International Journal of Integrated Engineering, Vol. 4 No. 3 (2012) p. 19-25

Covariance Bounds Analysis during Intermittent Measurement for EKF-based SLAM

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Received 1 October 2012; accepted 1 December 2012, available online 20 December 2012

Abstract: This paper deals with Extended Kalman Filter(EKF)-based SLAM estimation considering intermittent measurement. The study proposes a method to ensure that a designer is able to secure the state covariance or system uncertainties even though the mobile robot sometimes losses its measurement data during its observations. EKF which is based on Bayesian, is chosen in this work for estimation purposes in determining mobile robot position and the environment conditions. In addition, as most of estimation techniques uses Bayesian method, our propose results is expected to provide a general estimation guidelines especially on the filter statistical behavior. The analysis started from the review of theoretical studies of EKF including the expected condition occurred when mobile robot do not received information from its sensors. Simulation results have consistently supports our theoretical analysis which determines that a designer can determine the state error covariance explicitly whenever measurement data is not available.

Keywords: SLAM, EKF, Intermittent Measurement, Covariance

1. Introduction

History has shown that the role of robot has significantly improves and helps human in doing several tasks. Not limited to only industrial robots, mobile robots have extending their applications in household appliances such as vacuum cleaner robot and lawn-mover robot. However, in pursuing a truly autonomous robot has exhibits a lot of challenges to overcome. One of the research area that is believed to provide a solution for this problem is known as Simultaneous Localization and Mapping (SLAM) problem[1]-[6]. The problem defines a situation where a mobile robot attempts to observe its surroundings while consistently updates its location. Then, the robot continually constructs a map based on the information obtained during its observations.

In realizing a solution for SLAM problem, researcher has to deal with several issues such as uncertainties, data association, and feature extraction[6]. This paper reveals the uncertainties affect to the state covariance whenever robot losses its information.

Up to date, there are few approaches have been introduced such as Extended Kalman Filter(EKF), H infinity Filter, Unscented Kalman Filter(UKF), and also Particle Filter[6] for estimation purposes. Most of those techniques share the identical technical properties where they are based on Bayesian method. Between above mentioned approaches, EKF are the most celebrated method for SLAM solution. The reason could be due to the filter offers simple algorithm to follow and has lower computational cost compared to others.

One factor that can effects the mobile robot performance is the sensoring devices. Its existence is very important to provide information about the mobile robot location and the environment. Hence, if the mobile robot does not received any measurement data during its observations, then the probability of loosing confidence will be increased. Any sensor may not function as expected, intermittently or continuously after a period of time due to its life-span or due to the environment conditions. If information is not arriving to the controller during measurement, then this situation is known as intermittent measurement. This case will be analyzed in details in this paper later.

Motivated from above findings, this paper determines the condition of statistical bound during intermittent measurement[7]-[12] when EKF[13] is applied for SLAM problem. EKF that has been developed and proposed in various attempts for SLAM for example in [14] and [15] offers a reliable and satisfying result for estimation purposes. Until now, there are only few numbers of works have developed the explanations for intermittent measurement in mobile robot applications such as demonstrated by [16]-[17].

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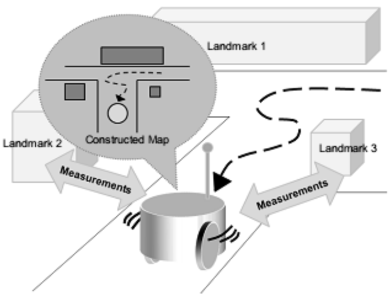

Fig.1 Illustration of SLAM problem

However both references did not explicitly determines how the state covariance behaves whenever measurement data are missing during mobile robot observations. Hence the paper attempts to discover the properties of estimation if this problem happens.

This paper is proposed to support the previous findings[16]-[18] to demonstrate the upper and lower bounds of state covariance behavior during intermittent measurement. Sinopoli et al.[10] and Plare et al.[11] have discovered a good references about the statistical bounds where the state covariance matrix are bounded to a defined level. Using those results as references, three conditions are analyzed to examine their effect and characteristics i.e initial state covariance, measurement noise covariance, and the state covariance statistical bounds. Previous results[18] which identifies the statistical bounds of the state covariance is extended to explain why the statistical bounds are approximating the lower bound or the upper bound. It is also worth to suggest that this work also able to provide essential information to H_∞ Filter based SLAM[19] as well due to the filter has almost same characteristics.

The remaining of this paper is organized as follows. Section two describes the kinematic model of the mobile robot together with a brief theoretical explanation about intermittent measurement. Our proposed theoretical results follow later in section three. Next, section four shows the simulation results and discussions. Finally, section five concludes our paper.

2. General Model

Process and measurement models are the best representation to provide a general picture on how actually SLAM problem is developed. The process model determines the mobile robot movements through an unknown environment. It calculates the mobile robot position based on two inputs i.e mobile robot velocity and angular acceleration which also incorporates associated noise during the movement. On the other hand, measurement model explains the measurement made by the mobile robot through its sensors regarding the relative angle and distance every time the mobile robot detects any objects. These two models are shown separately in Fig.2.

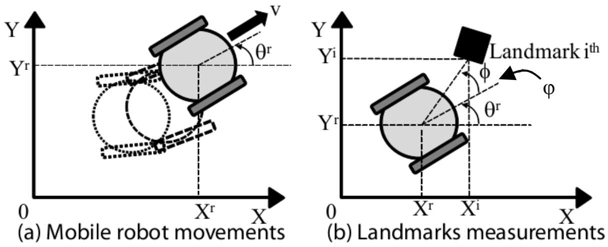

Fig. 2 SLAM general model showing the mobile robot movements (a) and landmarks measurements (b)

For process model, we consider a nonlinear discrete-time dynamical system as follows.

$$X_k = f(X_k, \omega_k, v_k, \delta\omega, \delta v) \quad (1)$$

where $X_k \in R^{3+2m}$ is the augmented state of mobile robot which consists of the heading angle θ_i^r and X_k^r, Y_k^r positions, together with any landmarks x_i, y_i location being observed. The mobile robot angular acceleration is defined by ω_k and its velocity by v_k. $\delta\omega$, δv are the associated process noise to the angular acceleration and velocity respectively. To simplify without losing generality, this paper considers a case for a stationary landmarks.

The measurement model is defined as below.

$$\begin{aligned} z_{k+1} &= \gamma_{k+1}\begin{bmatrix} r_i \\ \varphi_i \end{bmatrix} \\ &= \gamma_{k+1}\begin{bmatrix} \sqrt{(x_i - X_{k+1}^r)^2 + (y_i - Y_{k+1}^r)^2} + v_{r_i} \\ \arctan\dfrac{y_i - Y_{k+1}^r}{x_i - X_{k+1}^r} - \theta_{k+1}^r + v_{\theta_i} \end{bmatrix} \end{aligned} \quad (3)$$

From (3), r_i and φ_i are the relative distance and angle from mobile robot to any landmarks encountered during observations. v_{r_i}, v_{θ_i} defines the associated measurement noise to above equation both relative distance and angle. Notice that in (3), γ_{k+1} was added to the normal equation which makes the equation differs to the original measurement model[2]. With the existence of γ_{k+1}, (3) is now describing the condition of intermittent measurement. This characteristics is actually relies to the *Bernoulli process* as follow.

Now, the Extended Kalman Filter algorithm is presented to give an overview before going into details about the intermittent measurement scenario.

$$\begin{aligned} &\Pr\{\gamma = 1\} = p, \\ &\Pr\{\gamma = 0\} = 1 - p, \\ &E[\gamma] = E[\gamma^2] = p \end{aligned} \quad (4)$$

The addition of γ_{k+1} also explains the stochastic behavior of measurement data whether available or unavailable for a period of time. Equation (3) can also be represented in the following equation.

$$H_i = \begin{bmatrix} 0 & -\dfrac{dx_k}{r} & -\dfrac{dy_k}{r} & \dfrac{dx_k}{r} & \dfrac{dy_k}{r} \\ -1 & \dfrac{dy_k}{r^2} & -\dfrac{dx_k}{r^2} & -\dfrac{dy_k}{r} & \dfrac{dx_k}{r^2} \end{bmatrix} \quad (5)$$

where

$r = \sqrt{(x_i - x_{k+1})^2 + (y_i - y_{k+1})^2}$

$dx_k = x_i - x_{k+1},$

$dy_k = y_i - y_{k+1}$

Now, the EKF algorithm is presented. The predicted state of $\hat{X}^-_{k+1}$ is shown as

$$\hat{X}^-_{k+1} = F(\hat{X}^+_k, \omega_k, v_k, 0, 0) \quad (6)$$

Recognized that by comparing (1) and (6), $\delta\omega$, δv now have becomes zero as there are no process noise included in the prediction.

The associated a priori state covariance P_{k+1}^- is shown as follows.

$$P^-_{k+1} = f_r P^+_k f_k^T + f_{\omega v} \Sigma_k f_{\omega v}^T \quad (7)$$

where f_r, $f_{\omega v}$ are the mobile robot jacobian and control noise jacobian matrices evaluated from (1). Σ_k is the control noise covariance and P_k^+ is showing the previous updated state covariance. For $T=1$, the following equation is obtained.

$$f_r = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -v\sin\theta & 1 & 0 & 0 \\ v\cos\theta & 0 & 1 & 0 \\ 0 & 0 & 0 & I_n \end{bmatrix}, \quad f_{\omega v} = \begin{bmatrix} g_{\omega v} \\ 0 \end{bmatrix} \quad (8)$$

From above, $g_{\omega v}, I_n$ are the process noise and identity matrix with an appropriate dimension. The updated state covariance yields the following equation.

$$P^+_{k+1} = P^-_{k+1} - \gamma_{k+1} K_{k+1} H_i P^-_{k+1} \quad (9)$$

where

$$K_{k+1} = P^-_{k+1} H_i^T (H_i P^-_{k+1} H_i^T + R_{k+1})^{-1}$$

Finally, by using the EKF algorithm, the updated state can be derived where

$$\hat{X}^+_{k+1} = \hat{X}^-_{k+1} + \gamma_{k+1} K_{k+1} H_i (X_k - \hat{X}^-_{k+1})^{-1} \quad (10)$$

Based on the EKF algorithm, the prediction and update will be recursively done to obtain the newest location of both mobile robot and landmarks. As uncertainties are one issue in SLAM, this paper focuses on the updated state error covariance characteristics to identify its performance in certain statistical bounds through simulation. It is assumed that the data association is available at all time and the mobile robot is expected to moves in a planar environment.

3. Theoretical Results

Information is important for EKF to gain better picture about the state being updated. Fisher Information Matrix is applied in this work to analyze how the information behaves during estimation.

The FIM is given as follow.

$$J_{k+1} = (P^+_{k+1})^{-1} + H^+_{k+1} R^-_{k+1} H_{k+1} \quad (11)$$

This paper refers to (11) to understand how actually information being delivered when the mobile robot starts to observe its surroundings. Let the initial state covariance to be P_0 and consider a 1-D SLAM (a mobile robot with one axis) problem which makes the measurement matrix H_k instantly becomes an identity matrix. The measurement noise normally is set to be smaller than 1. If this case is referred, it can be explains by the above equation that if P_0, R is initially set to possess smaller covariance and smaller noise respectively, then J_k becomes bigger.

There is also other issue comes in if measurement data are missing. Theoretically, it should be identified that whenever data is not arriving at an update, the state covariance might become bigger than the previous state. However, is this true or the results will show opposite states? In our best knowledge, this characteristic is yet to be described by any preceding works[16]-[17]. Therefore, this paper attempts to clarify the state covariance behavior after the measurement data are missing.

To begin our analysis, it is worth to know that the updated state error covariance is converging to a steady state[2] such that

$$P_k > P_{k+1} > P_{k+2} .... > P_n$$

This relationship instantaneously describes that the FIM is increasing if the mobile robot is collecting more information each time it observes the environment.

$$J_n > J_{n-1} > J_{n-2} .... > J_n$$

Until now, researchers do not clearly understood that the efficiency of their estimation are actually depends on several aspects especially when measurement data are missing. Even if some of them might know the answers, the explanations are still lacking and require further descriptions. The next section unveils this condition. We propose three distinctive factors that are directly influenced the estimations.

3.1 Impact of Initial State Covariance

As mentioned earlier, the first objective of this paper is to explain that the initial state covariance has a significant effect to the overall estimation especially when measurement data are missing intermittently. This can be proven easily if (7) and (11) are referred. From (7), readers can identify that the updated state covariance is depend on the previous state covariance update. By substituting (7) into (11),

$$J_k = (f_r P_0 f_k^T + f_{\omega v} \Sigma_k f_{\omega v}^T)^{-1} + H_k^+ R_k^- H_k \quad (12)$$

Now, assume that the process noise is very small such that it can be neglected; this assumption is useful to find out the essential feature of covariance update. Then, we arrived in

$$J_k = (f_r P_0 f_k^T)^{-1} + H_k^+ R_k^- H_k \quad (13)$$

If a simple one dimensional localization is considered, then the only variable left for above equation becomes

$$J_k = (P_0)^{-1} + R_k^- \quad (14)$$

Above equation finally explains that, due to recursive update of FIM, the final updated state covariance is yielding smaller covariance. However, by analyzing further, if the initial state covariance is set to have big uncertainties in a situation where measurement data are not arrived such that the latter of right hand side of (9) is not available, then we exhibits erroneous results. The consequence becomes even worse if at the beginning of the estimation no measurement data are collectible during observations. Hence, researcher should take deep analysis at the initial start of mobile robot observations.

3.2 Measurement Noise Covariance

Continues from the initial state covariance explanations given in the earlier section, the measurement noise covariance is also contributes the same effects to estimation. Equation (14) expresses how the updated FIM is being influenced by the measurement noise covariance. It can be recognized that if the sensors are unable to provide less noise during inference, then estimation becomes unpredictable. Moreover, this factor produces even greater residue error if the noise statistics is completely unknown as EKF only capable to infer in environment that possesses Gaussian noise. Remark that localization may take place in various circumstances. It will be a wise decision to ensure that the designed system do not exhibit high measurement noise during measurement. H_∞ Filter[19] that depends on the measurement covariance can potentially become unreliable if this issue is not taken seriously.

3.3 EKF Statistical Bounds

When measurement data are not arriving for estimation purposes before updating the current state, EKF simply presume that the updated state holds the same state covariance calculated in earlier stage. This condition is identifiable through (9). . If this happen, then it can be expected that the mobile robot still has a good estimation. However, this case is not logically acceptable as when there is no information, the uncertainties should be increasing. In mobile robot localization, or even more complex problem such as SLAM, the uncertainties do has significant effect to the overall estimation. The uncertainties are subjected to the application of different sensors devices, feature recognition and dynamic environments. Unfortunately, there are no scientific explanations until now which discussing in detail regarding this matter. Hence it would be a good starts if there is a description about how the state covariance behaves during this problem. In addition, there are lot of unknown and unexpected factor exists that could influence the overall estimation such as modeling error, or dynamic obstacles. When these issues come in, how actually would the state covariance behaves? This paper reveals what happens to the updated state covariance in an intermittent measurement case.

Let the previous covariance to be P_k while the probability of when data is available is given by *p*. Therefore the probability of data is missing will be *1-p*. If *p* is 0.8, then the Bernoulli process for the opposite consequently becomes 0.2. This literally means that at time *k* when measurement data is arrived,

$$\Pr\{\gamma = 1\}_k = 0.8$$

Now at time *k+1*, measurement data becomes unavailable. Therefore, at state *k+1*, the updated state covariance seems to acquire previous covariance P_k with same probability of 0.8. However, based on Bernoulli process, now the state covariance P_{k+1} have the following characteristics.

$$\Pr\{\gamma = 0\}_{k+1} = 0.2$$

Above equation explains that the probability has becomes smaller or the failure rate is supposed to decrease than before. Unfortunately, from EKF algorithm, P_{k+1} refer to the P_k which has accidentally making the state covariance to exhibit the same value as P_k such that if the covariance is small at P_k then it is smaller at state P_{k+1} if claims by S.Huang are taken into account. As a result, the estimation becomes overconfident about the updated states even though the estimation becomes erroneous. Furthermore, we suggest that the measurement matrix H_k has a significant existence to the estimation. This characteristic will be shown later in this paper. A similar definition to [18] is made for evaluations purposes.

A theoretical explanation could further aid our above claims. When the intermittent measurement occurred, the updated state covariance becomes smaller than the previous state covariance. In similar means, $J_{k+1} > J_k$ or

$$J_{k+1} = (P_{k+1}^+)^{-1} > (P_k^+)^{-1} + H_k^+ R_k^- H_k \quad (15)$$

Unfortunately, this is not convincingly true as it shows contrast characteristics compared to the resulted performance. Referring to EKF algorithm, if the measurement data is not arriving, then $P_{k+1} = P_k$ which consequently denying the expected result exhibit by (15).

Driven by the above explanations on how the state covariance eventually ends, it is necessary to identify the statistical bounds. This paper suggests that, if measurement data are missing unexpectedly in a period of time, then the updated state covariance is bounded into a range of statistical bounds i.e upper and lower bounds. Moreover, interestingly, the state covariance is approximating the lower bound instead of the upper bound. Previous results[18] are referred for references. Some of them are included again to ease references.

Lemma 4[18] *Given P_o, Q_k, $R_k>0$. If a measurement data is missing in the interval of $1<k<N$ ($2<N<\infty$), then the FIM lower bound $\underline{J}_{k+1}$ and upper bound $\bar{J}_{k+1}$ becomes as*

$$\bar{J}_{k+1} = \rho_k^{-1} + \rho_k^{-1} Q_k (\rho_k + Q_k)^{-1} Q_k \rho_k^{-1} > 0$$
$$\underline{J}_{k+1} = \rho_k^{-1} - \rho_k^{-1} Q_k \rho_k^{-1} > 0$$

Theorem 1[18] *Assume that* $f_{\omega v}\Sigma_k f_{\omega v}^T$ *,* $P_o>0$ *and Assumption 1 are satisfied. If a measurement data is not arrived at any k+1, 1<k+1<N, then state error covariance* P_{k+1} *is bounded to the following if and only if* $\rho_{k+1} > \underline{Q}_{k+1}$.

$$\underline{P}_{k+1} \leq P_{k+1} \leq \bar{P}_{k+1}$$

such that

$$\underline{P}_{k+1} = \rho_k - Q_k(\rho_k + Q_k + Q_k\rho^{-1}Q_k)^{-1}Q_k$$
$$\bar{P}_{k+1} = \rho_k + (Q_k^{-1} - \rho_k^{-1})^{-1}$$

Our analysis show that the updated state error covariance going to the lower bound instead of upper bound. More interestingly, the result also shows that statistical bound proposed by above theorems can define opposite state i.e the upper bound becomes the lower bound and vice versa. This condition can be best described by referring to the FIM matrix in (11).

Above results can be further examined by considering (9) in identifying the conditions when measurement data are missing. As presented in [18], the state covariance suddenly reached the lower bound of state covariance. It is known that the state covariance must be positive semi definite at all time to preserve a reliable estimation. Then the previous state covariance before measurement data are missing must be a positive semi definite matrix i.e $P_k \geq 0$. As the measurement data are not available, it can be presume that the mobile robot sensors did not sense any objects nearby. Such that, the measurement matrix H_k during intermittent measurement is apparently exhibits higher relative distance and angle measurements than the H_k without any data losses.

Remark that some conditions can deteriorate our proposed *Theorem 1*. At time *k+1*, if measurement data are lost, then the landmark could be nearer or the mobile robot is leaving further the landmark. *Theorem 1* explains that if a mobile robot at time *k* finds a landmark which is far to it. Suppose that the mobile robot is approximating the landmark, then the measurement matrix becomes smaller such that $H_k > H_{k+1}$. The next theorem is presented to propose the outcomes.

Theorem 1 Assume that the R_k is bounded. The updated state covariance reaches the lower bound of state covariance matrix when measurement data is missing during mobile robot observations if and only if the measurement matrix satisfies the following.

$$H_{k+1} > H_k \tag{16}$$

However, if (17) is satisfied, then the updated state covariance shows the upper bound of state covariance.

$$H_k > H_{k+1} \tag{17}$$

Proof :

A numerical example could give better picture to simulate the actual results. Assume P_k , H_k are positive semi definite matrix and satisfies (16). If (9) is considered again, then

$$P_{k+1}^+ = P_{k+1}^- - K_{k+1}H_iP_{k+1}^- < P_k^- - K_kH_iP_k^-$$

which analogously shows that

$$J_{k+1} = (P_{k+1}^+)^{-1} + H_{k+1}^+R_{k+1}^-H_{k+1} > (P_k^+)^{-1} + H_k^+R_k^-H_k$$

Given above, it is clearly presented that as the FIM consumer a lot of information, then the updated state covariance becomes smaller than the previous condition. The same procedure can be applied to determine the condition if (17) is recognized during observation. Hence, the updated state covariance can exhibit both conditions whether going to the lower bound or otherwise which are depends exactly on (16) and (17).

□

The next section presents the simulation results to analyze our proposed *Theorem 1* stated above. Different movements are observed to differentiate the performance between each case.

4. Simulation and Discussion

Our preliminary simulation results have shown that most of the time, the updated state error covariance was approximating the lower bound of state error covariance. The reason relies on the basis that the FIM attempts to fully use its information obtained from observations as it is the only information available during mobile robot observations.

Table 1 shows the simulation settings that we define for our environment conditions.

Table 1 Simulation settings

Simulation variables	Settings
Sampling Time, *T*	0.1[s]
Process Noise, *Q*	1 X 10-6
Observation noise, R_θ, $R_{distance}$	R_θ=0.002, $R_{distance}$=0.02
Mobile Robot initial covariance, P_{vv}	1 X 102
Landmarks initial covariance, P_{mm}	100

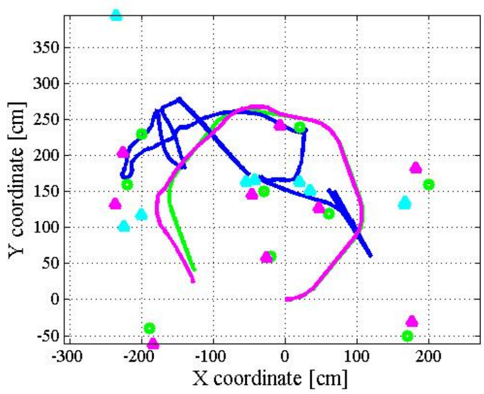

Fig. 3 Case 1: Mobile robot estimation and the constructed map based on the mobile robot observations. Green, magenta plots are showing the true estimation and estimation with EKF without losing any measurement data respectively. Blue and cyan plots are illustrating the estimation when mobile robot lost its measurement data

at 100[s], 400[s] for 1[s] duration while at 800[s] for 10[s].

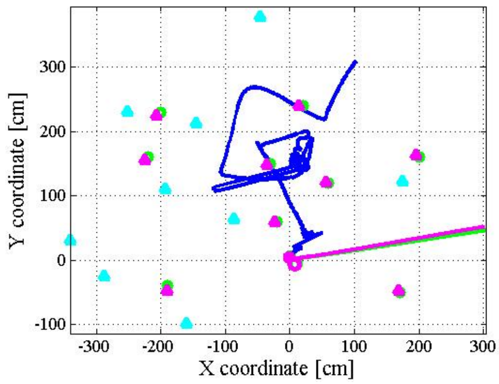

Fig. 4 Case 2: Mobile robot estimation and the constructed map based on the mobile robot observations with different movements. (Simulation settings configured for Fig. 3 are applied).

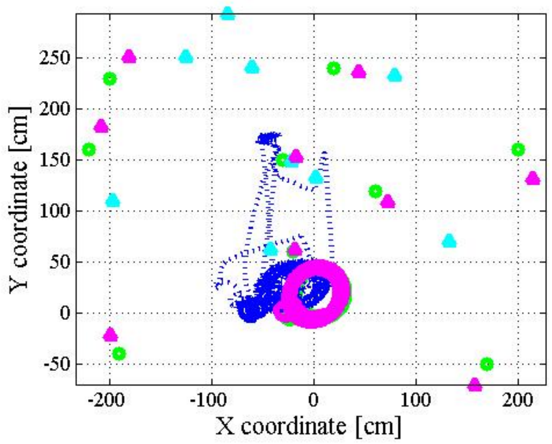

Fig. 5 Case 3: Mobile robot estimation and the constructed map based on the mobile robot observations with again different movements. The same simulation settings configured for Fig. 3 are applied to analyze the performance.

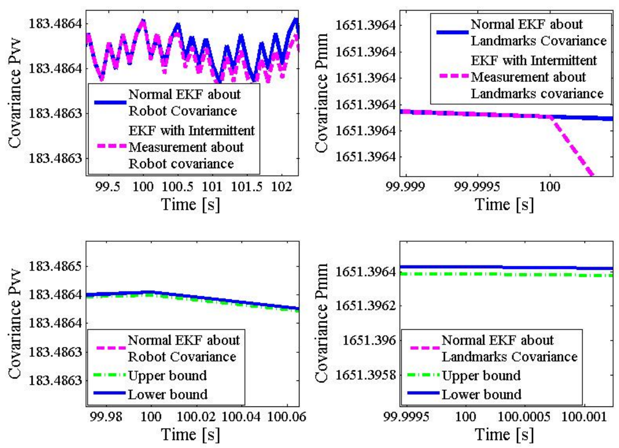

Fig. 6 Mobile robot and landmarks covariance characteristics for case 1.

The simulations are organized to loss measurement data at 100[s], 400[s] each for 1[s] and at 800[s] for 10[s].

Fig. 3, Fig.4, and Fig.5 shows estimation profiles for different mobile robot movements that consider same cases of true positions, as well as integrating the investigation of EKF estimation without information loss and EKF estimation with intermittent measurement. As presented in those figures, the estimation becomes erroneous as expected for EKF with intermittent measurement. To understand how effective the measurement made by mobile robot during its movement, state covariance is analyzed at each update. Fig.6, Fig.7, and Fig.8 describes the outcome. At 100[s], both mobile robot and landmarks state covariance exhibits lower or upper state covariance when intermittent measurement occurs. These profiles significantly defines and explains clearly what we have proposed in the section before regarding the state covariance characteristics that can exhibit either bigger or smaller state covariance than the state covariance in normal observations without any data losses. The rate of divergence, however, is not similar and unpredictable as it depends on the measurement matrix H_k.

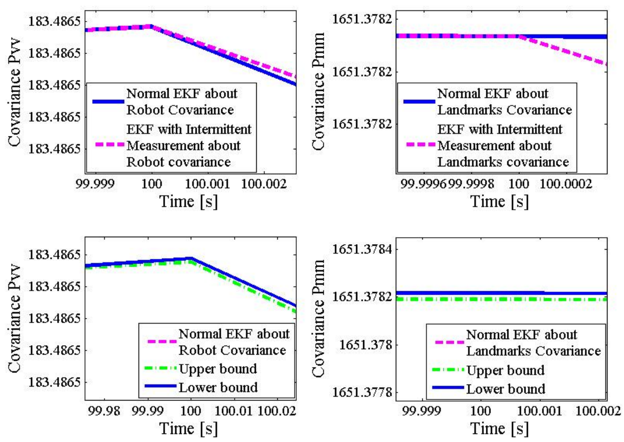

Fig. 7 Mobile robot and landmarks covariance results for case 2.

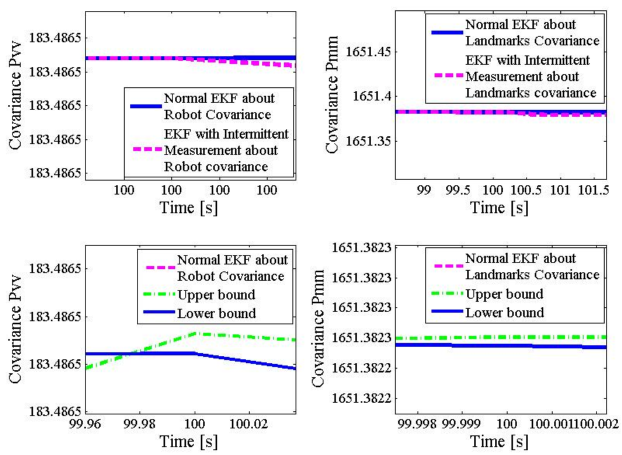

Fig. 8 Mobile robot and landmarks covariance results for case 3.

Analyzing further Fig.6 to Fig.8, readers can identify that there some inconsistency of the statistical bounds especially presented in Fig.5 and Fig.6. These figures determine that the upper and lower bounds are defining opposite means. Closely looking into the Theorem 1 suggested in this paper previously, and by incorporating theorems from literature reviews[18], the reason is actually relies on the behavior of H_k profile. Nevertheless, as demonstrated from Fig.6 to Fig.8, it is also can be easily recognized that the state covariance never exceed its given statistical bounds.

5. Conclusion

Results of theoretical and simulation analysis of intermittent measurement in EKF-based mobile robot have been reported in detail. Using almost the same analysis and simulation settings reported in Ahmad H. et al. [18], validation of theoretical analysis and simulations using EKF has been carried out. The prediction of the updated states are not actually relatively close to the true positions as some times the mobile robot gets to overconfident about its estimation. This means that the researchers must not only relying on the state covariance update without deeper investigations on how the estimation really becomes especially when measurement data are not arriving in a longer period of time. Theoretical analysis also proposed that the measurement matrix H_k effects the performance of EKF based SLAM as well as the statistical bounds. Currently this is unavoidable as the measurement matrix H_k cannot be easily controlled; as it is function to consistently measure the relative angle and distance between mobile robot and landmarks observed.

References

[1] Dissanayake M. W. M. G, Newman P., Clark S., Durrant-Whyte H.F, Csorba M., A solution to the Simultaneous Localization and Map Building(SLAM) Problem", *IEEE Trans. on Robotics and Automation*, Vol. 17, No. 3, (2001), pp. 229-241.

[2] Huang S., Dissayanake G., Convergence and consistency Analysis for Extended Kalman Filter Based SLAM, *IEEE Transaction on Robotics*, Vol. 23, No. 5, (2007), pp. 1036-1049.

[3] Ahmad H., Namerikawa T., Robotic Mapping and Localization Considering Unknown Noise Statistics, *Journal of System Design and Dynamics,* Vol.5, No.1, (2011), pp. 70-82.

[4] Ahmad H., Namerikawa T., Feasibility Study of Partial Observability in H infinity filtering for Robot Localization and Mapping Problem, *American Control Conference*(ACC2010), (2010), pp. 3980-3985.

[5] Ahmad H., Namerikawa T., Robot Localization and Mapping Problem with Unknown Noise Characteristics, *2010 IEEE Multi-conference on Systems and Control*, (2010), pp. 1275-1280.

[6] Thrun S., Burgard W., and Fox D, Probabilistic Robotics. MIT Press, Cambridge, (2005), MA.

[7] Nahi N.E., Optimal Recursice Estimation With Uncertain Observation, *IEEE Transactions on Information Theory,* Vol.15, No.4, (1969), pp. 457–462.

[8] Lynch P.M, Figueroa J.F., Position Estimation with Intermittent Measurements*, American Control Conference*, (1991), pp.2280–2285.

[9] Bingham B., Predicting the Navigation Performance of Underwater Vehicles, *IEEE/RSJ Intelligent Robots and Systems*, (2009), pp. 261-266.

[10] Sinopoli Schenato B., L., Franceschetti M., Poolla K., Jordan M.I, Sastry S.S, Kalman Filtering With intermittent measurements, *IEEE Trans. on Automatic Control*, Vol. 49, Issue 9, (2004), pp. 1453-1464.

[11] Plarre K., Bullo F., On Kalman Filtering for Detectable Systems With intermittent measurements, *IEEE Trans. on Automatic Control*, Vol. 54, No. 2, (2009), pp. 386-390.

[12] Kluge S., Reif K., Brokate M., Stochastic Stability of the Extended Kalman Filter with Intermittent Observations, *IEEE Trans. on Automatic Control*, Vol. 55, No. 2, (2010), pp. 514-518.

[13] Bar-Shalom Y., Li X. R., Kirubarajan T., Estimation with Applications to Tracking and Navigation, John Wiley and Sons Inc., (2001).

[14] Andrade-Cetto J., Sanfeliu A., Environment Learning for Indoor Mobile Robots; A Stochastic State Estimation Approach to Simultaneous Localization and Map Building, *Springer tracts in advanced robotics*, Springer, (2006).

[15] Vidal-Calleja T., Andrade-Cetto J., Sanfeliu A., Conditions for Suboptimal filter Stability in SLAM, *Proc. of 2004 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, (2004), pp. 771-777.

[16] Payeur P., Dealing with Uncertain Measurements in Virtual Representations for Robot Guidance, *IEEE International Symposium on Virtual and Intelligent Measurement Systems*, Vol. 14, No. 12, (2008), pp. 2151-2158.

[17] Muraca P., Pugliese P., Rocca G., An Extended Kalman Filter for the state estimation of a mobile robot from intermittent measurements, *16th Mediterranean Conf. on Control and Automation*, (2008), pp. 1850-1855.

[18] Ahmad H., Namerikawa T., Intermittent Measurement in Robotic Localization and Mapping with FIM Statistical Bound", *IEEJ Transaction on Electronics, Information and Systems*, Section C, Vol.131, No.5, (2011), pp.1223-1232.

[19] Ahmad H., Namerikawa T, H_∞ Filter-SLAM: A Sufficient Condition for Estimation, *18^{th} World Congress of the International Federation of Automatic Control (IFAC)*, Milan, Italy, August 28-Sept. 2 2011, (2011).