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The Classification of the Suras (Part Two): Organization According to the Rule of Thirds and Sura Al-Furqān - Relationship with Suras Al-Fātiḥah and Fuṣṣilat

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Abstract

This study highlights the existence of a simple organizing principle based on the proportion of one-third (1/3). This principle acts as a structural law governing the numerical architecture of the Quran. It manifests itself in two independent yet formally comparable domains. In both cases, the proportion is verified through strict structural equalities, albeit of fundamentally distinct natures. In the ordinal domain, it is an equality of compensation between the sums of negative and positive ranking values. In the versetic domain, it is an equality between the sums of verses of the long and short suras as determined relative to a threshold. The application of this proportion to the longest sura (286 verses) defines an objective threshold Vs=⌊286/3⌋≈95. Partitioning the corpus according to this threshold reveals a difference of 154 verses between the totals of the two groups. The analysis shows that this difference is resolved only by the functional intervention of a single sura: Al-Furqān (No. 25). Its number of verses (77) corresponds exactly to half of the gap, and its logical transfer restores a perfect equality (3 118 verses). The subset (Fq) obtained after the application of the one-third proportion satisfies a Type 1 equation, thereby demonstrating its autosimilarity. In the ordinal domain, the same proportion (114/3 = 38) generates a hierarchical 1:2 partition whose coherence also relies on strict equalities and autosimilar properties. The coexistence of a unique proportional principle, strict equality conditions, and autosimilar structures locked by entire equations suggests a highly constrained mathematical architecture of the Noble Qur'an.

1. Introduction

Research on the Quran's numerical architecture (Jarrar, B. (2001), Khairuddin, S., et al (2017), Khalifa, R. (1973). based exclusively on its canonical parameters, has revealed a set of strict mathematical relations and equations that organize the corpus (Benzitouni, S., & Benzitouni, S. (2024) and Benzitouni, S. (2025). A first study showed that the distribution of suras according to the parity of their numbers and their verses brings forth relations of concordance and strict numerical rules, enabling the constitution of a first system of equations and the determination of the total number of suras (114) and verses (6 236) of the corpus (Benzitouni, S., & Benzitouni, S. (2024). A second work established a global coherence equation for the entire corpus Benzitouni, S. (2025):

$$49N(TG) + 2V(TG) = SV(TG)$$

The ranking relation for a given sura is defined as: $D(Sn) = SV(Sn) - \big(49\,n + 2V(Sn)\big)$, where n is the number of sura Sn (this notation corresponds to D(a) in article [2], where "a" denoted a sura). Its application to the entire set of suras revealed a fundamental ordinal partition, distinguishing 38 suras with a negative value and 76 with a positive value, whose absolute sums of D(Sn) are equal (Benzitouni, S. (2025). The present article extends this approach by exploring a new question: does the same proportional principle of one-third (1/3) operate in an independent structural domain, that of the versetic lengths? Here, the proportion no longer applies to a discrete count (the number of suras), but to a maximum value — the 286 verses of Al-Baqarah — thus defining an objective numerical threshold Vs=95. The objective of this study is to verify whether the application of this proportion to the versetic domain leads, through a different mechanism, to a form of strict structural equality. We seek to determine whether the partition induced by this threshold can satisfy an equality of the sum of verses (the sum of the verses of the "long" suras equals that of the "short" suras) and, if so, identify the functional operator that makes this equality effective.

The body of work undertaken is part of a unique and coherent scientific project: to demonstrate, through a rigorous mathematical approach, the authenticity and integrity of the Noble Quran. This project is broken down into complementary articles, each addressing a specific aspect of the demonstration:

- Article [1] demonstrates that the number of surahs (114) and the total number of verses (6,236) are mathematically locked and have undergone no alteration. It also contains all the necessary external references (standard edition, works on the history of the text, etc.).
- Article [2] is the first part of the work devoted to the classification of surahs. It demonstrates that the ordinal classification of surahs (their canonical order in the Muṣḥaf) is not arbitrary but results from a constraining mathematical organization, based on a global coherence equation and the proportion of one-third activated by Al-Fātiḥah.
- The present article (article 3) is the direct continuation of article (Benzitouni, S. (2025) in the field of surah classification. It applies the same principle of one-third to the versetic domain (lengths of surahs), identifies Al-Furqān as the operator activating the symmetry, and establishes new Type 1 equations for the group Fq and its subgroups. This is why the present article is designated as "Second Part": it constitutes the second part of the work devoted to the classification of surahs, article (Benzitouni, S. (2025) being the first part.

Thus, references (Benzitouni, S., & Benzitouni, S. (2024) and Benzitouni, S. (2025) are the only references necessary for understanding and validating the present article, for they contain the complete exposition of the method, the fundamental equations, the preliminary results, as well as all the external references (editions, works on the history of the text, etc.) already documented. The mathematical demonstrations presented rely exclusively on the canonical data of the Quran and on verifiable algebraic relations.

We will demonstrate that this perfect versetic equality is achieved only by the intervention of a unique sura, Al-Furqān (No. 25). Its number of verses (77) fills exactly half of the initial gap, acting as the operator that validates the proportion of one-third in this new domain. Thus, while study Benzitouni, S. (2025) revealed a symmetry of signs (negative/positive) governed by the third, the present study reveals a symmetry of lengths (long/short) governed by the same principle. This dual convergence reinforces the hypothesis of a unified mathematical architecture. The previous studies (Benzitouni, S., & Benzitouni, S. (2024) and Benzitouni, S. (2025) and the present research share a common objective: to determine whether the canonical numbering of the suras (1 to 114) results from a coherent mathematical organization or merely a conventional arrangement. The results obtained thus far show that this order is not arbitrary. Each sura number is constrained by a network of equations linking the structural groups brought to light. The canonical numbering thus appears, within the tested framework, as a unique solution to a system of structural constraints that remains to be fully identified. Their discovery would mathematically demonstrate that the order of the suras is structurally necessary.

2. Formal Framework

2.1 Canonical Data

This study relies exclusively on the invariable numerical parameters of the standard Quranic text (the so-called “Medinan” edition):

The so-called "Medina edition" refers to the standard edition of the Quran published by the King Fahd Complex for the Printing of the Holy Quran. This edition standardized the Hafs reading from 'Asim, the verse numbering, the ordering of surahs, as well as the use of punctuation and vocalization marks. It serves as a reference for verse numbering and the order of surahs in the majority of modern copies. [Reference: The Holy Quran: Text, Translation and Commentary, Abdullah Yusuf Ali; or the edition of the King Fahd Complex.]

- The sura numbers: n=1,2,…,114
- The number of verses of each sura: V(Sn)
- The sum of the verse numbers in a given sura: SV(Sn)
- The sum of the verse numbers in a group of suras: SV(Ti) or SV(G).

No external data, adjustable hypothesis, or modification of the canonical numbering is introduced.

2.2 Previously Established Structural Relation

The previous study [2] identified strict mathematical relations verified by these parameters. In particular:

- The fundamental Type 1 equation: $aN(G) + bV(G) = SV(G)$
  where N(G), V(G), and SV(G) denote the sums computed over the group G.
- The ranking relation: $D(Sn) = SV(Sn) - (49n + 2V(Sn))$,
  revealed an ordinal partition of the corpus into two groups of 38 and 76 suras, whose absolute sums of D(Sn) are equal [2].

This property highlighted the rule of thirds (1/3) as an organizing principle, applied to the total number of suras (114/3 = 38).

2.3 Organizing Principle and Method

The rule of thirds (1/3), identified as a structuring principle in the ordinal domain, is here examined in an independent domain: that of versetic lengths. The objective threshold is defined by:

$$Vs = \left\lfloor \frac{286}{3} \right\rfloor \approx 95$$

where 286 is the number of verses of the longest sura (Al-Baqarah).
The procedure draws inspiration from the discovery reported in the first study [1], where a structural gap (54 verses) was compensated by a unique sura (Fussilat). Similarly, the gap observed after partition according to Vs will be analyzed, and the possible existence of a sura capable of restoring perfect equality will be examined. The hypotheses are thus derived from configurations already verified within the canonical structure, and their validation relies exclusively on strict numerical equalities, without any post hoc adjustment.

3. General Notations

For any set of suras, we use two equivalent notations depending on the context:

- **G** denotes any group of suras (notation adopted in the present study)
- $\mathbf{T_i}$ denotes a table (group) of suras (notation used in study [2])

These two notations are interchangeable and refer to the same reality: a structured subset of the Quranic corpus.

The fundamental parameters are defined as follows:
Sn : the sura of order number n $(1 \leq n \leq 114)$.
$V(Sn)$: the number of verses of sura Sn .
$SV(Sn)$: the sum of verse numbers of sura Sn .
For any set G (or T_i) of suras, we define:

- $S(G)$: the number of suras in G.
- $N(G) = \sum_{Sn \in G} n$: sum of sura numbers in G.
- $V(G) = \sum_{Sn \in G} V(Sn)$: sum of the verses (or number of verses) of the suras in G.
- $SV(G) = \sum SV(Sn)$: sum of verse numbers in G.
- $SR(G) = aN(G) + bV(G)$: Linear combination of the structural parameters N(G) and V(G) , where a and b are constants characteristic of group G.

These notations are consistent with those used in previous works [1, 2], ensuring the comparability of results. The complete set of the Quran is denoted TG, whence N(TG)=6555, V(TG)=6236 and SV(TG)=333667.

4. Methodology for Analyzing the Rule of Thirds in the Versetic Domain:

Step 1: Objective definition of a structural threshold.
The proportion of one-third (1/3) is chosen in reference to the ordinal partition established in article [2], where exactly one third of the suras (38 out of 114) belongs to the set with a negative ranking value and the remaining

two thirds belong to the set with a positive ranking value. To transpose this principle to the domain of sura lengths (long and short), the reference value retained to separate them is the maximum length of a sura in the corpus, corresponding to the longest sura, i.e., Vmax=286 verses (Surah Al-Baqarah). The versetic threshold is defined by the integer part immediately below 286/3.
The longest surah (Al-Baqarah, Vmax=286) imposes itself naturally because:

- It represents the maximum extremity of the length scale from 3v to 286v.
- It is unique and objectively defined.
- Its use makes it possible to define a threshold (Vs) that partitions the set of surahs into two groups: those whose length exceeds this threshold, and those which are below it.

Justification of the integer threshold: The calculation 286/3=95.3 is not an integer. To obtain an objective and operational threshold within the framework of a discrete partition (surahs have an integer number of verses), two options are available: rounding down to the lower integer (95) or up to the higher integer (96). The choice of Vs=95 (default integer part) is justified by:

- No surah has exactly 95 verses, which eliminates any ambiguity of assignment.
- The partition thus defined (V>95 and V<95) produces two non-empty groups (21 and 93 surahs), allowing a meaningful analysis.
- Rounding up (96) would lead to a non-compliant delimitation of the groups V>96 and V<96, because surah 56 (V=96) would create an ambiguity as to its membership.
- A posteriori, validation by the results (gap of 154, correction by Al-Furqān, Type 1 equations) confirms the relevance of this choice.

$$Vs = \left\lfloor \frac{Vmax}{3} \right\rfloor = \left\lfloor \frac{286}{3} \right\rfloor \approx 95$$

Step 2: Partition of the corpus and calculation of the sum of verses.
The corpus is partitioned into two disjoint subsets:

- $G_{>95} = \{ Sn \in TG \mid V(Sn) > Vs \}$, ("long" suras)
- $G_{<95} = \{Sn \in TG \mid V(Sn) < Vs\}$, ("short" suras)

No sura has exactly 95 verses.

Step 3: Test of symmetric equality and identification of the gap.
If $V(G_{<95}) \neq V(G_{>95})$, the gap is defined as: $\Delta = V(G_{<95}) - V(G_{>95})$.

Step 4: Search for an internal correcting operator.
If a unique operator corrects this gap by transfer from one group to the other, its value must satisfy: δ = Δ/2. This condition follows from the requirement that transferring a sura of size δ simultaneously increases one sum and decreases the other by δ, thereby cancelling the gap. The corpus is examined for a sura with $V(S_n) = \delta$.

Step 5: Validation of the operator and redefinition of the groups.
If such a sura S_k exists, its transfer defines:
$Fq_{sym} = G_{>95} \cup \{S_k\}$, (Versetic Symmetry Group)
$F'q = G_{<95} \backslash \{S_k\}$, (Complementary Group)
The equality $V(Fq_{sym}) = V(F'q)$ is then verified.

Step 6: Analysis of the coherence of the obtained group.
The internal coherence of the group Fq is tested according to the principle established in previous works [1, 2]. This consists of:
a) Verifying whether it obeys a Type 1 equation of the form $aN(Fq) + bV(Fq) = SV(Fq)$, where a and b are positive integer constants, or whether it satisfies another characteristic relation allowing the determination of the sum of its constituent numbers.
b) Testing the autosimilarity property by partitioning Fq according to the parity of numbers and verses. It is then verified that, for each subgroup obtained, either it satisfies a Type 1 equation, or its sum parameters can be determined by deduction from the equations of the other groups or by other specific properties (such as proportionality relations).
This systematic methodology allows moving from a proportional observation (the threshold of 95) to the identification of a structured subset (Fq), whose emergence depends on the existence of a unique operator (S_k).

5. Results: Application of the Methodology and Identification of the Fq Structure

The sequential application of the methodological steps defined in Section 4 to the canonical data of the Quran yields the following results.

5.1 Step 1: Definition of the Threshold and Step 2: Initial Partition

$$Vs = \left\lfloor \frac{Vmax}{3} \right\rfloor = \left\lfloor \frac{286}{3} \right\rfloor \approx 95$$

5.2 Step 3: Equality Test and Identification of the Gap

The calculation of the versetic sums for each group gives:
$V(G_{>95}) = 3041\,,\quad V(G_{<95}) = 3195$

Table 1 *Sum of verses before operator intervention*

Group	Criterion	Sum of Verses V(G)
$G_{>95}$	$V(Sn) > 95$	3 041
$G_{<95}$	$V(Sn) < 95$	3 195

The hypothesis of symmetric equality $V(G_{>95}) = V(G_{<95})$ is not verified. The gap is:
$\Delta = V(G_{>95}) - V(G_{<95}) = 3\,195 - 3\,041 = 154$
The values $V(G > 95) = 3041$ are obtained by directly summing the verse counts of the surahs belonging to each group, according to the partition defined by the threshold Vs=95.
For example, the sum for G>95 is calculated as follows:
$V(G > 95) = 286 + 200 + 176 + 120 + 165 + 206 + 129 + 109 + 123 + 111 + 99 + 128 + 111 + 110 + 98 + 135 + 112 + 118 + 227 + 182 + 96 = 3041$
The complementary group G<95 is obtained by difference, and its sum satisfies:
$V(G < 95) = V(TG) - V(G > 95) = 6236 - 3041 = 3195$

5.3 Step 4: Search for the Correcting Operator

The target value for a correcting operator is half of the gap:

$$\delta = \frac{\Delta}{2} = \frac{154}{2} = 77$$

A systematic examination identifies a unique sura (the only one with 77 verses in the canonical corpus): Al-Furqān.

5.4 Step 5: Final Symmetry

The transfer of sura S_{25} from group $G_{<95}$ to group $G_{>95}$ defines the symmetry group Fq_{sym} and its complement F′q:
$Fq_{sym} = G_{>95} \cup \{S_{25}\}, F'q = G_{<95} \backslash \{S_{25}\}$
The new sums of the verses are:
$V(Fq_{sym}) = V(G_{>95}) + V(S_{25}) = 3041 + 77 = 3118$
$V(F'q) = V(G_{<95}) - V(S_{25}) = 3195 - 77 = 3118$
Perfect equality is thus established:
$V(Fq_{sym}) = V(F'q) = 3118.$
This equality confirms that the symmetry condition is satisfied under the defined transfer operation.

Table 2 *Perfect equality after intervention*

Groups	Composition	Sum of Verses V(G)
Fq_{sym}	$G_{>95} \cup \{S_{25}\}$	$3041 + 77 = 3118$
$F'q$	$G_{<95} \backslash \{S_{25}\}$	$3195 - 77 = 3118$

5.5 Definition and Coherence of Fq

$Fq = G_{>95}$ (before transfer)

- $N(Fq) = 337$
- $V(Fq) = 3041$
- $SV(Fq) = 247\ 309$

These parameters satisfy $337a + 3041b = 247309$.
Solving this equation in $\mathbb{N}^+$ yields a unique positive solution: a = 409, b = 36**.**
Direct verification gives: $(409 \times 337) + (36 \times 3\,041) = 137833 + 109476 = 247\ 309$
This corresponds exactly to $SV(Fq)$, confirming the integration of Fq as a coherent, locked subsystem within the global architecture [2]. This uniqueness is all the more significant as it is repeated for the subgroups of Fq, $(Fq_2, Fq_3, Fq_2 \cup Fq_4)$, each verifying an equation of the same form with unique coefficients. This systematic replication confirms that the Type 1 equation is a reliable structural marker: every coherent group in the system verifies it, with its own specific coefficients.

Thus, far from being an artificially constructed solution for Fq, the pair (409,36) is merely the particular manifestation, for this group, of a mathematical rule that applies to certain groups in the corpus structure.

Table 3 – Detailed Composition and Parameters of Group Fq

This table consists of the 21 surahs whose number of verses is greater than 95, defined by the criterion V(Sn)>95. These surahs compose the group Fq=G>95.

For each surah, the following are indicated: its order number (No.), its name, its number of verses (V), the sum of its verse numbers (SV), its Regularization value SR(Fq), and its Ranking value D(Fq) (the last two columns being derived from the master equation [2]). The total rows at the bottom of the table present the aggregated sums N(Fq)=337, V(Fq)=3041 and SV(Fq)=247309."

Surahs not listed in Table 3 (such as Ar-Ra'd, No. 13, with 43 verses) satisfy V(Sn)<95 and therefore belong to the complementary group G<95. Their exclusion from Table 3 is therefore a direct consequence of the selection criterion based on the threshold Vs=95

Table 3 *Fq*

N°	Sura Name	V	SV	SR(Fq)	D(Fq)
2	Al-Baqarah	286	41041	11114	-29927
3	Ali 'Imran	200	20100	8427	-11673
4	An-Nisa	176	15576	7972	-7604
5	Al-Ma'idah	120	7260	6365	-895
6	Al-An'am	165	13695	8394	-5301
7	Al-A'raf	206	21321	10279	-11042
9	At-Tawbah	129	8385	8325	-60
10	Yunus	109	5995	8014	2019
11	Hud	123	7626	8927	1301
12	Yusuf	111	6216	8904	2688
15	Al-Hijr	99	4950	9699	4749
16	An-Nahl	128	8256	11152	2896

J. of Quranic Sciences and Research Vol. 7 No. 1 (2026) 49-62

N°	Sura Name	V	SV	SR(Fq)	D(Fq)
17	Al-Isra	111	6216	10949	4733
18	Al-Kahf	110	6105	11322	5217
19	Maryam	98	4851	11299	6448
20	Taha	135	9180	13040	3860
21	Al Anbya	112	6328	12621	6293
23	Al-Mu'minun	118	7021	13655	6634
26	Ash-Shu'ara	227	25878	18806	-7072
37	As-Saffat	182	16653	21685	5032
56	Al-Waqi'ah	96	4656	26360	21704
337		3041	247309	247309	0

$$\boldsymbol{N(Fq)} = X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_9 + X_{10} + X_{11} + X_{12} + X_{15} + X_{16} + X_{17} + X_{18} + X_{19} + X_{20} + X_{21} + X_{23} + X_{26} + X_{37} + X_{56} = 337 \quad (1)$$

The value SV(Fq)=247309 is obtained by summing the verse number sums from the SV column of Table 3. For each surah Sn, the sum of its verse numbers is calculated using the formula for the sum of an arithmetic series:

$$SV(Sn) = \frac{V(Sn) \times \big((V(Sn) + 1)\big)}{2}$$

For example, for Surah Al-Imran (No. 3), which has V=200 verses, we obtain:
$SV = (200 \times 201)/2 = 20100$
The total sum is then:
$SV(Fq) = 41041 + 20100 + 15576 + 7260 + 13695 + 21321 + 8385 + 5995 + 7626 + 6216 + 4950 + 8256 + 6216 + 6105 + 4851 + 9180 + 6328 + 7021 + 25878 + 16653 + 4656 = 247309$
The details of the calculations for each surah are presented in Table 3.

5.6 Internal Autosimilarity: Subgroups of Fq

Partitioning Fq according to the parity of N and V reveals its internal structure. Each subgroup verifies either a Type 1 equation, confirming an internal autosimilarity of the system, or another specific property (such as a parity relation or a relation derived from the Type equation). These equations admit, in each case, a unique solution in $\mathbb{N}^+$. (The term "autosimilarity" is used here in a structural sense and does not imply geometric autosimilarity). Notably, we distinguish:

Table 4 *Subgroup* Fq_2
Fq_2 : suras with even number and odd verse count

$74a' + 747b' = 60\,964$
Solution: $(a', b') = (521, 30)$
Verification: $74 \times 521 + 747 \times 30 = 38\,554 + 22\,410 = 60\,964$

Fq_2

$N(Fq_2)$	Sura Name	$V(Fq_2)$	$SV(Fq_2)$	$SR(Fq_2)$	$D(Fq_2)$
6	Al An'am	165	13695	8076	-5619
10	Yunus	109	5995	8480	2485
12	Yusuf	111	6216	9582	3366
20	Taha	135	9180	14470	5290
26	Ash-Shu'ara	227	25878	20356	-5522
74		747	60964	60964	0

$$\boldsymbol{N(Fq_2)} = X_6 + X_{10} + X_{12} + X_{20} + X_{26} = 74 \quad (2)$$

Table 5 *Subgroup Fq_3*
Fq_3 : suras with odd number and even verse count

$115a' + 1036b' = 83\,534$
Solution: $(a', b') = (330, 44)$
Verification: $115 \times 330 + 1036 \times 44 = 37\,950 + 45\,584 = 83\,534$

Fq_3

$N(Fq_3)$	Sura Name	$V(Fq_3)$	$SV(Fq_3)$	$SR(Fq_3)$	$D(Fq_3)$
3	Ali 'Imran	200	20100	9790	-10310
5	Al-Ma'idah	120	7260	6930	-330
7	Al-A'raf	206	21321	11374	-9947
19	Maryam	98	4851	10582	5731
21	Al Anbya	112	6328	11858	5530
23	Al-Mu'minun	118	7021	12782	5761
37	As-Saffat	182	16653	20218	3565
115		1036	83534	83534	0

$$\boldsymbol{N(Fq_3)} = X_3 + X_5 + X_7 + X_{19} + X_{21} + X_{23} + X_{37} = 115 \quad (3)$$

Table 6 *Subgroup $Fq_2 \cup Fq_4$*
$Fq_2 \cup Fq_4$: set of suras with even number

$170a' + 1543b' = 136\,598$
Solution: $(a', b') = (386, 46)$
Verification: $170 \times 386 + 1543 \times 46 = 65\,620 + 70\,978 = 136\,598$

Fq_2 U Fq_4

$N(Fq_2UFq_4)$	Sura Name	$V(Fq_2UFq_4)$	$SV(Fq_2UFq_4)$	$SR(Fq_2UFq_4)$	$D(Fq_2UFq_4)$
2	Al-Baqarah	286	41041	13928	-27113
4	An-Nisa	176	15576	9640	-5936
6	Al-An'am	165	13695	9906	-3789
10	Yunus	109	5995	8874	2879
12	Yusuf	111	6216	9738	3522
16	An-Nahl	128	8256	12064	3808
18	Al-Kahf	110	6105	12008	5903
20	Taha	135	9180	13930	4750
26	Ash-Shu'ara	227	25878	20478	-5400
56	Al-Waqi'ah	96	4656	26032	21376
170		1543	136598	136598	0

$$\boldsymbol{N(Fq_2\ U\ Fq_4)} = X_2 + X_4 + X_6 + X_{10} + X_{12} + X_{16} + X_{18} + X_{20} + X_{26} + X_{56} = 170 \quad (4)$$

Table 7 *Subgroup Fq_4*
Subgroup Fq_4 (N even, V even) – Equation 5:

Fq_4

$N(Fq_4)$	Sura Name	$V(Fq_4)$	$SV(Fq_4)$	$SR(Fq_4)$	$D(Fq_4)$
2	Al-Baqarah	286	41041	670	-40371
4	An-Nisa	176	15576	548	-15028
16	An-Nahl	128	8256	1040	-7216
18	Al-Kahf	110	6105	1102	-5003
56	Al-Waqi'ah	96	4656	2936	-1720
96		796	75634	6296	-69338

$$\boldsymbol{N(Fq_4)} = X_2 + X_4 + X_{16} + X_{18} + X_{56} = 96 \quad (5)$$

Table 8 *Subgroup Fq_1*
Subgroup Fq_1 (N odd, V odd) – Equation 6:

		$\mathbf{Fq_1}$			
$N(Fq_1)$	Sura Name	$V(Fq_1)$	$SV(Fq_1)$	$SR(Fq_1)$	$D(Fq_1)$
9	At-Tawbah	129	8385	699	-7686
11	Hud	123	7626	785	-6841
15	Al-Hijr	99	4950	933	-4017
17	Al-Isra	111	6216	1055	-5161
52		462	27177	3472	-23705

$$\boldsymbol{N(Fq_1)} = X_9 + X_{11} + X_{15} + X_{17} = 52 \quad (6)$$

5.7 Structural Determinism and Inverse Calculation of Parameters

These equations allow for an inverse calculation of the parameters. For example:

- **Sum of numbers for Fq_2** :
  $N(Fq_2) = (60\,964 - (30 \times 747)) / 521 = (60\,964 - 22\,410) / 521 = 38\,554 / 521 = 74$
- **Sum of numbers for Fq_3** :
  $N(Fq_3) = (83\,534 - (44 \times 1036)) / 330 = (83\,534 - 45\,584) / 330 = 37\,950 / 330 = 115$
- **Sum of numbers for $Fq_2 \cup Fq_4$** :
  $N(Fq_2 \cup Fq_4) = (136\,598 - (46 \times 1543))/386 = (136\,598 - 70\,978)/386 = 65\,620/386 = 170$
- **Sum of numbers for Fq_4 (deduction)**:
  $N(Fq_4) = N(Fq2 \cup Fq_4) - N(Fq2) = 170 - 74 = 96$
- **Sum of numbers for Fq_1 (deduction)**:
  $N(Fq_1) = N(FqG) - N(Fq2) - N(Fq_3) - N(Fq_4) = 337 - 74 - 115 - 96 = 52$

These calculations reveal a locked architecture where each parameter is constrained by precise relations

6. Comparative Analysis of the Structural Roles of Al-Furqān and Al-Fatihah

6.1 Two Distinct Manifestations of the Same Principle

The rule of thirds manifests differently in two structural domains:

- Ordinal domain: conditioned by the sign of $D(Sn) = SV(Sn) - (49N(Sn) + 2V(Sn))$; the effect (38/76 partition) becomes visible only after activation by Al-Fātiḥah [2].
- Versetic domain: direct application to $Vmax = 286$ defines threshold 95; equality $\sum V > 95 = \sum V < 95$ is achieved through the transfer of Al-Furqān.

6.2 Al-Fatihah (S_1): Operator for Actualizing the Ordinal Partition

The functional displacement of Al-Fātiḥah towards BZ — specifically towards subgroup IZ_1 — activates the 1/3 partition within the lower-level groups and generates a hierarchical structure governed by proportional and linear constraints.

1. Internal Partitions:

The same proportion reproduces in all these sets:

- In the total set $TG = (TA \cup BZ1) \cup (TB \cup BZ2)$:
  - $TA \cup BZ1 = 38\ suras$ (⅓ × 114)
  - $TB \cup BZ2 = 76\ suras$ (⅔ × 114)
- In the set $TA \cup TB$ (87 $suras$):
  - $TA = 29\ suras$ (⅓ × 87)
  - $TB = 58\ suras$ (⅔ × 87)
  - Relation: $S(TB) = 2 \times S(TA)$
- In the set $BZ = BZ1 \cup BZ2$ (27 $suras$):
  - $BZ1 = 9\ suras$ (⅓ × 27)
  - $BZ2 = 18\ suras$ (⅔ × 27)

  - Relation: $S(BZ2) = 2 \times S(BZ1)$
- In the set $TA' \cup TB'$ (78 $suras$):
  - $TA' = 26\ suras$ (⅓ × 78)
  - $TB' = 52\ suras$ (⅔ × 78)
  - Relation: $S(TB') = 2 \times S(TA')$
- In the set $PI1 \cup PI2$ (9 $suras$):
  - $PI1 = 3\ suras$ (⅓ × 9)
  - $PI2 = 6\ suras$ (⅔ × 9)
  - Relation: $S(PI2) = 2 \times S(PI1)$

2. Specific Structural Balances:

Its influence also operates through its number (N(S1) = 1) and its verse count (V(S1) = 7), generating precise linking equations:

- Balance of number sums in BZ: $N(BZ2) = 2 \times N(BZ1) + N(S1)$
- Cross ordinal/versetic balance in IZ: $N(IZ2) = 2 \times V(IZ1)$ where $V(IZ1) = V(IZ1') + V(S1)$

Activation of hierarchical structural coherence: This configuration allows the emergence of coherent groups (TA, TA', PI_1, PI), each verifying a Type 1 equation $(aN + bV = SV)$

6.3 Al-Furqān (S_{25}): Operator for Activating Versetic Symmetry

The initial inequality ($\sum V > 95 = 3041,\ \sum V < 95 = 3195$).
Transfer of Al-Furqān from $G < 95\ to\ G > 95$ is the operation that:

- Establishes symmetric equality: $\sum V > 95 = \sum V < 95 = 3118$ verses.
- Defines the symmetry subset $Fqsym = G > 95 \cup \{S_{25}\}$, while preserving the structural coherence of the subsystem Fq = G>95, whose subgroups (Fq_1, Fq_2, Fq_3, Fq_4) continue to satisfy Type 1 equations.

6.4 Functional Convergence: A Unified Architectural Model

The parallel between the two operators is systematic and reveals a coherent two-level model:

Table 9 *Comparison between the two operator suras*

Aspect	Al-Fatihah (Sura 1)	Al-Furqān (Sura 25)
Domain of Action	Ordinal	Versetic
Activated Principle	*D(Sn) = SV(Sn) − (49 n + 2 V(Sn))* → third of 114	Third of 286
Initial State	Arbitrary partition	Non-symmetric
Operator Action	Displacement	Transfer
Main Structural Effect	38/76 partition	∑V=3 118
Secondary Structural Effect	Hierarchical structure of TA (ordinal group) [2]	Hierarchical structure of Fq (versetic group)
Verification	Type 1 equations	Type 1 equations

This convergence highlights a system of operators structured around a single generative principle — the rule of thirds (1/3) — whose application to each domain is carried out by a specialized operator. The complementarity of these roles, together with the precision of the results (strict equalities, Type 1 equations, autosimilarity), argues in favor of a unified and intentional design.

J. of Quranic Sciences and Research Vol. 7 No. 1 (2026) 49-62

7. Discussion

7.1 Numerical Precision and Semantic Coherence

The analysis demonstrates that the symmetric equality $V(Fqsym) = V(F'q)$ emerges only through the precise functional intervention of Al-Furqān. This mechanism rests on four objective elements:

- **Numerical precision:** The gap between the two groups is Δ = 154. Its half δ = 77 corresponds exactly to the number of verses Al-Furqān.
- **Structural specificity:** Within the canonical corpus, Al-Furqān is the only sura consisting of 77 verses.
- **Name coherence:** The name Al-Furqān ("The Criterion", from the root *F-R-Q*: to separate, to distinguish, to re-establish the difference) corresponds semantically to the mathematical function observed in this configuration.
- **Robustness of threshold 95:** Threshold 95 was tested between values 84 and 97. For each threshold, we calculate the gap $\Delta = |V(G <) - V(G >)|$, its half δ = Δ/2, then verify whether a sura possesses this number of verses

Table 10 *Robustness test around threshold 95*

Threshold	V(G>)	V(G<)	Δ = |G< - G>|	δ = Δ/2	Sura δ exists?
84	3484	2752	732	366	No
86/87	3399	2837	562	281	No
90/92	3134	3102	32	16	No
95	**3041**	**3195**	**154**	**77**	**Yes (Al-Furqān)**
97	2945	3291	346	173	No
100/108	2748	3488	740	370	No

Result:
Only threshold 95 yields δ = 77, corresponding to Al-Furqān. For all other tested thresholds, no sura corresponds to δ. These results indicate that threshold 95 is structurally determined rather than arbitrarily selected.

7.2 Dual Structural Analogy: An Organizing Principle

The Study Identifies a Structural Correspondence Between The Versetic Domain and The Ordinal Domain [2]:

- **Same organizing principle:** The rule of thirds (1/3).
- **Two different applications:**
  - **Ordinal:** 114 suras → 38/76 partition, symmetry of signs.
  - **Versetic:** 286 verses → threshold of 95, symmetry between verse counts.
- **Two structurally decisive suras:**
  - For the ordinal architecture: Al-Fātiḥah (S_1).
  - For the versetic architecture: Al-Furqān (S_{25}).

7.3 Nature of the Established Equations

The main results of this study, derived through a formal approach identical to previous works (Benzitouni, S., & Benzitouni, S. (2024) and Benzitouni, S. (2025), determine the sum of surah numbers of objective structural groups (parity, ranking sign, length...). A significant part of these results takes the form: $aN(G) + bV(G) = SV(G)$ (Type 1 Equation) where G is a suras group defined by an objective structural criterion and a, b are positive integer constants specific to this group. This equation allows the determination of the sum of sura numbers for a group: $N(G) = (SV(G) - b \cdot V(G))/a$ (Derived Equation). Each structural group thus imposes an exact mathematical constraint on the sum of the numbers of a specific subset of suras.

7.4 Final Objective

The final objective of this cumulative approach is to identify a complete set of groups G_1, G_2, ..., G_k, each verifying either an independent Type 1 equation or another specific property (such as a proportional relationship or a fixed sum). The set of derived equations from the Type 1 equation, along with the other identified properties, forms a system of linear equations whose unknowns are the individual numbers of the 114 suras ($X_1, X_2, \ldots, X_{114}$). The central hypothesis is that a system of 114 independent equations would admit the canonical numbering (1, 2, 3, ..., 114) as its unique solution.

7.5 Specific Contribution of This Study

This study makes a crucial contribution to the construction of this system:

- It identifies the versetic group Fq and its equation: $409N + 36V = SV$.
- It demonstrates the autosimilarity of Fq through similar equations for its subgroups ($Fq_1, Fq2, Fq_3, Fq_4, Fq2 \cup Fq_4$), thereby adding additional linear equations to the global system.
- It highlights the role of Al-Furqān as a versetic operator, parallel to Al-Fātiḥah in the ordinal domain.

7.6 Systemic Coherence

The accumulation of satisfied constraints significantly reduces the plausibility of a purely random configuration under simple probabilistic assumptions:

- Threshold 95 derived from one-third of 286.
- Even gap Δ = 154 whose half δ = 77 corresponds to Al-Furqān (unique).
- Perfect balance after transfer: $\sum V > 95 \; = \; \sum V < 95 \; = \; 3118$.
- Strict equation of group Fq: ($409N \; + \; 36V \; = \; SV$).
- Autosimilarity of subgroups ($Fq_1, Fq2, Fq_3, Fq_4$)
- Same principle (1/3) in two independent domains with distinct operators.
- Global validation by Fuṣṣilat (Benzitouni, S., & Benzitouni, S. (2024).

Historical objection:
How could such an architecture have been conceived in the 7th century?
This is not a material construction modifiable through trial and error, but a text revealed and transmitted orally then in writing, in a context:

- Without formalized algebra
- Without developed positional numeration
- Without complex calculation tools
- In a predominantly oral culture

Three hypotheses:

1. **Chance:** The accumulation of constraints makes this hypothesis statistically implausible.
2. **Conscious human construction:** Would suppose the capacity to plan a global equation system without adapted tools, and to ensure its intact transmission – with no historical testimony.
3. **Architecture intrinsic to the text:** The equations brought to light result neither from subsequent human calculation nor from a gradual organization of the corpus. They are inscribed in the very structure of the text since its origin, and merely awaited contemporary mathematical tools to be discovered.

This third hypothesis offers a coherent explanatory framework for:

- The absence of historical mention (inaccessible to contemporaries)
- Preservation over fourteen centuries (intrinsic to the text)
- Late discovery (made possible by modern mathematical tools)

This is therefore a **global textual architecture**, whose systemic complexity appears difficult to reconcile with the documented mathematical tools and transmission practices of the 7th century.

7.7 Conclusion and Perspectives

The identification of Al-Furqān's role and the equations of group Fq constitutes a major methodological advancement. Each equation obtained is not an isolated curiosity but a verifiable mathematical constraint adding to a cumulative system. The objective is to progressively constitute a global network of constraints whose resolution aims to demonstrate that the canonical order of the Muṣḥaf is the unique configuration satisfying all internal relations brought to light.

Future work:

- Identify other objective structural groups
- Establish their equations
- Test their linear independence
- Complete the system until obtaining a fully constraining network

This study is not a definitive endpoint but a structuring step in a cumulative demonstration.

8. Conclusion

Summary of key elements:

"This study has highlighted three constitutive elements of the Quran's mathematical architecture:

1. **A unique proportional principle**: the rule of thirds (1/3), which manifests in both the ordinal and versetic domains.
2. **Strict equality conditions**: a compensation equality between positive and negative ranking values in the ordinal domain, and an equality of verses between long and short surahs in the versetic domain.
3. **Locked autosimilar structures**: in the versetic domain, the group Fq and its subgroups obey Type 1 equations of the form $aN + bV = SV$, with uniquely determined positive integer coefficients." Likewise in the ordinal domain with the groups TA, TA' and PI_1.

This study provides evidence that the rule of thirds (1/3) structurally organizes a part of the Quran's numerical architecture, manifesting independently in the versetic and ordinal domains.

Versetic domain: The application of one-third to the longest sura (286 verses) defines an objective threshold of 95. The observed symmetry $\sum V > 95 = \sum V < 95$ is achieved only through the unique intervention of Al-Furqān (Sura 25), whose 77 verses fill exactly half of the initial gap (154). Its name ("The Criterion") precisely reflects its role of separation and balancing.

The group Fq = G>95 verifies a strict Type 1 equation ($409N + 36V = SV$) and exhibits recursive structural coherence: its subgroups (defined by parity) obey equations of the same form with their own specific coefficients. Al-Furqān, integrated into subgroup Fq_1, respects and does not break this system; its sum is deduced from the equations of the replicated groups.

Ordinal domain Benzitouni, S. (2025): The same principle of one-third, activated by Al-Fātiḥah (Sura 1), generates an analogous hierarchically organized and linearly constrained. This dual correspondence supports the conceptual unity of the underlying structural mode.

Scope and Implications

- **Systemic coherence:** The convergence of several elements—(a) simple mathematical laws (thirds, concordance), (b) recursive structural replication verified by linear equations, (c) semantic adequacy of names— contributes to a cumulative argument against a purely fortuitous configuration. Roles specialize: Fuṣṣilat (S41) as global validation key (Benzitouni, S., & Benzitouni, S. (2024), Al-Fātiḥah and Al-Furqān as internal organization operators.
- **Proof system:** Each Type 1 equation adds an independent linear constraint on suras numbers. The accumulation of these constraints builds a system whose targeted unique solution is the canonical numbering. The progressive identification of these constraints – at least 114, one equation per group – would constitute a strong mathematical argument. It would suggest that this architecture exceeds the documented capacities of 7th-century human planning, particularly with regard to the simultaneous coordination of multiple structural, numerical, hierarchical, and semantic constraints at the scale of the entire corpus**.**
- **Textual preservation:** The interdependence of the system – where an alteration would break multiple balances – attests to exceptionally faithful transmission.
- **Research field:** The Quran opens a unique study perspective, inviting exploration of other regularities and mapping of its numerical structure.

Ultimately, the hierarchical and linearly constrained architecture revealed by suras Al-Fātiḥah and Al-Furqān presents a structurally coherent configuration whose internal consistency raises fundamental questions about its origin and preservation, calling for further in-depth investigation.

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Conflict of Interest

The author declares that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The author is responsible for the study conception, research design, data collection, data analysis, result interpretation and manuscript drafting.

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