.123	0.210	0.083
A3	0.057	0.067	0.104	0.123	0.210	0.107
A4	0.172	0.040	0.074	0.053	0.163	0.060
A5	0.019	0.093	0.074	0.123	0.070	0.036
A6	0.172	0.093	0.044	0.158	0.023	0.060
A7	0.057	0.067	0.074	0.053	0.163	0.083
A8	0.134	0.040	0.104	0.088	0.117	0.107
A9	0.057	0.093	0.104	0.053	0.163	0.060
A10	0.019	0.120	0.133	0.123	0.070	0.036
A11	0.096	0.040	0.015	0.088	0.163	0.012
A12	0.172	0.013	0.104	0.088	0.070	0.036
A13	0.057	0.120	0.104	0.053	0.023	0.083
A14	0.019	0.040	0.104	0.158	0.070	0.012
A15	0.096	0.040	0.074	0.018	0.163	0.107

A16	0.057	0.013	0.044	0.123	0.210	0.060
A17	0.172	0.040	0.015	0.088	0.070	0.083
A18	0.019	0.120	0.044	0.053	0.023	0.107
A19	0.134	0.093	0.104	0.088	0.070	0.060
A20	0.096	0.040	0.133	0.018	0.210	0.036

The negative ideal solution (NIS) vector $v_j = \min_{i=1\ldots m} v_{ij}$ is determined as given in Table 6.

Table 6 – The negative ideal solution (NIS) vector

	C1	**C2**	**C3**	**C4**	**C5**	**C6**
NIS ($v_j = \min_{i=1\ldots m} v_{ij}$ **)**	0.019	0.013	0.015	0.018	0.023	0.012

Table 7 - The Euclidean distance (E_i **) between an alternative *i* and NIS (** v_j **)**

	E_i	**Rank (** E_i **)**	R_i	**Rank (** R_i **)**
A1	0.188	14	0.351	15
A2	0.260	1	0.561	2
A3	0.259	2	0.567	1
A4	0.225	5	0.461	4
A5	0.154	19	0.315	17
A6	0.230	4	0.451	5
A7	0.184	15	0.397	12
A8	0.211	7	0.489	3
A9	0.197	10	0.429	8
A10	0.198	8	0.401	10
A11	0.176	16	0.313	18
A12	0.197	9	0.382	13
A13	0.164	18	0.340	16
A14	0.175	17	0.303	19
A15	0.197	11	0.397	11
A16	0.225	6	0.407	9
A17	0.190	13	0.368	14
A18	0.150	20	0.266	20
A19	0.192	12	0.448	6
A20	0.236	3	0.432	7

The Euclidean distance (E_i) and Rectilinear distance (R_i) between an alternative *i* and the negative ideal solution (NIS) (v_j) are computed and the ranking orders of alternatives are presented in Table 7.

The Spearman's Rank Correlation Coefficient (*ρ*) between the Euclidean distance **(** E_i **)** and Rectilinear distance **(** R_i) ranking orders is approximately 0.736181.

The weighted sum model (Φ_i) and weighted product model (Ψ_i) computations for each alternative *i* and the corresponding criterion weight (ω_j) are computed, and the resulting ranking orders of alternatives are presented in Table 8.

The Spearman's Rank Correlation Coefficient (*ρ*) between the weighted sum model (Φ_i) and weighted product model (Ψ_i) ranking orders is approximately 0.9653266.

Table 8 - The weighted sum model (Φ_i **) , weighted product model (** Ψ_i **) , and the resulting ranking orders of alternatives**

	Φ_i	**Rank (** Φ_i **)**	Ψ_i	**Rank (** Ψ_i **)**
A1	0.451	15	0.345	15
A2	0.661	2	0.626	1
A3	0.667	1	0.622	2

A4	0.561	4	0.517	5
A5	0.415	17	0.341	16
A6	0.551	5	0.413	9
A7	0.497	12	0.467	7
A8	0.589	3	0.562	3
A9	0.529	8	0.493	6
A10	0.501	10	0.384	13
A11	0.413	18	0.329	18
A12	0.482	13	0.397	12
A13	0.440	16	0.337	17
A14	0.403	19	0.294	19
A15	0.497	11	0.408	11
A16	0.507	9	0.413	10
A17	0.468	14	0.381	14
A18	0.366	20	0.248	20
A19	0.548	6	0.520	4
A20	0.532	7	0.414	8

The correlation analysis matrix in Table 9 shows the correlation coefficients between different methods and metrics used for ranking alternatives:

Table 9 - The correlation analysis matrix between different methods and metrics used for ranking alternatives

	WSM	WPM	EUC	REC
WSM	1			
WPM	0.95	1		
EUC	0.90	0.79	1	
REC	1	0.95	0.90	1

Correlation between WSM and WPM: The correlation coefficient is 0.95, which indicates a very strong positive correlation between the Weighted Sum Model (WSM) and the Weighted Product Model (WPM). This suggests that these two methods produce highly similar rankings for the alternatives.

Correlation between WSM and EUC: The correlation coefficient is 0.90, indicating a strong positive correlation between the Weighted Sum Model (WSM) and the ranking based on Euclidean distance (EUC). This suggests that there is a significant degree of similarity in the rankings produced by these two methods.

Correlation between WSM and REC: The correlation coefficient is 1.0, indicating a perfect positive correlation between the Weighted Sum Model (WSM) and the ranking based on rectilinear distance (REC). This means that the rankings are identical for these two methods.

Correlation between WPM and EUC: The correlation coefficient is 0.79, indicating a strong positive correlation between the Weighted Product Model (WPM) and the ranking based on Euclidean distance (EUC). While the correlation is strong, it is slightly lower than the correlation between WSM and EUC.

Correlation between WPM and REC: The correlation coefficient is 0.95, indicating a very strong positive correlation between the Weighted Product Model (WPM) and the ranking based on rectilinear distance (REC). This suggests that these two methods produce highly similar rankings.

Correlation between EUC and REC: The correlation coefficient is 0.90, indicating a strong positive correlation between the ranking based on Euclidean distance (EUC) and the ranking based on rectilinear distance (REC). This suggests that there is a significant degree of similarity in the rankings produced by these distance metrics.

Overall Analysis: The correlation matrix shows that the rankings produced by WSM and WPM are highly similar, with a very strong positive correlation. Both WSM and WPM have strong positive correlations with the rankings based on EUC and REC, indicating consistency in the rankings among these methods and metrics. The perfect correlation between WSM and REC suggests that they produce identical rankings. This analysis indicates that the chosen methods and metrics are consistent in their ranking of alternatives, which can provide confidence in the decision-making process.

The FC-MCDM approach is conducted for both the Euclidean distance (E_i) and Rectilinear distance (R_i). The results of these analyses are presented in Table 10 and Fig. 1, respectively.

Table 10 - The FC-MCDM for both the Euclidean distance (E_i) and Rectilinear distance (R_i) and the resulting ranking orders of alternatives

λ_i	0	0,1	0,3	0,5	0,7	0,9	1
A1	15	15	15	15	15	15	14
A2	2	2	2	2	2	1	1
A3	1	1	1	1	1	2	2
A4	4	4	4	4	4	5	5
A5	17	18	18	19	19	19	19
A6	5	5	5	5	3	4	4
A7	12	12	12	12	13	14	15
A8	3	3	3	3	6	7	7
A9	8	8	8	9	9	8	10
A10	10	10	10	10	10	9	8
A11	18	17	17	17	16	16	16
A12	13	13	13	13	12	12	9
A13	16	16	16	16	17	18	18
A14	19	19	19	18	18	17	17
A15	11	11	11	11	11	11	11
A16	9	9	9	8	7	6	6
A17	14	14	14	14	14	13	13
A18	20	20	20	20	20	20	20
A19	6	6	7	7	8	10	12
A20	7	7	6	6	5	3	3

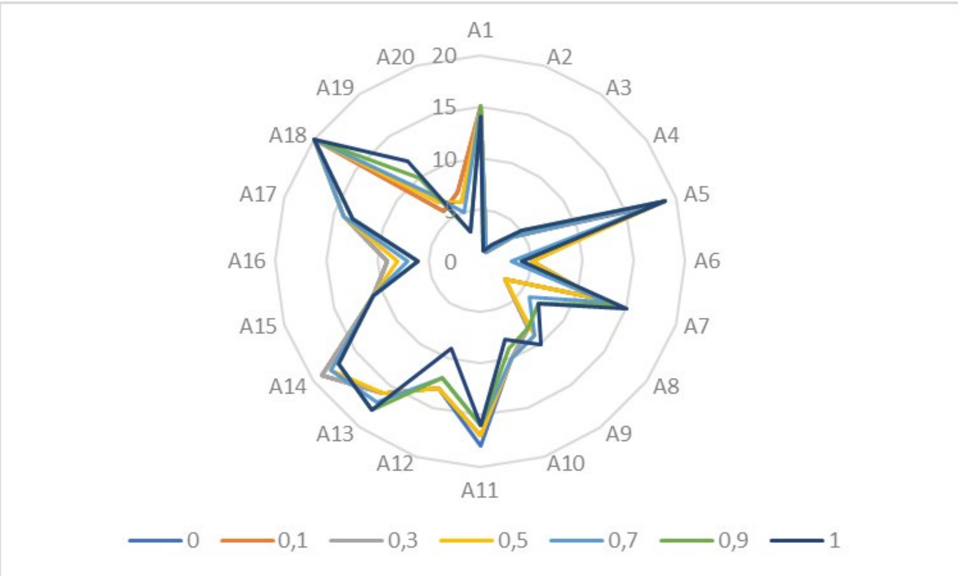

Fig. 1 - The FC-MCDM for both the Euclidean distance (E_i) and Rectilinear distance (R_i) and the resulting ranking orders of alternatives

In Table 10, there are two sets of ranking orders based on different criteria: one set based on λ_i values and the other set based on λ_i values in the context of the FC-MCDM technique. Let's compare and analyze the ranking orders based on λ_i Values in FC-MCDM approach (Euclidean and Rectilinear Distances):

In this set of rankings, the alternatives are ranked based on the Euclidean distance (E_i) and Rectilinear distance (R_i) values, considering different λ_i values. The rankings of alternatives are influenced by the λ_i values and the specific aggregation FC-MCDM method used.

In Table 10, we have the ranking orders of alternatives based on the FC-MCDM for different values of the parameter λ_i, ranging from 0 to 1. The FC-MCDM method combines the Euclidean distance (E_i) and Rectilinear distance (R_i) calculations for each alternative and criterion, varying the parameter λ_i to control the weighting between (E_i) and (R_i). Here are some observations and analysis based on the ranking orders in Table 10:

Sensitivity to Parameter λ_i: As λ_i changes from 0 to 1, the ranking orders of alternatives can vary significantly. This demonstrates the sensitivity of the FC-MCDM method to the choice of λ_i. Different values of λ_i place different emphasis on either (E_i) or (R_i), affecting the final rankings.

Trade-Off Between (E_i) and (R_i): When λ_i is close to 0 (e.g., λ_i = 0.1), the rankings tend to be more influenced by (R_i), resulting in some alternatives moving up or down in the rankings compared to when λ_i is closer to 1 (e.g., λ_i = 0.9), where (E_i) has more influence.

Consistency Across λ_i Values: Despite the variations in rankings for different λ_i values, some alternatives consistently perform well or poorly across a range of λ_i values.

Ranking Stability: Alternatives that have relatively stable rankings across different λ_i values may be considered more robust or less sensitive to changes in the weighting between (E_i) and (R_i).

Decision Sensitivity: The choice of the best alternative can vary depending on the λ_i value used. Decision-makers should consider the implications of these variations when selecting the most suitable alternative.

Contextual Considerations: The choice of λ_i should align with the decision context and the relative importance of (E_i) and (R_i) in the decision-making process. A lower λ_i may be chosen when emphasizing RD, while a higher λ_i may be selected to prioritize (E_i).

In summary, the FC-MCDM method provides a flexible approach for decision-makers to explore different trade-offs between Euclidean distance (E_i) and Rectilinear distance (R_i) in multiple-criteria decision-making. The choice of λ_i should align with the specific objectives and preferences of the decision problem at hand.

The FC-MCDM approach is conducted for both the Weighted Sum Model (Φ_i) and Weighted Product Model (Ψ_i). The results of these analyses are presented in Table 11 and Fig. 2, respectively.

Table 11 - The FC-MCDM for both the Weighted Sum Model (Φ_i) and Weighted Product Model (Ψ_i) and the resulting ranking orders of alternatives

λ_i	0	0,1	0,3	0,5	0,7	0,9	1
A1	15	15	15	15	15	15	15
A2	1	1	1	2	2	2	2
A3	2	2	2	1	1	1	1
A4	5	5	4	4	4	4	4
A5	16	16	17	17	17	17	17
A6	9	8	8	5	7	6	5
A7	7	7	7	12	9	10	12
A8	3	3	3	3	3	3	3
A9	6	6	6	8	6	7	8
A10	13	13	13	10	12	11	10
A11	18	18	18	18	18	18	18
A12	12	12	12	13	13	13	13
A13	17	17	16	16	16	16	16
A14	19	19	19	19	19	19	19
A15	11	11	11	11	11	12	11
A16	10	10	10	9	10	9	9
A17	14	14	14	14	14	14	14

A18	20	20	20	20	20	20	20
A19	4	4	5	6	5	5	6
A20	8	9	9	7	8	8	7

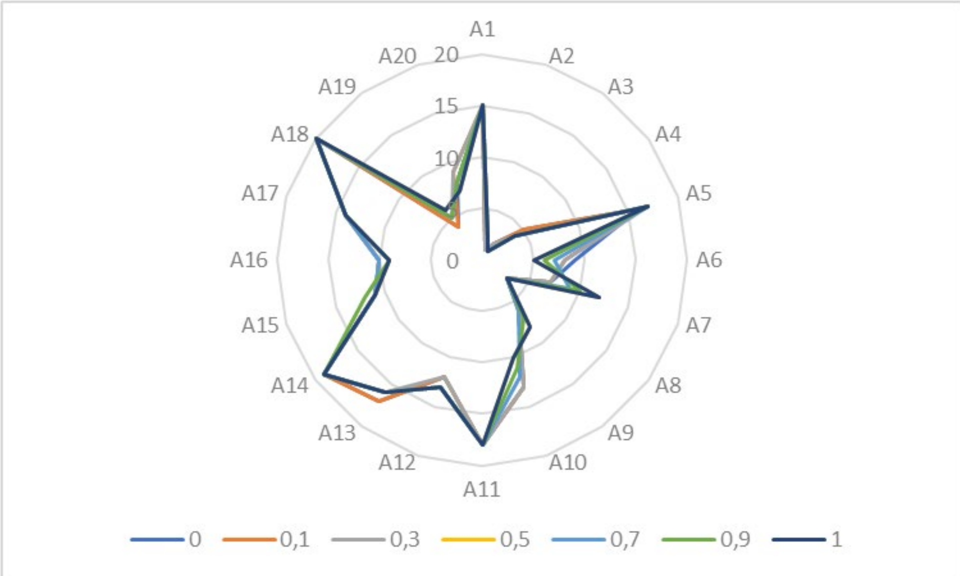

Fig. 2 - The FC-MCDM for both the Weighted Sum Model (Φ_i) and Weighted Product Model (Ψ_i) and the resulting ranking orders of alternatives

In Table 11, there are two sets of ranking orders based on different criteria: one set based on λ_i values and the other set based on λ_i values in the context of the FC-MCDM. In this set of rankings, we have the FC-MCDM ranking orders of alternatives based on the Weighted Sum Model (WSM) and Weighted Product Model (WPM) for different values of the parameter λ_i, ranging from 0 to 1. Here are the observations and analysis based on the ranking orders in Table 11:

Consistency Across λ_i Values: Unlike Table 10, where the rankings were more sensitive to changes in λ_i, Table 11 shows a higher degree of consistency. For most alternatives, the rankings remain the same across different λ_i values.

Dominance of WPM Rankings: The WPM rankings (λ_i = 1) tend to dominate the rankings across all alternatives. This suggests that the WPM places a strong emphasis on certain criteria, leading to consistent rankings.

Limited Sensitivity to λ_i: While there is some variation in rankings for λ_i values between 0 and 0.7, the differences are relatively small, indicating that the choice of λ_i has less impact on the rankings in this case.

Similar Top Performers: Alternatives such as A2, and A3 consistently rank at the top, regardless of the λ_i value. Similarly, A14 and A18 consistently rank at the bottom.

Influence of Models: The choice between WSM and WPM has a significant impact on the rankings. WPM tends to reward alternatives that perform well in all criteria, while WSM is more flexible in accommodating different performance profiles.

Decision Stability: The relatively stable rankings may provide decision-makers with more confidence in the selected alternatives, as they are less sensitive to changes in the parameter λ_i.

In summary, Table 11 demonstrates that the FC-MCDM method applied to the WSM and WPM models results in more consistent rankings across different λ_i values compared to Table 10. The choice between WSM and WPM should align with the decision context and the desired emphasis on criteria. Overall, Table 11 provides a more stable basis for decision-making when using the FC-MCDM method. The choice of ranking method and the specific parameter values λ_i can significantly impact the ranking orders of alternatives. Some alternatives show consistent rankings across different parameter values λ_i, suggesting their stability in the decision process. Other alternatives exhibit sensitivity to parameter changes, resulting in varying rankings. It's important to consider the context and decision criteria when choosing the most appropriate ranking method and parameter values, as they can lead to different recommendations for alternative selection.

4. Conclusion

In fuzzy decision analysis process, numerical investigations in the realm of Fuzzy Multiple Criteria Decision Making (MCDM) involved the utilization of the FC-MCDM method for commercial aircraft selection purposes. The weights assigned to criteria were determined through the introduction of an innovative fuzzy weight approach. The outcomes obtained from the analysis of Euclidean and Rectilinear distances were compared with the results of the Weighted Sum Model (WSM) and Weighted Product Model (WPM), revealing a significant agreement between rectilinear distance (REC) analysis and the WSM. Furthermore, the correlation analysis of alternative rankings indicated that REC analysis and the WSM were practically indistinguishable in these two methods. Notably, the correlation coefficient between WSM and WPM, as well as between WPM and REC, stood at a robust 0.95, signifying a very strong positive correlation. In conclusion, the novel FC-MCDM method was effectively applied to address the selection of the optimal commercial aircraft, demonstrating the applicability and efficiency of the proposed model. In the future, it is aimed to apply the proposed FC-MCDM method in the solution of complex engineering, technological, and industrial problems and compare it with other MCMD methods.

Acknowledgement

The authors fully acknowledged National Aviation Academy for supporting this work.

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