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Mathematical Modelling Concerning Compressibility of Air In Porosity During Semi-Dry Pressing Process Of Ceramic Powder

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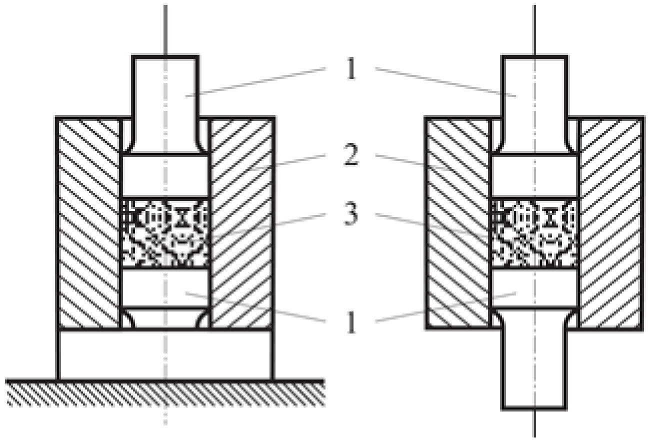
Abstract

In this article, during the pressing of a cylindrical ceramic product from ceramic powder masses with a moisture content of 8...12%, the powder mass is considered as a model consisting of two: elastic (gas-air) and incompressible (mineral particle, water) phases. Based on this, a mathematical model was developed that depends on the initial properties of the powder mass, the allowable residual porosity in the structure of the ceramic product, and the displacement value of the punch during pressing during semi-dry pressing of the ceramic powder mass. This mathematical model allows for analytical calculations of gas-air compression level in powder pores, compressed air pressure, pressing force and pressure.

1. Introduction

The use of structural ceramic materials (parts) in operating conditions with high temperature, abrasive and aggressive environment allows to increase the service life of machines and mechanisms and reduce their production cost [1]. Physico-mechanical properties of ceramic materials depend on their chemical and structural phase composition along with the technology of detail preparation. Usually, the widely used technology for obtaining machine parts from ceramic construction materials consists of the following operations: Semi-dry pressing of ceramic powder raw material to obtain a press briquette; sintering a press briquette at high temperatures. Semi-dry pressing of ceramic powders is a method of pressing mineraloceramic powders with a moisture content of 7% to 12% in closed press-molds under the influence of mechanical loading and obtaining pressbriquettes. The essence of this method is as follows: First, slightly moistened mineral powder is poured into the matrix of the press mold, then it is unilateral or bilateral pressed with a punch (Fig. 1).



(a) **(b)**

Fig. 1 *Scheme of semi-dry pressing of mineralceramic powders: (a) Unilateral pressing; (b) Bilateral pressing; 1 – punch; 2 – matrix; 3 – mineralceramic powders*

When the powder is pressed, the initial structure is rearranged by compression and complex processes with mechanical and molecular character take place in it, as a result, the initial volume of the powder decreases and forms a pressbriquette with the properties of a solid body [2]. In the initial period of pressing, i.e. at relatively low pressures, the particles that make up the powder begin to densify by moving freely relative to each other and relative to the matrix wall. If we consider ceramic powder as a system consisting of three phases: solid (mineral), liquid (water) and gas (air), at this stage, a certain part of the gas phase in the powder begins to escape to the atmosphere through the gaps between the powder particles and the volume of the powder decreases sharply, then the density increases sharply, and the surface of the mineral particles is covered by a film of water.

At the next stage, when the pressing pressure increases sharply, the reduction of the powder volume decreases sharply. The interparticle gaps in the powder layer close to the matrix wall are closed by the water contained in the powder. As a result, a certain amount of gas-air in the powder begins to compress inside the pressbriquette. If the pressing of the powder is continued further (or over-pressing occurs), the gas-air is compressed to such an extent that an expansion stress is created in the press briquette. This, in turn, creates cracks perpendicular to the pressing direction when the press briquette is removed from the matrix. Over-pressing of powder occurs when pressing is carried out at pressures higher than the critical pressure of pressing. Therefore, preliminary determination of the critical pressure is important in the production of ceramic products.

2. Literature Review

Based on the experimental results, the researchers usually expressed the empirical dependences of the densification process in the pressing of powder products in the form of logarithmic, degree, exponential and other such mathematical equations [3]. Logarithmic equations are mainly used in the theory of static pressing of metal and non-metallic powder products. The logarithmic equation obtained on the basis of the theory of interparticle connection discrete media for the pressing process of ceramic material powders was first developed by A.S. Berezhny in 1947:

$$\Pi = a - b \cdot lgP \quad (1)$$

Where, Π – porosity of press briquette; a and b – pressing coefficients; P – pressing pressure. Equation (1) has been thoroughly analyzed by Popilski, R.P. who is a contemporary expert in the field of pressing ceramic powder materials. According to the analysis, the values of a and b - pressing coefficients in equation (1) can be experimentally determined from the porosities P_1 and P_2 - achieved at pressing pressures R_1 and R_2 - in the form of the following equations:

$$a = P_1 + \left(\frac{lgP_1}{lgP_1/P_2}\right) \cdot (\Pi_1 - \Pi_2) \quad (2)$$

$$b = \frac{\Pi_1 - \Pi_2}{lgP_2/P_1} \quad (3)$$

When pressing the powders of ceramics, metals, and metal-based alloys, along with logarithmic equations, degree equations are also widely used to express the densification law of press briquettes. Balandin, P.P. was one of the first to present the following exponential equation for the densification process during pressing of ceramic powders:

$$\Delta = \frac{H}{\eta}[1 - \exp(-aP^n)] \tag{4}$$

Where, Δ – deposition of powder; *N* – the height of the powder placed in the press mold; *R* – pressing pressure; *a, η* and *n* – coefficients. Based on the results of experimental tests, Kunin, N.F. and Yurchenko, B.Y. showed that it is possible to express the process of densification of powder through K (pressing coefficient) with the following exponential equation:

$$K = \frac{d\gamma}{dp} = K_0 e^{-aP} \tag{5}$$

Where, K_0 – initial pressing coefficient (when the pressing pressure is equal to P = 0); *a* – coefficient of loss of compressibility of powder, that is, it explains the relative decrease of the *K*-pressing coefficient with an increase in *r*-pressing pressure by one unit; *γ* – density of press briquettes at *r* - pressure of pressing. Integrating the equation (5) gives the following equation of pressing:

$$\gamma = \gamma_{c.f.d} - \left(\frac{K_0}{a}\right)e^{-ap} \tag{6}$$

Where, $\gamma_{c.f.d}$ – conditional final density when pressing the powder at very high pressure, it may be slightly greater than the theoretical density of the powder material. The given equation (6) was used to take into account the change of the pressing curve depending on the speed of movement of the punch. All K_0, *α* and $\gamma_{c.f.d}$ coefficients representing powder compressibility in the equations given above by Kunin, N.F. and Yurchenko, B.Y. should be determined experimentally. In addition to the equations considered above, scientists have proposed many other types of pressing equations. For example, based on theoretical and experimental studies, Jdanovich, G.M. created an equation that reflects the compaction of powders by ideal pressing (without considering the effect of external friction):

$$p = p_k \frac{(\vartheta^n - \vartheta_0^n)}{(1 - \vartheta_0^n)} = p_k \frac{(\beta_0^n - \beta^n)}{[\beta^n(\beta_0^n - 1)]} \tag{7}$$

Where, ϑ_0 – relative density of the powder material when the pressing pressure (p) is equal to zero; *β* – relative volume of the pressbriquette; β_0 – relative volume of the powder material when the pressing pressure (p) is equal to zero. The value of the degree (n) in the equation depends on the mechanical properties of the powder particle material (σ_{ok}, σ_v, r_k), the coefficient of interparticle friction, the initial relative density of the powder (ϑ_0), and its "integrated" average value is approximately expressed as follows:

$$n = 1 + \frac{2}{\Pi_0} \tag{8}$$

Where, P_0 – relative porosity of the powder. At the same time, G.M. Zhdanovich gave the following relationship between the pressing pressures of powders under ideal and real conditions. For unilateral pressing method,

$$p_x = p_u(1 + 0.5A\beta) \tag{9}$$

For bilateral pressing method,

$$p_x = p_u(1 + 0.25A\beta) \tag{10}$$

Where, r_x – pressure required for actual pressing of powders; r_i –pressure required for ideal pressing of powders; *A* – constant value; *β* – relative volume of press briquette. G.M. Jdanovich gave the following equation depending on the conditions of powder pressing to determine the constant value (A):

$$A = \xi f(L_o - L_i)\frac{h_k}{S_n} \quad (11)$$

Where, L_o and L_i – the outer and inner contour length of the press briquette, respectively (if there is a hole in the center of the part being pressed); f – coefficient of friction of the powder particle with the wall of the press mold; ξ – pressure acting on the wall of the press mold; S_n – the surface of the nominal section of the press-briquette. Assuming that as a result of pressing the powders under high pressure, the interparticle spaces (porosity) in the powders will completely disappear, Nikolaev, A.N. proposed the following equation, which is similar to the equation that describes the process of pressure treatment of metals:

$$p = C\sigma_{y.s.}\vartheta \ln\left[\frac{\vartheta}{(1-\vartheta)}\right] \quad (12)$$

Where, C – experimentally determined coefficient; $\sigma_{y.s.}$ – yield strength of the powder material; ϑ – the relative density of the powder material. A.N. Nikolaev's equation (12) has no physical meaning for the initial and marginal critical conditional periods of pressing, because according to the equation, when the pressing pressure is r = 0, the relative density of the powder material is ϑ= 0,5, however, the relative density of most metal powders is ϑ< 0,5. At the same time, the relative density $\vartheta \rightarrow 1$, pressing value $p \rightarrow \infty$, and in real conditions, the pressing pressure tends to the critical pressure ($p \rightarrow p_c$). Konopitsky, K. (Austria) proposed the following logarithmic equation for the compression process of powders:

$$p = A\ln\left(\frac{\Pi_0}{\Pi}\right) \quad (13)$$

Where, A – constant; Π – the current relative porosity of the pressbriquette; Π_0 – initial (powder) relative porosity. Torre, C. (Germany) approved the equation (13) derived by Konopitsky and obtained the following similar equation based on his experimental results:

$$p = 2\sigma_{y.s.}\ln\left(\frac{1}{\Pi}\right) + C \quad (14)$$

Where, $\sigma_{y.s.}$ – yield strength of a powder particle; C – constant. Torre, C. expressed the constant (C) in this equation according to the initial conditions of the powder pressing process $p_0 = 0$, $\Pi = \Pi_0$, as follows:

$$C = 2\sigma_{y.s.} \cdot \ln(\Pi_0) \quad (15)$$

And substituting it into the equation (14), he found that $A = 2\sigma_{y.s.}$ in Konopitsky equation (13). World scientists named equation (14) as "Konipitsky-Torre equation". In 2001, Brazilian researchers Panelli, R. and Filo, F.A. proposed a phenomenological equation for compaction of metal powders and mineral salts [4]. The authors concluded that the relationship between the pressing pressure and the (ε) porosity of the press briquettes can be expressed by a differential equation based on the results of the analysis of the dependence curves obtained during the pressing of the powders of these materials:

$$\frac{d\varepsilon}{dP} = k_4\frac{\varepsilon}{\sqrt{P}} \quad (16)$$

Where, ε – porosity of the press briquette; k – proportionality coefficient; P – pressing pressure. The solution of this equation leads to the corresponding Panelli-Filo equation:

$$\ln\left(\frac{1}{1-D}\right) = a\sqrt{P} + \ln\left(\frac{1}{1-D_0}\right) \quad (17)$$

Where, $1 - D = \varepsilon$ – porosity of the press briquette; a – constant taking into account the plastic deformation of the powder; D_0 – relative density of the pressbriquette. The Panelli-Filo equation, like the equation of the scientists mentioned above, is that the pressing pressure (P) tends to an infinitely large value in the production of press-briquettes that do not have absolute porosity. Modern pressing equations include the differential equation developed by R. D. Ge in 1995 for pressing powders of high hardness materials (titanium and others).

$$\frac{dD}{dP} = k\frac{(1-D)D^n}{P^m} \quad (18)$$

Where, n and m – constants introduced by the author to express the dependence of pressing pressure on the degree of densification of the pressbriquette in a differential form. The expression of this equation after integration and a number of modifications by Ge, R.D. is as follows:

$$\log\left[\ln\left(\frac{1}{1-D}\right)\right] = a\log P + b \tag{19}$$

Where, a and b – constants that take into account the property of the powder material. The physical meaning of the constants in this equation is the same as in the Panelli-Filo equation. According to sources [5, 6], the density of powders during the pressing process is estimated by the coefficient K_{comp}. At the same time, this indicator is not an empirical coefficient, but a kinematic indicator. For example, when studying the compressibility of gases in physics, this indicator is called the degree of compressibility and is determined by the following formula:

$$\delta = \frac{H_g}{h} \tag{20}$$

Where, H_g – the height of the gas chamber; h - the size of the gas after final compression. If we apply this formula to the compaction of the powder, then the size of the height of the powder before pressing in the matrix (H_i) becomes the size of the final height of the pressbriquette (h), and δ is the degree of compaction of the powder.

If the degree of compression of the powder is high, for example $\delta \approx 2$, then there is very little space left for the gas phase in the powder, and the pressure of the compressed air is very large. At very high compression levels, the press briquette in the press mold behaves almost like an elastic medium. It is known that the elastic expansion of the press briquette after stopping the pressing pressure consists of the sum of the elastic expansions of mineral particles, gas-air phase and water. If we take into account the incompressibility of the water contained in the powder, the formation of cracks perpendicular to the pressing direction in the super-pressed press briquette is caused by the pressure of the mineral particles in the powder and the compressed air.

In practice, a multi-stage pressing procedure is used to avoid delamination and cracks in the ceramic pressbriquette [5]. The density of the press briquette depends on the degree of compression of the raw material and its density in free powder form and is expressed by the following formula:

$$p_c = p_k \delta \tag{21}$$

Where, ρ_c- pressbriquette density; ρ_k- density of the powder embedded in the matrix in free form. The effect of pressing pressure on the degree of compaction of powder can be determined from the empirical formula of Kazakevich, S.S.:

$$\delta = ap^n \tag{22}$$

Where, a and n – coefficients depending on the properties of raw materials, usually they are in the range of values 1.2...2.6 and 0.07...0.01 respectively; p – pressing pressure, *MPa*. In semi-dry pressing, the densification of ceramic powder is influenced by the pressing speed. The pressing speed is expressed by the speed of movement of the punch, the slower the punching powder is compressed, the more time is spent on the release of the gas-air phase from the pressbriquette, as a result of which the density of the pressbriquette increases and the value of the residual stress in the body decreases. [6]. According to the results of the research, semi-dry pressing of ceramic powders at a speed of 9 mm/second and pressures of 10...11 MPa leads to the formation of cracks in the body of the press briquette. In the research work [5], the following formula was proposed to determine the sediment value of the powder, poured into the matrix during pressing by the punch:

$$x_{max} = \left[\frac{H}{1.45}\left(1 - e^{-0.15p^{0.2}}\right) + 0.001\omega^4\right]\lambda\varphi \tag{23}$$

Where, ω – powder moisture, *%;* λ – coefficient taking into account the granulometric composition of the powder, when $\omega < 11.5\%$, λ is in the range of values 1.1…1.13; φ – coefficient depending on the plasticity of semi-dry or fully wetted powder raw materials, for raw materials with $\omega < 10.5\%$ moisture, $\varphi = 1.0$; $\varphi = 1.04$ for raw materials with high plasticity; H – the height of the powder poured in the matrix. The given formula (23) does not take into account the residual porosity in the finished product (Π - absolute or e_p - relative porosity), but takes into account the moisture content of the raw material.

In the theory of scientists such as Cramer, R. and Kaiser, A., the concept of the limiting density of the material is accepted [7]. The initial volume of the total gas-air phase in the powder poured in the matrix is expressed by the following formula:

$$V_{g0} = V_0\left(1 - \frac{\rho_0}{\rho_l}\right) \quad (24)$$

Where, V_0 – the volume of the powder in the matrix; ρ_0 – density of powder raw materials; ρ_l – the limiting density of the material. The total volume of the gas-air phase in the finished product (pressbriquette) is expressed by the following formula:

$$V_{g1} = V_1\left(1 - \frac{\rho_f}{\rho_l}\right) \quad (25)$$

Where, V_1 – finished product volume; ρ_f – final density of raw materials; ρ_l – the limiting density of the material. In the presence of these quantities, the degree of compaction of the powder is expressed as follows:

$$\delta = \frac{V_0}{V_1} = \frac{\rho_f}{\rho_0} \quad (26)$$

Based on this, the authors of the study [7] proposed a formula for determining the degree of gas-air compression in pores:

$$\delta_{\Pi} = \frac{V_{g0}}{V_{g1}} = \frac{1 - \rho_0/\rho_l}{1 - \rho_f/\rho_l} \quad (27)$$

Analysis of the given formulas shows that all formulas are considered empiric, because all of them have ρ_l (limiting density of powder) and it needs to be determined experimentally. In addition, the disadvantage of the formula is that they do not take into account the geometric and structural parameters of the pressing process.

3. Theoretical Part

In the research work carried out by the author [8], mathematical modeling of the degree of compressibility in the process of pressing the ceramic powder mass was carried out based on the fact that mineral particles in the powder and at the same time water behaves as a solid phase when the volume is compressed. This article uses the definitions, formulas and terms given in the research paper [8]. The current volume of the powder mass placed in the matrix can be qualitatively expressed as the total volume of solid mineral particles that make up the powder (V_1), including the volume of water in the powder (usually 8...12%) and the volume of air-filled pores between the powder particles (V_2):

$$V' = V_2 + V_1 \quad (28)$$

In order to facilitate mathematical operations, we express the formula (28) in the following relative quantities:

$$1 = \frac{V_1}{V'} + \frac{V_2}{V'} \quad (28a)$$

Then, according to [8], the relative porosity of the powder (θ) and the relative volume of the solid phase (v) can be expressed as:

$$\theta = \frac{V_1}{V'} \quad (29)$$

$$v = \frac{V_2}{V'} \quad (30)$$

From the given formulas (28), (28a), (29) and (30), we write the following:

$$\theta + v = 1 \quad (31)$$

The amount of relative final residual porosity (θ') in structural ceramic materials is allowed to be in the range of 0.005...0.02, depending on the level of loading affecting it [1]. As the amount of residual porosity decreases in the ceramic material, its physical and mechanical properties and quality indicators increase. Therefore, residual porosity in ceramic material can be considered as a predictable quantity. We assume that the ceramic powder is completely compacted without the release of water, which corresponds to the non-porous condition.

If we take into account that the volume of the solid phase in the powder (mineral particles and water) does not change during the pressing process, then the degree of air compression in the pores can be determined as the ratio of the initial volume of the porosity to its current volume:

$$\Omega_{air} = \frac{V_{por}}{V_1} = \frac{V_0 - V_2}{V' - V_2} \tag{32}$$

Where, Ω_{air} – degree of compressibility of the air in the porosity; V_{por} – the initial volume of the porosity in the powder; V_o – the initial volume of the powder in the matrix. By dividing the derivative and denominator of the expression (32) by the current volume of the powder, we get the following:

$$\Omega_{air} = \frac{\Omega - v}{1 - v} \tag{32a}$$

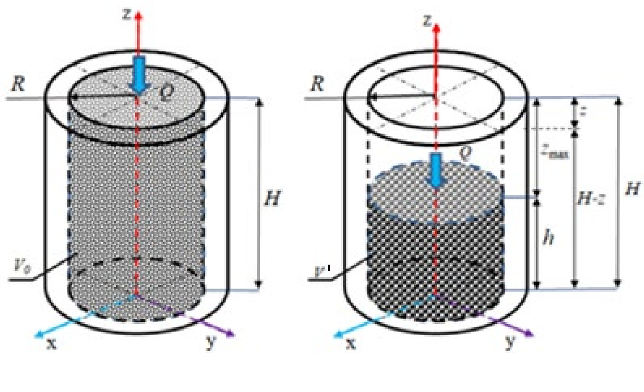
Where, Ω – the current degree of compaction of the powder mass. Taking (31) into account, we express (32a) as follows:

$$\Omega_{air} = \frac{\Omega - 1 + \theta}{1 - \theta} \tag{32b}$$

The formula (32b) is an analytical expression that shows the obvious nonlinear change of the degree of air compression in the pores of the powder in the matrix, depending on the value of the displacement of the punch. If we put the current compression level of the powder $\left(\Omega = \frac{H}{H-z}\right)$ and the current relative porosity when the powder is compressed $\left(\theta = 1 - \frac{1}{\Omega'}(1 - \theta')\frac{L}{L-x}\right)$ into the expression (32b) given in [8], then we get the basic kinematic equation of the pressing process:

$$\Omega_{air} = \frac{1 - \frac{1 - \theta'}{\Omega'}}{\frac{H - z}{H} - \frac{1 - \theta'}{\Omega'}} \tag{33}$$

Where, Ω' – the degree of final compaction of the powder. The obtained equation (33) allows to determine the degree of air compression in the powder pores while the mass of ceramic powder with height H inside the matrix depend on the current displacement value of the punch (z) (Fig. 2).



(a) **(b)**

Fig. 2 *Geometric parameters of the ceramic mass pressing process [8]: a – before pressing; b – after pressing; H – powder height; h- the final height of the pressbriquette; zs – inner radius of the die; z –current displacement of the punch; zmax – maximum displacement value of the punch; V0 – the initial volume of the powder in the matrix; V'– current volume of the powder*

A ceramic powder with a final compression degree of $\Omega' = 2$ is shown in Fig. 2: For the case *H = 67,7 mm;* z_{max} *= 33,7 mm; R = 10,0 mm; h = 34 mm*, according to the equation (33), the change in the degree of air compression in the pores depending on the displacement value of the punch (z) is shown in Fig. 3.

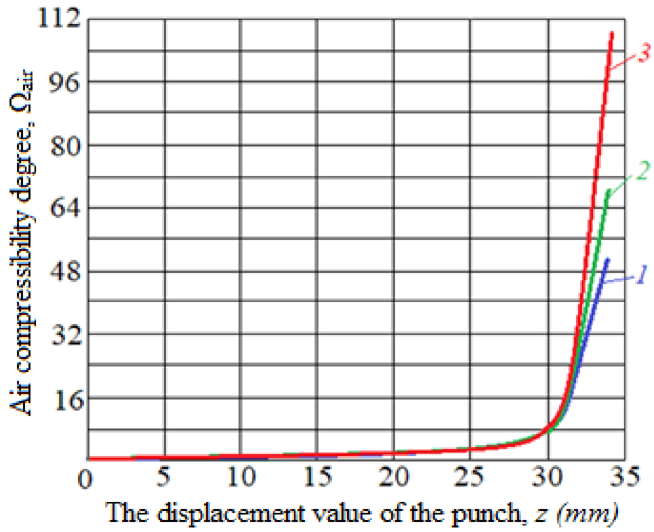


Fig. 3 *The graph of the change in the degree of air compression in the pores depending on the value of punch's displacement (z): 1 - θ' = 0,005; 2 - θ' = 0,01; 3 - θ' = 0,02*

If the degree of compressibility of air in the porosity is known, then it is possible to determine the pressure of the compressed air and its temperature using the formula for the polytropic compressibility of gases [4]:

$$p_{air} = p_{n.air} \cdot \Omega_{air}^{n_1} \quad (34)$$

$$T_{air} = T_{n.air} \cdot \Omega_{air}^{n_1 - 1} \quad (35)$$

Where, $p_{n.air}$ – normal initial air pressure (101325 Pa); $T_{n.air}$ – initial air temperature, ^oK; n_1 – polytropic degree of compressed air in porosity. The mass of the pressed powder contains very little air mass, so its temperature change is not noticeable, and we consider the compression of air to be close to an isothermal process, then it can be assumed that $n_1 \to 1$. The resistance force exerted on the punch by the pressure generated by the compression of the air in the pores of the powder can be determined from the following formula:

$$Q_{air} = F_{S.P.} \cdot p_{air} \quad (36)$$

Where, $F_{S.P.}$ – the surface of the punch, the surface of the punch for the pressing process shown in Fig. 2 are $F_{S.P.} = \pi R^2$. For the case seen above, the final residual porosity (θ') is 0.02; When the polytropic degree is n = 1.0 and n = 1.15, according to the formula (34), the change in the pressure of compressed air (p_{air}) as a function of the value of the current displacement of the punch (z) is shown in Fig. 4.

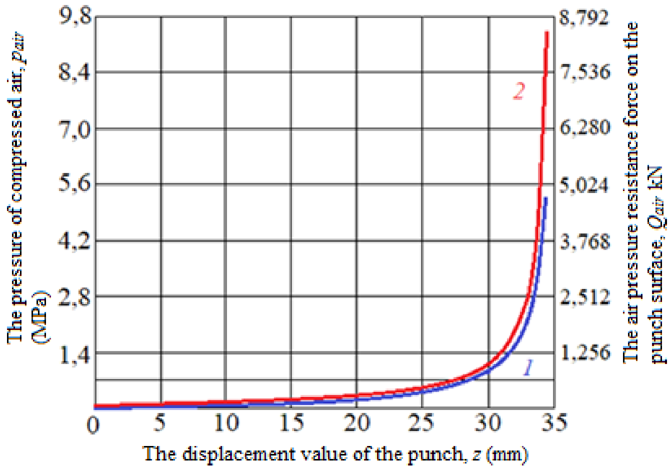


Fig. 4 *A graph of the change in pressure of compressed air depending on the displacement value of the punch, θ'= 0,02: 1 – n = 1,0; 2 – n = 1,15*

According to most scientists, a certain (up to 30-50%) part of the air in the powder comes out of the slits of the die and punch wall during the pressing process [5, 6, 9]. The compression of air from the powder mass depends on the pressing pressure and the gas permeability of the system, where the gas permeability of the system depends on the moisture content of the powder and decreases sharply with the increase in the degree of compression.

At the end of the pressing process, when the pressing force increases to the maximum level, it is reasonable to assume that the pressed powder will have the properties of a solid body. Then we accept the hypothesis that the law of dependence of longitudinal and transverse deformations known for solid bodies can be applied to the mass of pressed powder in the matrix. Considering that the maximum pressing force generated at the last stage of pressing is important, the accepted hypothesis is reasonable.

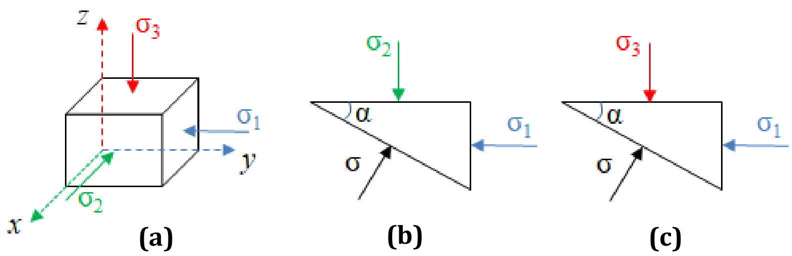


Fig. 5 *The state of volumetric stress at the points of the mass being pressed (a) The scheme of changing the volume stress state to the stress state in two horizontal; (b) And vertical; (c) Planes: σ1, σ2, σ3 – the principal stresses; σ, τ – normal and tangensial stresses*

The complex stress state at the points of the pressed body is the volumetric stress state [10] and it is characterized by the principal stresses σ_1, σ_2, σ_3. In the theory of compaction of powder masses [5, 6] and soil mechanics [11], many authors use the plane stress state model instead of the volumetric stress state. Such a simplified approach to solving the problem is appropriate if, based on the principle of independent movement of forces, we accept the hypothesis of replacing the volumetric stress state with the state of stress in two orthogonal planes (Fig. 5).

At the end of the process of pressing the powder mass, that is, when there are no tangensial stresses acting on the side wall of the matrix, transverse (lateral) stresses σ_2, σ_3 become principal stresses. In this case, the following mechanical equation can be used for the process of compression of powder masses in a matrix [11]:

$$\frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2} = \sin\alpha \tag{37}$$

It is possible to express the principal stress from equation (37):

$$\sigma_2 = \sigma_1 \frac{1 - \sin\alpha}{1 + \sin\alpha} = \sigma_1 k_b \tag{38}$$

Where, k_b – the lateral pressure coefficient acting on the inner wall of the matrix.

$$k_b = \frac{1 - \sin\alpha}{1 + \sin\alpha} \quad (39)$$

The angle of internal friction of ceramic powder (α) varies in the range of 11...23°, and the k_b value varies in the range of 0.68...0.44. The boundaries of variation of the lateral pressure coefficient calculated from the formula (39) correspond to the results obtained experimentally in the research work [5]. The formula (39) and the average value of the lateral pressure coefficient $k_b \approx 0.5$ show that, the force of air compression into the pores and the force of friction on the surface of the inner wall of the matrix and between the powder particles are the main components of the total pressing force in the punch:

$$Q = Q_{air} + Q_T \quad (40)$$

Where, Q – the total resistance force exerted on the punch; Q_{air} – the resistance force exerted on the punch as a result of compressing the air into the porosity; Q_f – the force of friction on the inner wall of the matrix and between the powder particles. We determine the total pressing resistance using the principle of independent movement of forces:

$$Q = Q_B + k_f Q_{air} = Q_{air}(1 + k_f) \quad (40a)$$

Where, k_f – a coefficient that takes into account the frictional forces during pressing, which establishes the relationship between the total pressing force in the punch and the force resulting from the air pressure in the pores. In order to facilitate theoretical research, we introduce general relative magnitudes. To do this, by dividing the coefficients of equation (40) by the surface of the punch, we obtain the expression of the total pressing pressure corresponding to the punch:

$$p = p_{air}(1 + k_f) \quad (41)$$

If we replace p_{air} in formula (41) by $p_{n.air} \cdot \Omega_{air}^{n_1}$ according to formula (34), then formula (41) will have the following form:

$$p = p_{n.air} \cdot \Omega_{air}^{n_1} \cdot (1 + k_f) \quad (41b)$$

Where, p – the average pressure corresponding to the punch, which reflects the air pressure in the pores and the friction on the surface of the inner wall of the matrix, *Pa*.

k_f – coefficient of friction force can be determined according to the formula (41b):

$$k_f = \frac{p}{p_{air}} - 1 \quad (42)$$

Where, p – the maximum pressing pressure at the end of the pressing process, which is determined experimentally; p_{air} – maximum calculated pressure of compressed air in pores. Then the total force on the punch surface is as follows:

$$Q = pF_{S.P.} \quad (43)$$

Unlike certain empirical formulas describing the powder pressing process [5, 6], The proposed formulas are a mathematical model, which allows to calculate the characteristics of pressing processes and to determine the degree of powder compaction, air in porosity, compressed air pressure, pressing pressure and power during powder pressing using an analytical method by introducing the experimentally determined coefficient of friction force for specific pressing conditions (k_f).

4. Experimental Methodology

"Angren" kaolin (Uzbekistan) powder was used to obtain ceramic pressbriquettes, which was crushed and then sifted through a sieve with a cell size of 1.5 mm. Kaolin powder was first dried at a temperature of 150 °C, 8.00% of water was mixed with the dried powder, and a total mass of 1 kg of powder was prepared, from the prepared powder, the powder with a mass of *m = 17.68 g* was measured on an electronic scale with an accuracy of *0.01* and was placed in the matrix. In this case, the initial height of *m = 17.68 g* of powder placed in a matrix with an inner radius of *R = 10.0 mm* was assumed to be *H=67.7 mm.* The koalin powder and the parts of the press mold prepared for its pressing are presented in Fig. 6.

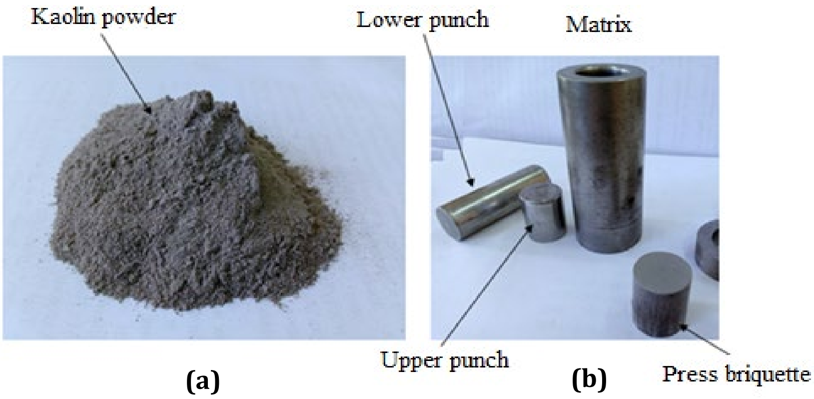


Fig. 6 *(a) The kaolin powder; and (b) The parts of the press mold*

The process of pressing the powder mass in a press mold was carried out in a "UTC-5727.FPR" hydropress (Turkey) with a maximum pressing force of 2000 kN (Fig. 7). The process of pressing the powder mass in a press mold in a hydropress was carried out in automatic control mode using a touch screen. For this, the automatic control touch screen of the hydropress was turned on, and data on the dimensions of the press briquettes was previously entered (Fig. 8.a), then the limit load values, the rate of load increase, the current and final heights of the punch were entered (Fig. 8.b), then the measurement units were selected for determining the results (Fig. 8.c).

The pressed briquettes were dried for 2 hours at a temperature of 70...80 °C and then sintered for 3 hours in a STA-1700 (Turkey) furnace at a temperature of 1300 °C. The amount of final residual porosity ($e_{r.p.}$) in the sintered ceramic samples was determined by photographing the surface of the grind using an Oxion metallographic microscope at a magnification of x200 to x600 and using the Image Focus Alpha computer software.

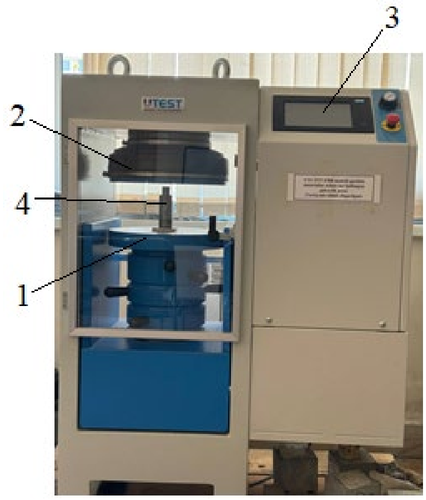


Fig. 7 *UTC-5727.FPR" (Turkey) model hydropress: 1 – base; 2 – top plate with fixed tensioner; 3 – automatic press control touch screen; 4 – press mold*

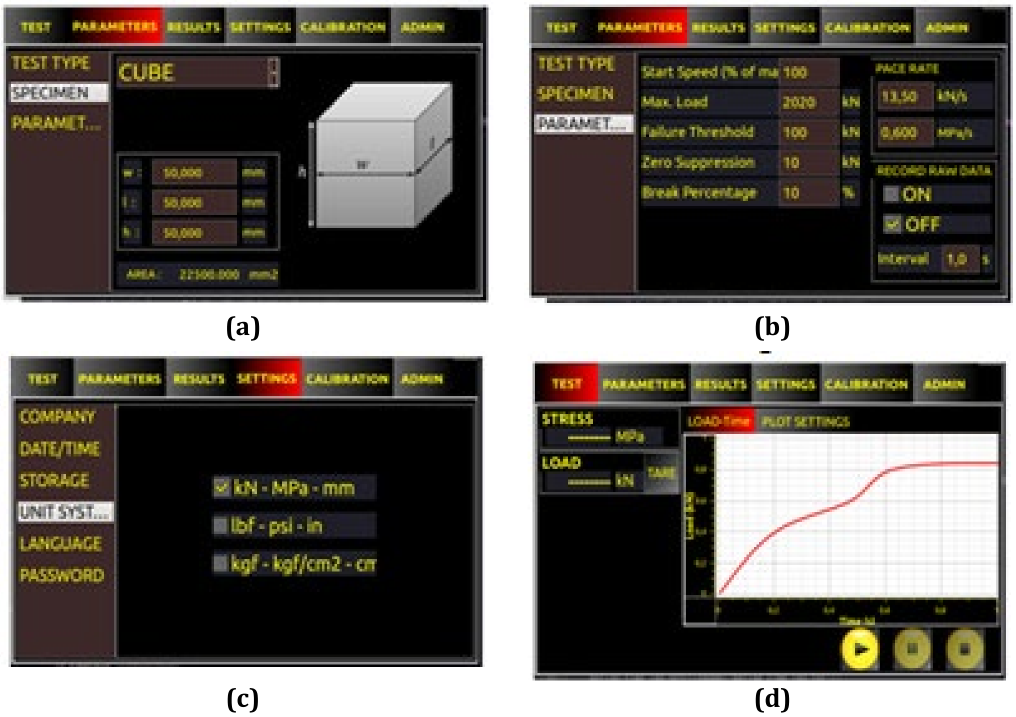


Fig. 8 *Information on the hydropress touch screen: a – enter the dimensions of the pressbriquette; б – setting the level of loads; c – choose a unit of measure; d – the results of pressing the powder mass*

The purpose of the experimental research is to justify the value of the final porosity of the ceramic product (θ'), the coefficient of friction force (k_f) and the values of other parameters of the pressing process in the obtained analytical expressions.

5. Experimental Results and Discussion

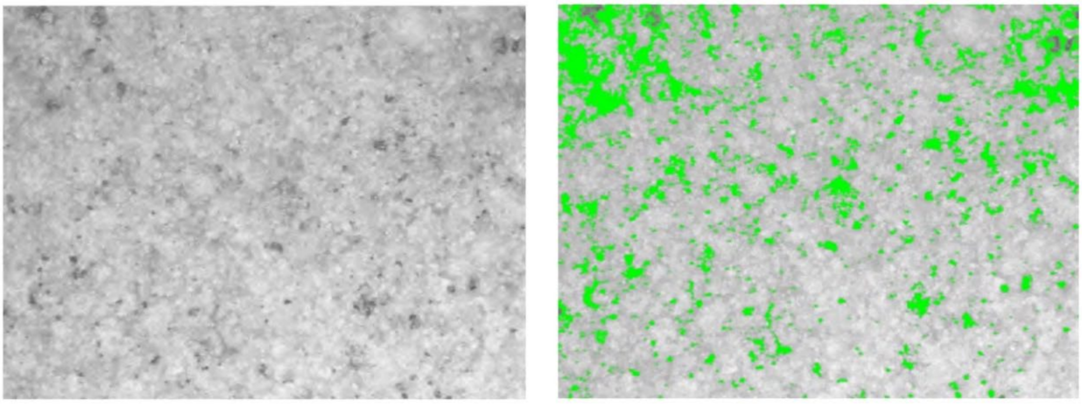
In the process of pressing the kaolin powder mass with 8% moisture content in the "UTC-5727.FPR" hydropress, on the touch screen, the displacement value of the punch (z) and the corresponding pressing force (Q), as well as the pressure on the surface of the punch (p), are shown in table 1. According to the experimental results, during semi-dry pressing of "Angren" kaolin (Uzbekistan) powder mass with 8% moisture content, the maximum displacement value of the punch stopped at 33.7 mm, in which the corresponding pressing force (on the hydropress touch screen) was 74,493 kN. A total of 5 samples were taken in this order. In this case, for a constant z_{max} = 33.7 mm, the average value of the pressing force (Q) was 77.015 kN.

The obtained press briquette samples were dried and sintered, and the external surfaces were inspected for defects. Grinds were prepared from the samples to determine the final residual porosity (θ') and The microstructure was photographed using an Oxion metallographic microscope at a magnification of x200 to x600 and The images were processed using the computer software "Image Focus Alpha" and the amount of final residual porosity (θ') was determined for each sample (Fig. 9). According to the microstructural analysis carried out on 5 grinds, the amount of residual porosity in the ceramic sample was found to be 0.027 on average. The results of determining the residual porosity in the ceramic sample using the "Image Focus Alpha" computer software are presented in Table 2.

Table 1 *The kaolin powder mass with 8% moisture content in the "UTC-5727.FPR" hydropress, on the touch screen, the displacement value of the punch (z) and pressing force (Q), as well as the pressing pressure (p)*

z, mm	Q, kN	p, MPa	z, мм	Q, kN	p, MPa
1	1,105	0,703	18	3,207	2,245
2	1,596	0,704	19	3,491	2,665
3	1,656	0,705	20	3,849	2,945
4	1,700	0,809	21	4,043	3,228
5	1,775	0,811	22	4,267	3,785
6	1,820	0,825	23	4,520	4,623

7	1,909	0,839	24	5,132	5,378
8	1,954	0,955	25	5,490	6,163
9	1,984	0,975	26	6,415	7,423
10	2,118	1,055	27	6,997	8,826
11	2,252	1,123	28	8,459	11,225
12	2,327	1,259	29	9,817	14,033
13	2,476	1,354	30	12,846	17,567
14	2,566	1,453	31	15,367	23,813
15	2,745	1,684	32	25,692	33,605
16	2,834	1,826	33	38,493	50,421
17	3,088	2,129	33,7	76,987	77,055



(a) **(b)**

Fig. 9 *Microstructure of a ceramic sample pressed at a pressing force of Q = 156 kN and sintered at 1300 oC: a – microstructure taken using an Oxion metallographic microscope; b - microstructure processed in a computer software "Image Alpha Focus"*

The average pressing pressure determined in the experiment (table 1) for z_{max} = 33.7 mm is 77.015 MPa and from the formula (33) for the case of $\theta' = 0{,}02$; $n_1 = 1{,}0$ $\Omega' = 2$, the theoretically determined maximum air pressure in the pores (p_{air} = 5.16 MPa) according to the formula (42), the friction force coefficient is as follows:

$$k_f = \frac{p}{p_{air}} - 1 = \frac{77.015}{5.16} - 1 = 13.92$$

Based on the determined coefficient of friction force (k_f), we calculated the air pressure in the pores (p) and the average pressure corresponding to the poinson, which reflects the friction on the surface of the inner wall of the matrix, according to the formula (41). The calculated results are presented in Table 3.

Table 2 *The residual porosity in the ceramic sample using the "Image Focus Alpha" computer software*

Index	Center	Perimeter	Area	Porosity, %
935	(3432.36;2520.00)	1649.43	10216.3	0.2
1305	(640.00;2512.30)	2744.45	13036.5	0.18
2001	(261.00;2480.50)	6012.32	40231.5	0.49
2350	(3347.00;2306.58)	3524.34	20399	0.31
3610	(3247.82;2120.00)	3650.71	44129	0.34
4160	(244.00;2192.40)	8361.61	123215	1.54
6320	(2387.62;630.12)	1002.38	12910	1.56
The rest	6840 objects	201934.3	763468.3	9.34
Total	6851 objects	229331.2	1010149	13.96

Table 3 *The average theoretical-experimental calculated pressure (p) and resistance force (Q) corresponding to the punch, which reflects the air pressure in the pores and the friction on the surface of the inner wall of the matrix*

z, mm	p, MPa	Q, kN	z, mm	p, MPa	Q, kN
1	1,105	0,329	18	3,207	1,006
2	1,596	0,501	19	3,491	1,096
3	1,656	0,519	20	3,849	1,208
4	1,700	0,533	21	4,043	1,269
5	1,775	0,557	22	4,267	1,339
6	1,820	0,571	23	4,520	1,419
7	1,909	0,599	24	5,132	1,611
8	1,954	0,613	25	5,490	1,723
9	1,984	0,622	26	6,415	2,014
10	2,118	0,665	27	6,997	2,197
11	2,252	0,707	28	8,459	2,656
12	2,327	0,730	29	9,817	3,082
13	2,476	0,777	30	12,846	4,033
14	2,566	0,805	31	15,367	4,825
15	2,745	0,861	32	25,692	8,067
16	2,834	0,889	33	38,493	12,086
17	3,088	0,960	33,7	76,987	24,173

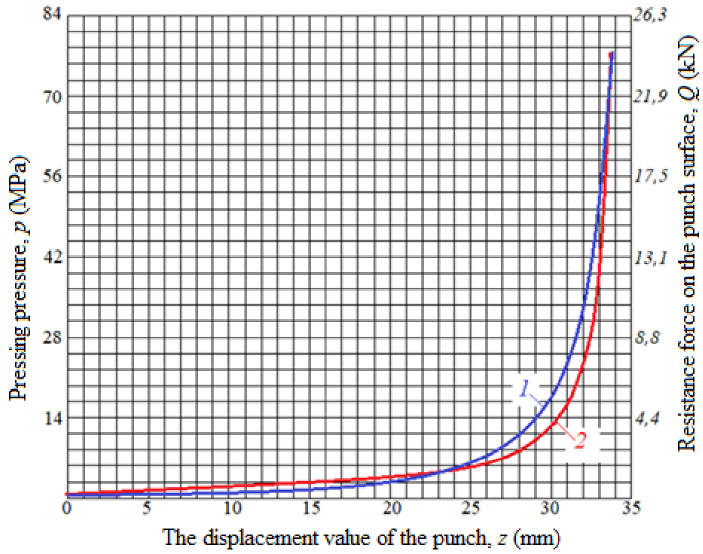


Fig. 10 *Graph of change of pressing pressure (p) and pressing force (Q) during semi-dry pressing of "Angren" kaolin powder as a function of poinson displacement value (z): 1 – experimental result; 2 – theoretical result*

Figure 10 shows the graph of the average pressing pressure (p) and pressing force (Q) depending on the displacement value of the punch (z). Curve 1 in the graph is made on the basis of the average value of the result obtained experimentally in 5 samples according to table 1, Curve 2 is obtained based on theoretical results based on Table 3.

From the graph presented in Figure 10, it can be seen that the theoretical results obtained in the displacement range of the punch from 0 mm to 23 mm (Figure 10, curve 2) are slightly larger than the average results obtained experimentally, but in the displacement range from 25 mm to 32 mm, they have a slightly smaller value. In our opinion, it is difficult to evenly distribute 8% moisture in 1 kg of powder mass when preparing "Angren" kaolin for pressing. Therefore, it is natural that the moisture content of 1 kg of powder mass varies by a certain amount, which causes the pressing parameters to differ in one way or another in the process of pressing the powder.

In general, experimental studies have shown that the mathematical model developed for the analytical determination of pressing pressure and force depending on the displacement value of the punch, including the degree of air compression in the pores of the ceramic powder mass, its pressure, and the coefficient of friction, for the process of pressing the ceramic powder mass, has shown itself to be adequate.

Conclusion

Accepting the mass of ceramic powder with a moisture content of 8...12% as a model consisting of compressible gas-air and incompressible solid (mineral particles and water) phases made it possible to develop a mathematical model of the ceramic powder mass pressing process. The developed mathematical model allows determining the degree of compression of the powder mass, the degree of compression of air in the pores, the pressure of compressed air, the total pressing force and pressure, depending on the displacement value of the punch. In this case, the degree of air compression in the pores (Ω_{air}) is a non-linear function of the displacement of the punch (z) and the parameters of the pressing process (H, θ', Ω'), the pressure of the compressed air (p_{air}) is a function of the degree of compression of the air, which depends on the physical parameters of the pressing process, such as the normal initial pressure of the air in the pores ($p_{n.air}$), the polytropic degree of the compressed air in the pores (n_1). The theoretical and experimental adequacy of the dependence of the pressing force (Q) on the punch displacement (z) is ensured by the coefficient of friction forces ($k_f=13.92$) and the value of the polytropic index ($n_1 = 1.0$) as well as the values of the relative final porosity of the ceramic product ($\theta'=0.02$).

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Shukhrat Shakirov, Begali Bektemirov;* ***data collection:*** *Shukhrat Shakirov, Begali Bektemirov, Sanobar Sadaddinova, Ulugbek Umirov, Mukhlisakhon Abdurakhmonova;* ***analysis and interpretation of results:*** *Begali Bektemirov, Kamoliddin Urokov, Zukhra Mirzarakhimova;* ***draft manuscript preparation:*** *Shukhrat Shakirov, Begali Bektemirov. All authors reviewed the results and approved the final version of the manuscript.*

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