INTERNATIONAL JOURNAL OF INTEGRATED ENGINEERING
ISSN: 2229-838X e-ISSN: 2600-7916

IJIE

Vol. 17 No. 1 (2025) 140-153
https://publisher.uthm.edu.my/ojs/index.php/ijie

Multi-Item Flow Shop Batch Scheduling with Two Stages Dedicated Machines to Minimize Total Actual Flow Time

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DOI: https://doi.org/10.30880/ijie.2025.17.01.012

Article Info

Received: 12 September 2024
Accepted: 6 March 2025
Available online: 30 April 2025

Keywords

Total actual flow time, batch scheduling, flowshop, dedicated machine

Abstract

A cigar manufacturing company produces two products using both shared and specialized machinery. There are 14 production stages, featuring distinct machines in Stage 2 and Stage 4. The company currently lacks a schedule, resulting in requests being fulfilled beyond the deadline. To address this issue, we developed a batch scheduling model for the flow shop, which prioritizes dedicated machines to reduce Total Actual Flow Time (TAFT). We selected the TAFT performance criteria to ensure timely completion and delivery of all orders, taking into account the maturity date. By implementing this model, the company aims to streamline operations, improve efficiency, and ultimately enhance customer satisfaction through the timely fulfillment of requests. The developed model integrates with an algorithm to address current challenges. The calculations conducted indicate that the developed model is capable of addressing batch scheduling issues for multi-stage processes with a focus on minimizing TAFT. We conducted tests on the proposed model, varying the common due dates for scenario 1 and the unit counts for scenario 2. The test results indicated that a type of product does not need to be scheduled sequentially, allowing for the ordering to be alternated with other types of products. Additionally, the batch size produced is not required to be uniform across all existing batches.

1. Introduction

Competition in the manufacturing industry generally includes aspects of quality, price, production costs, and delivery time [1]. Scheduling is one of the efforts that may be made to accomplish this. Scheduling also helps with decision-making processes that are used regularly in many manufacturing and service industries [2]. Therefore, good production scheduling is a must. However, over time, scheduling becomes a complex problem due to the influence of due dates, the product structure becomes more developed, the number of machines increases, and so on. Scheduling problems occur in various industrial sectors, such as the semiconductor industry, the chemical industry, and the cigar-producing consumption industry.

A cigar manufacturing company has two products manufactured on a common machine and a unique machine. There are 14 stages of production, with unique machines in Stage 2 (rolling) and Stage 4 (wrapping) that process products in different work batches. These companies consist of more than three stages of production, commonly referred to as multi-stages [3]. The company does not yet have a schedule so that the fulfilment of requests passes the due date. Due dates are important and must be fulfilled because fulfilling due dates can maintain customer loyalty [4].

Research conducted by [4], [5] carried out the development by proposing a schedule with the criteria of minimizing Total Actual Flow Time (TAFT) by taking into account the due date. TAFT criteria make jobs do not have to arrive at the same time (at time zero) but can arrive when the job is about to start processing or can be called Just in Time (JIT). So, using TAFT criteria will ensure scheduling does not exceed the due date and minimize processing time on the production floor [6]. Research on the batch scheduling model to minimize the total actual flow time was studied by several researchers before such as [7]–[17] has proven effective in generating schedules that meet due dates.

This research develops based on the research conducted by [18] who conducted a two-stage batch scheduling study with common machines in stage one and unique machines in stage two to minimize TAFT. This study aims to develop a scheduling model to solve batch scheduling problems for the Flowshop with the criteria of minimizing TAFT.

2. Research Method

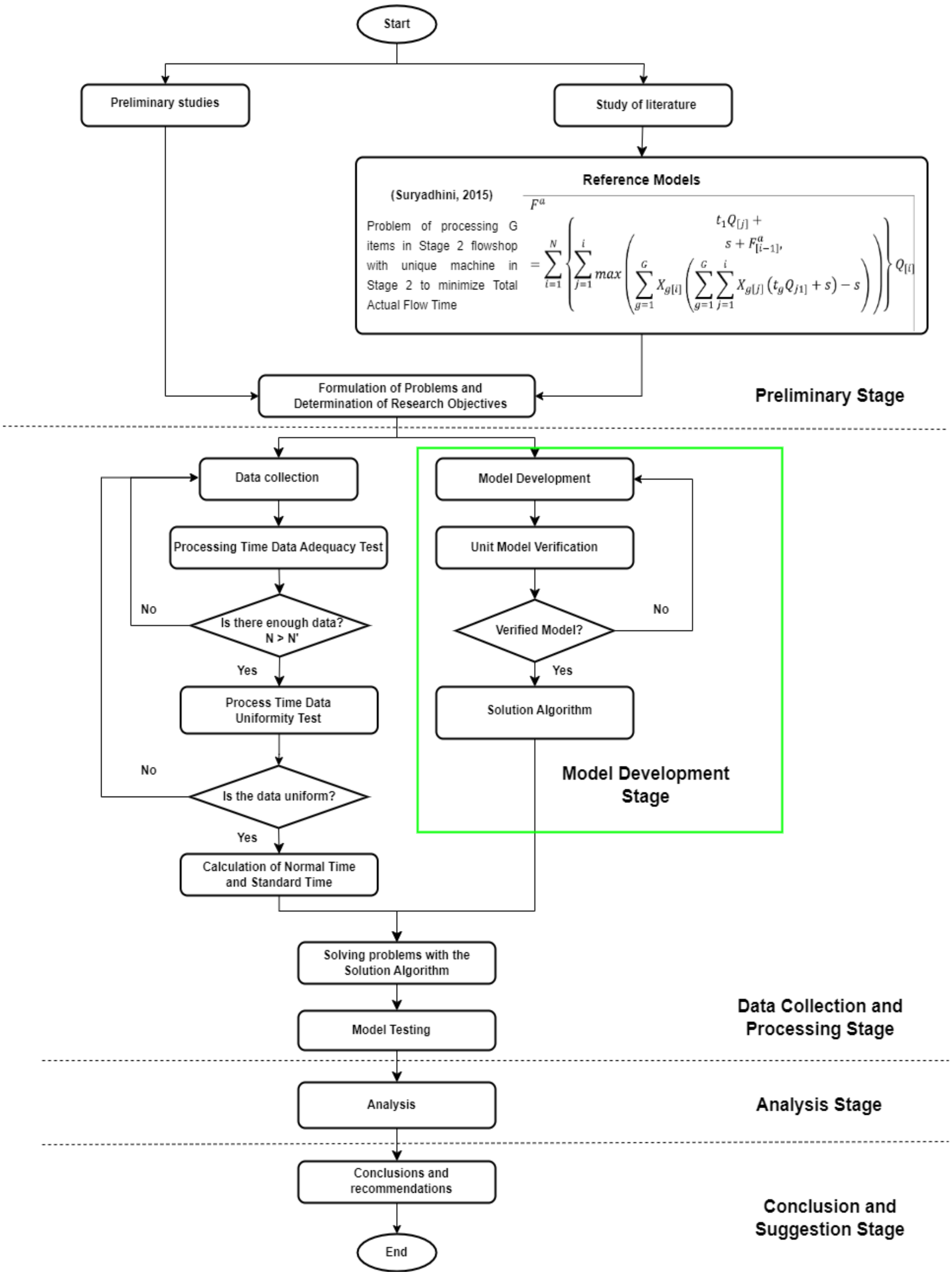

Fig. 1 *Research methodology*

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The research was conducted in multiple phases: preliminary, model development, data collection and processing, analysis, and conclusion and suggestion (refer to Figure 1). The preliminary phase consists of: a preliminary study, which involves initial research and model development, where observations are conducted to understand the production process and identify field obstacles; a literature study, conducted alongside the preliminary study to facilitate the identification of references for model development, serving as a guide in addressing field-related issues; and finally, the formulation of the research problem and questions.

The model development stage involves defining the objective function, decision variables, and constraints. The model undergoes verification through unit testing. The solution algorithm is an algorithm developed based on [6] and this algorithm is used in problem-solving in this study.

During the data collection and processing phase, we collect data on the processing time of each machine, historical demand data, and due date data. The next step is to solve the problem using a solution algorithm. Then the model that has been developed is tested with two scenarios: the first is testing a common due date variation, while the second is testing a different number of units for each product type.

The next step after data processing is analysis, which involves carrying out model testing. The last stage of this research is drawing conclusions from data processing and suggestions for model development from the results of this study.

3. Model Development

3.1 Description of the Production System

The problem with scheduling batches in this study is that there are many items of different types ($g = 1, 2, \dots, G$) and their respective quantities (n_g) which will be processed in a flow shop production system that has several machines ($m = 1, 2, \dots, M$). Setup time is required before the batch is processed, and the amount is the same for each type of machine, denoted by (s). The processing time for general machines is stated with (t_{m0}), while the processing time for unique machines is stated with (t_{ma}) for product type 1 and (t_{mb}) for product type 2. The delivery time for orders is carried out at the same time(d). The number of batches (N) is not included in the model formulation, because the model will be more complex, so batch quantity (N) must be determined in advance. The start time of the batch being processed on the machine is indicated by ($B_{[i]}$). The batch size is indicated by ($Q_{[i]}$). The performance criterion used, namely minimizing the total actual flow time, is expressed by (F^a).

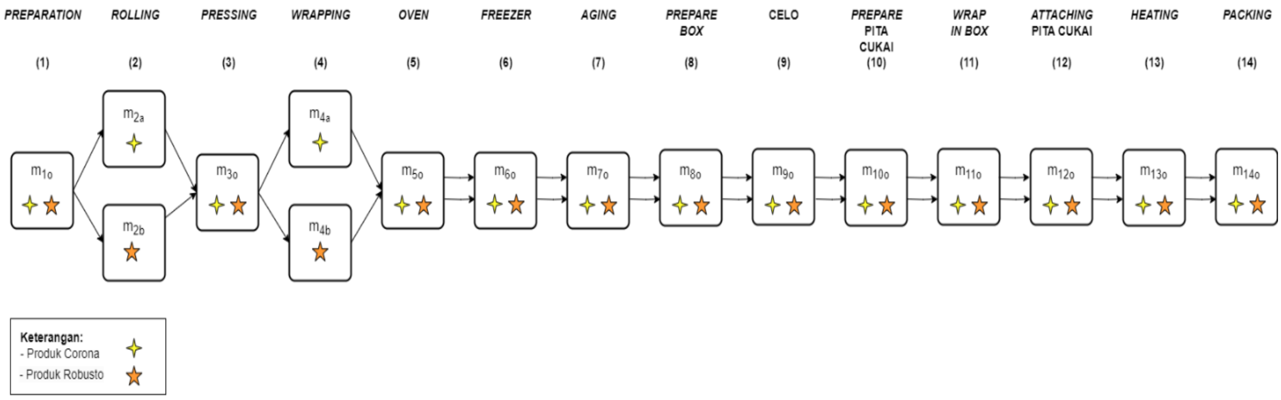

Fig. 2 *System's conceptual description*

Figure 2 illustrates that product 1 (corona) and product 2 (robusto) are processed on the same general machine during stages 1, 3, 5, …, and 14. Unique machines are currently positioned at stages 2 and 4. Machine (a) exclusively produces type 1 products, whereas machine (b) exclusively produces type 2 products.

Figure 3 depicts the conceptual scheduling issue addressed in this study. The production process is summarized as follows: Each product consists of two batches. Type 1 is classified as Corona (N_1=2) while Type 2 is categorized as Robusto (N_2=2). Batches of Type 1 are organized in sequences 1 and 3, whereas batches of Type 2 are organized in sequences 2 and 4.

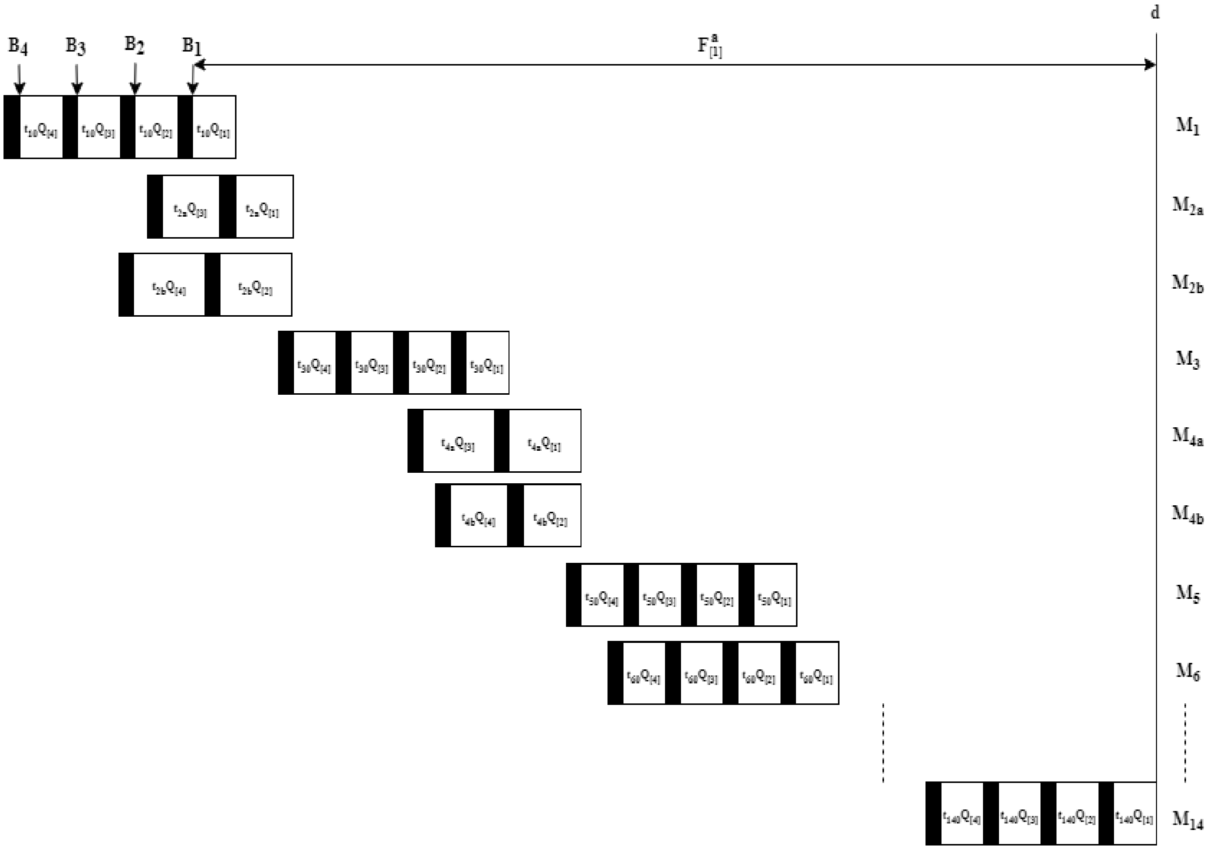

Fig. 3 *Gantt chart for multiproduct flowshop with dedicated machine*

3.2 Model Development

The steps for developing the model in this study are as follows:

a. Define the objective function
b. Determine the decision variables
c. Defines the limiting function

Model development in this study uses the following notation

a. Model index

m = The index denotes the machine (1, 2, ..., k; k is the number of machines)
i, j = An index that represents the batch order
g = An index declaring the type of item

b. Model parameters

d = Due date (time)
M = Number of machines (units)
G = Number of item types (units)
n_g = Number of requests of item type g (units)

c. Model variables

t_{mo} = Processing time on a common machine m (time/unit)
t_{ma} = Processing time of an item a on a unique machine m (time/unit)
t_{mb} = Processing time of an item b on a unique machine m (time/unit)

d. Decision variable

$B_{[i]}$ = At the start of processed the- i batch (time)
$Q_{[i]}$ = Batch size at the i position (units)
N = Number of batches (units)
$X_{g[i]}$ = A binary number indicating type g in position to - i

The total actual flow time for scheduling problems in this batch on the m-stage flow shop is represented by Equation (1).

$$F^a = \left[\sum_{i=1}^{N}\sum_{j=1}^{i} t_{10}Q_{[j]} + (s + F^a_{[i-1]})\right]Q_{[i]} \quad (1)$$

See Figure 2, there is $B_{[i]}$, which is the start time of the batch. In this problem, the start time of the batch can be found using Equation (2).

$$B_{[1]} = d - \begin{pmatrix} t_{10}Q_{[1]} + max\left[(X_{(1)[1]} \times t_{2a}Q_{[1]}), (X_{(2)[2]} \times t_{2b}Q_{[2]})\right] + 3s + \sum_{j=1}^{i} t_{30}Q_{[j]} + \\ max\left[(X_{(1)[1]} \times t_{4a}Q_{[1]}), (X_{(2)[2]} \times t_{4b}Q_{[2]})\right] + 3s + \sum_{j=1}^{i} t_{50}Q_{[j]} + t_{60}Q_{[1]} + \\ t_{70}Q_{[1]} + t_{80}Q_{[1]} + t_{90}Q_{[1]} + t_{100}Q_{[1]} + t_{110}Q_{[1]} + t_{120}Q_{[1]} + t_{130}Q_{[1]} + t_{140}Q_{[1]} \end{pmatrix} \quad (2)$$

The right-hand side in Equation (2) is the result of $F^a_{[1]}$, so that the value of $B_{[i]}$ are as shown in Equation (3).

$$B_{[i]} = d - \left(t_{10}Q_{[1]} + s + F^a_{[i-1]}\right) \quad (3)$$

The 1st term in Equations (2) and (3) shows the deadline for submission. In the 2nd term Equation (2) shows the duration batch in the first position is on the production floor. Meanwhile, in the 2nd term Equation (3) shows the duration batch to $i - 1$ on the production floor plus time setup plus duration batch to i on Stage1.

If, at the start, batches are included in the total actual flow time formula, the model formulation in this study becomes as shown in Equation (4).

$$\text{Minimize } F^a = \sum_{i=1}^{N}\left[\{d - B_{[i]}\}Q_{[i]}\right] \quad (4)$$

With,

$$B_{[1]} = d - \begin{pmatrix} t_{10}Q_{[1]} + max\left[(X_{(1)[1]} \times t_{2a}Q_{[1]}), (X_{(2)[2]} \times t_{2b}Q_{[2]})\right] + 3s + \sum_{j=1}^{i} t_{30}Q_{[j]} + \\ max\left[(X_{(1)[1]} \times t_{4a}Q_{[1]}), (X_{(2)[2]} \times t_{4b}Q_{[2]})\right] + 3s + \sum_{j=1}^{i} t_{50}Q_{[j]} + t_{60}Q_{[1]} + \\ t_{70}Q_{[1]} + t_{80}Q_{[1]} + t_{90}Q_{[1]} + t_{100}Q_{[1]} + t_{110}Q_{[1]} + t_{120}Q_{[1]} + t_{130}Q_{[1]} + t_{140}Q_{[1]} \end{pmatrix} \quad (5)$$

$$B_{[i]} = d - \left(t_{10}Q_{[1]} + s + F^a_{[i-1]}\right), i \geq 2, \dots, N \quad (6)$$

$$B_{[N]} \geq 0 \quad (7)$$

$$F^a_{[i]} = t_{10}Q_{[i]} + (s + F^a_{[i-1]}) \quad (8)$$

$$X_{(g)[i]} = 0 \; or \; 1, \forall \; g \; and \; i \quad (9)$$

$$\sum_{i=1}^{N} Q_{[i]} X_{(g)[i]} = n_{(g)}, \forall \; g \quad (10)$$

$$Q_{[i]} \geq 1, \; integer \; \forall \; i \quad (11)$$

$$N \geq G \quad (12)$$

Equation (4) states the objective function of this research model, namely minimizing the total actual flow time of all parts to be processed.
Equation (5) states that the last batch processed must be completed on the due date.
Equation (6) states the constraints when starting the batch, for greater than or equal to 2.
Equation (7) states the constraint when starting the first batch to be processed must be at zero or after zero.
Equation (8) states the actual flow time constraint for the batch at position.
Equation (9) states the constraints on the existence of a product type in the batch and the sequence of the batch, if $X_{(g)[i]} = 1$ then the batch is a batch of Type product g and is in position i, but if $X_{(g)[i]} = 0$ then the batch is in position to i not a batch of product Type g.
Equation (10) shows the batch size constraint on the batch for the type of product on the order and states that the number of parts for all batches must be equal to the total number of parts that must be processed.
Equation (11) states that the batch size constraint must be greater than or equal to 1 and an integer.
Equation (12) states that the number of batches must be more than or equal to the number of product types to be processed.

3.3 Model Unit Verification

We verify the consistency of the units to determine if the performance criteria units are suitable for the constraints used in the mathematical model. Based on the results of unit verification, it is known that the left and right sides of both the objective function and constraints in the mathematical model are the same. We can conclude that the developed mathematical model has been verified based on the results of unit verification. For example, we use equation 4 as follows:

$$F^a = \sum_{i=1}^{N} \left[\{ d - B_{[i]} \} Q_{[i]} \right]$$

$$time\ unit = (time - time) \times unit$$

$$time\ unit = time\ unit$$

In the equation above, it can be seen that the units on the left side and the right side are the same, namely, they have time units. The above equation is therefore verified.

3.4 Solution Algorithm

Batch scheduling in the m-stages flow shop is obtained by following the design algorithm of batch scheduling in a two-stage flow shop whose design is based on [6]. The basic structure of the algorithm used in the outline is as follows:

a. The initial solution is obtained by calculating the total actual flow time for the number of batches equal to the number of product types ($N = G$).
b. The improvement direction was carried out by breaking up the batch it originally consisted of G batch become $G + 1$.
c. The stop rule is set when the minimum total actual flow time value has been found ($F_N^a \geq F_{N-1}^a$) or the number of batches is equal to the total number of parts ($N = n_{total}$).

In this research, the solution algorithm is implemented to solve the problem with the help of LINGO software. The algorithm for solving batch problems on a flowshop with unique machines in Stages 2 and 4 is as follows:

Step 1 : Specify the number of types of products to be produced with the notation g, for $g = 1, 2, \dots, G$.
Step 2 : Express these products as batches, in this step, the number of batches is equal to the number of product types (G).
Step 3 : Set $N = G$, and solve Equation (4) to Equation (12) to get batch order and total actual flow time. Does the resulting schedule start before time zero?
- If not, then the solution is considered feasible and go to step 4.
- If yes, then the solution is said to be infeasible, then the order received cannot be processed (order rejected).

Step 4 : Set $N = G + 1$. Go to step 5.
Step 5 : Solve Equation (4) to Equation (12) to get the batch size and order and total actual flow time. After that go to step 6.
Step 6 : Does the resulting schedule start before time zero?
- If not, then the solution is considered feasible and go to step 7.
- If yes, then the solution is said to be infeasible and proceed to step 10.

Step 7 : Is $F_N^a \leq F_{N-1}^a$?
- If yes, go to step 8.
- If not, stop the algorithm, $N_{selected} = N-1$.

Step 8 : Is $N = n_{total}$?
- If yes, stop the algorithm,.
- If not, next to step 9.

Step 9 : Set $N = N + 1$, back to steps 5.
Step 10 : Set $N = N + 1$.
Solve Equation (4) to Equation (12) to get the batch size and order and total actual flow time.
Step 11 : Does the resulting schedule start before time zero?
- If not, then the solution is considered feasible and returns to step 8.
- If yes, then the solution is said to be not feasible, stop the algorithm, $N_{selected}$ is N decent last.

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4. Results and Discussion

4.1 Numerical Experience

We demonstrated the implementation of the proposed model and algorithm using an example from cigar manufacturing. Two types of products are produced: Type 1, consisting of 250 units, and Type 2, consisting of 400 units. The setup duration for each machine is 5 minutes, with a due date of the 10000th minute. The standard processing time utilized in this research is presented in Table 1.

Table 1 *Processing time*

Stage	Machine	Standard Time (minute)
Preparation	m_1	2.8
Rolling corona	m_{2a}	3
Rolling robusto	m_{2b}	3.2
Pressing	m_3	0.6
Wrapping corona	m_{4a}	3
Wrapping robusto	m_{4b}	3.5
Oven	m_5	2.4
Freezer	m_6	1.9
Aging	m_7	2.3
Prepare Box	m_8	1.7
Celo	m_9	3
Prepare Tax Tape	m_{10}	0.6
Wrap in box	m_{11}	1.7
Attaching Tax Tape	m_{12}	1.7
Heating	m_{13}	1.4
Packaging	m_{14}	0.3

Step 1 : Specify the number of types of products to be produced with the notation g. $G = 2$

Step 2 : Express these products as batches, in this step, the number of batches is equal to the number of product types (G). $N = G$

Step 3 : Set $N = G$, and solve Equation (4) to Equation (12) to get batch order and total actual flow time. The result for batch size to be produced is $Q_{[1]} = 250$ and $Q_{[2]} = 400$. Then check whether the resulting schedule started before time zero. The start time is $B_{[2]} = 2920$. Since the start time is more than time zero the solution is considered feasible and goes to step 4.

Step 4 : Set $N = G + 1$.
$N = 2 + 1 = 3$. Go to step 5.

Step 5 : Solve Equation (4) to Equation (12) to get the batch size and order and total actual flow time. $F^a = 3821000$, batch sizes are $Q_{[1]} = 250$, $Q_{[2]}150$, and $Q_{[3]} = 250$. After that go to step 6.

Step 6 : Does the resulting schedule start before time zero?

Batch Sequence	**Type of Product**	**Stage 1**		...	**Stage 14**	
		Start	**End**	...	**Start**	**End**
1	1	4935	5635	...	9925	10000
2	2	4230	4930	...	9860	9920
3	2	3525	4225	...	9795	9855

The start time is $B_{[3]} = 3525$. Since the start time is more than time zero the solution is considered feasible and goes to step 7.

Step 7 : Is $F_N^a \leq F_{N-1}^a$?
$F_2^a = 5052000$ and $F_3^a = 3821000$, so that $F_3^a \leq F_2^a$. Go to step 8.

Step 8 : Is $N = n_{total}$?
$n_{total} = 650$
$N = 3$
Go to step 9.

Step 9 : Set $N = 3 + 1 = 4$, back to steps 5.

Step 10 : Set $N = 4$.
Solve Equation (4) to Equation (12) to get the batch size and order and total actual flow time. $F^a = 1848000$, batch sizes are $Q_{[1]} = 50, Q_{[2]} = 200$, $Q_{[3]}$ =200, and $Q_{[4]} = 200$. Continued to the next step.

Step 11 : Does the resulting schedule start before time zero?

Batch Sequence	**Type of Product**	**Stage 1**		**...**	**Stage 14**	
		Start	**End**	**...**	**Start**	**End**
1	1	8200	8340	...	9985	10000
2	2	7635	8195	...	9920	9980
3	2	7070	7630	...	9855	9915
4	1	6506	7066	...	9790	9850

The start time is $B_{[4]} = 6506$. Since the start time is more than time zero the solution is considered feasible and goes to step 7.

Step 12 : Is $F_N^a \leq F_{N-1}^a$?
$F_3^a = 3821000$ and $F_4^a = 1848000$, so that $F_4^a \leq F_3^a$. Go to step 8.

Step 13 : Is $N = n_{total}$?
$n_{total} = 650$
$N = 4$
Go to step 9.

Step 14 : Set $N = N + 1 = 4 + 1 = 5$.
Solve Equation (4) to Equation (12) to get the batch size and order and total actual flow time. $F^a = 2947875$, batch sizes are $Q_{[1]} = 125, Q_{[2]} = 125$, $Q_{[3]}$ =125, $Q_{[4]} = 125$and $Q_{[5]} = 150$. Continued to the next step.

Step 11 : Does the resulting schedule start before time zero?

Batch Sequence	**Type of Product**	**Stage 1**		**...**	**Stage 14**	
		Start	**End**	**...**	**Start**	**End**
1	1	6147	6497	...	9962.5	10000
2	2	5792	6142	...	9920	9957.5
3	1	5437	5787	...	9877.5	9915
4	2	5082	5432	...	9835	9872.5
5	2	4727	5147	...	9785	9830

The start time is $B_{[5]} = 4727$. Since the start time is more than time zero the solution is considered feasible and goes to step 7.

Step 12 : Is $F_N^a \leq F_{N-1}^a$?
$F_4^a = 1848000$ and $F_4^a = 2947875$, so that $F_5^a \geq F_4^a$. The solution is said to be not feasible, stop the algorithm, $N_{selected}$ is N decent last. $N = 4$.

The results of the calculations are presented in Table 2 and illustrated in Figure 4.

Table 2 *Data total actual flow time cigar manufacturing*

N	Total Actual Flow Time
2	5052000
3	3821000
4	1848000
5	2947875

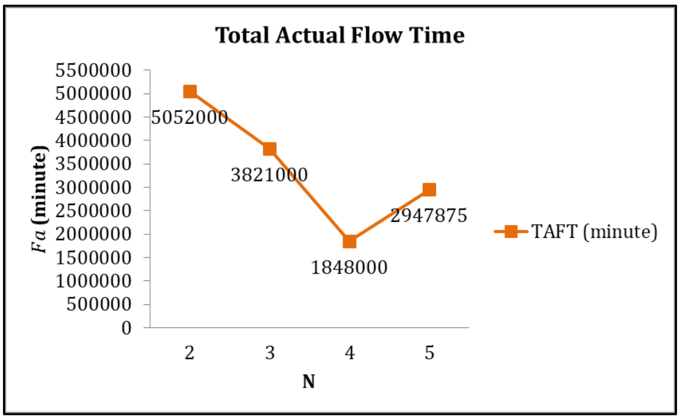

Fig. 4 *Total actual flow time graph for the problem*

Table 2 and Figure 4 show the comparison between the total actual flow time and the number of batches. The results obtained for the smallest total actual flow time are $F^a = 1848000$ with the resulting batches of 4. The size of each batch produced is $Q_{[1]} = 50$, $Q_{[2]} = 200$, $Q_{[3]}$ =200, and $Q_{[4]} = 200$ by the sequence of *batch* $b_{1[1]} - b_{2[2]} - b_{2[3]} - b_{1[4]}$. Product type 1 is produced in batches 1 and 4, whereas type 2 is produced in batches 2 and 3. The minimum total actual flow time value has been achieved with N=4.

Figure 4 illustrates that the total actual flow time results in a fluctuating graph. Beginning at N = 2, the graph shows a decrease at N = 4, followed by an increase at N = 5. The minimum value is still achieved when the batch value N = 4.

4.2 Model Testing

The model was tested in several scenarios with different data sets on numerical experience; we reduced the quantity of each type of product by 1/50 from numerical experience.

4.2.1 Scenario 1

The model was tested in scenario 1 using different due dates while keeping the parameters for processing time and setup time the same. The N that was selected in the calculation with the solution algorithm is N = 4. The due date in scenario 1 is determined arbitrarily. The due dates used are the 500th minute, the 50th minute, and the 10th minute. The results of scenario 1 can be seen in Table 3.

Table 3 *Results of data processing scenario 1*

d	*N*	Number of Batches		F^a
		Type 1	Type 2	
500	2	1	1	48.844
	3	1	2	39.682
	4	2	2	24.78
	5	1	4	44.02
50	2	1	1	48.844
	3	1	2	39.682
	4	2	2	24.78
	5	1	4	44.02
10	2	1	1	48.844
	3	1	2	39.682
	4	2	2	24.78
	5	1	4	44.02

Table 3 shows whether the three due dates achieved a value of F^a=24.78. This value is obtained from the number of batches formed, N = 4, with details of size and type in each batch shown in Table 4.

Table 4 *Batch size and sequence for scenario 1*

Sequence	d= 500		d= 50		d= 10	
	$Q_{[i]}$	Type	$Q_{[i]}$	Type	$Q_{[i]}$	Type
1	1	1	1	1	1	1
2	4	2	4	2	4	2
3	4	1	4	1	4	1
4	4	2	4	2	4	2

Values F^a for the three due dates are the same because the batch sizes and product types in each batch produced are the same. Thus, it can be stated whatever the value of the requested due date, the value F^a will stay the same. However, it is necessary to pay attention to the processing time because if the due date being tested is less than the total production process time, the model cannot be run or results are not obtained.

4.2.2 Scenario 2

Scenario 2 involves evaluating the model with a dataset comprising a varying number of units. In scenario 2, the number of units is arbitrarily chosen. The following rules apply to the comparison of the number of units of the two products:

a. The number of units of Type 1 is greater than the number of units of Type 2 ($n_1 > n_2$).
b. The number of units of Type 1 is smaller than the number of units of Type 2 ($n_1 < n_2$).
c. The number of units of Type 1 products is equal to the number of units of Type 2 ($n_1 = n_2$).

Data set 1 (SD1), data set 2 (SD2), and data set 3 (SD3) are arranged as shown in Table 5.

Table 5 *Scenario 2 data sets*

Data Set 1			Data Set 2			Data Set 3		
Test	n_1	n_2	Test	n_1	n_2	Test	n_1	n_2
SD1-1	6	1	SD2-1	1	6	SD3-1	1	1
SD1-2	5	2	SD2-2	2	5	SD3-2	2	2
SD1-3	4	3	SD2-3	3	4	SD3-3	3	3

SD1-1 shows data set 1 for test 1, and so on until SD3-3 which shows data set 3 for test 3.

The results of data set 1 (SD1) are shown in Table 6.

Table 6 *Results of data sets 1 for scenario 2*

d	N	Number of Batches		F^a
		Type 1	Type 2	
SD1-1	2	1	1	17.4
	3	2	1	13.122
	4	3	1	11.896
	5	4	1	16.008
SD1-2	2	1	1	17.762
	3	2	1	16.14
	4	3	1	10.86
	5	3	2	16.008
SD1-3	2	1	1	18.236
	3	2	1	11.148
	4	2	2	11.896

The data presented in Table 6 shows that numerical examples using data set 1(SD1) yield varying values for N and F^a. N=4 was acquired in SD1-1 with $F^a = 11.896$. The SD1-2 has $N = 4$ with $F^a = 10.86$. $N = 3$ with $F^a = 11.148$ was found in SD1-3. The size and type of each batch in numerical example data set 1 (SD1) are provided in Table 7.

Table 7 *Batch size and sequence for data sets 1 in scenario 2*

Batch Sequence	SD1-1		SD1-2		SD1-3	
	$Q_{[i]}$	Type	$Q_{[i]}$	Type	$Q_{[i]}$	Type
1	1	2	1	1	1	1
2	2	1	2	1	3	2
3	2	1	2	1	3	1
4	2	1	2	2		

Based on the numerical example with data set 1 (SD1), it is known that the number of batches, batch size, and composition of each product are not always the same and do not follow a certain pattern.

The results of data set 2 (SD2) are shown in Table 8

Table 8 *Results of data sets 2 for scenario 2*

d	N	Number of Batches		F^a
		Type 1	Type 2	
SD2-1	2	1	1	12.612
	3	1	2	11.148
	4	1	3	10.86
	5	1	4	14.426
SD2-2	2	1	1	14.752
	3	2	1	14.496
	4	3	1	11.896
	5	3	2	14.426
SD2-3	2	1	1	17.004
	3	1	2	13.122
	4	2	2	10.86
	5	2	3	14.426

Table 8 indicates that the numerical example using data set 2 (SD2) yields the same value for N, specifically N = 4. However, there are differences in the resulting F^a values. The results of SD2-1, SD2-2, and SD2-3 were as follows: $F^a = 10.86$, $F^a = 11.896$, and $F^a = 10.86$, respectively. Table 9 provides the batch size and product type information for each batch in Data Set 2 (SD2).

Table 9 *Batch size and sequence for data sets 2 in scenario 2*

Batch Sequence	SD2-1		SD2-2		SD2-3	
	$Q_{[i]}$	Type	$Q_{[i]}$	Type	$Q_{[i]}$	Type
1	1	1	1	2	1	1
2	2	2	2	2	2	1
3	2	2	2	2	2	2
4	2	2	2	1	2	2

Based on the numerical example with data set 2 (SD2), it is known that the number of batches is the same with N = 4, but the batch size and composition of each product are not always the same and do not follow a certain pattern.

The results of data set 3 (SD3) are shown in Table 10.

Table 10 *Results of data sets 3 for scenario 2*

d	N	Number of Batches		F^a
		Type 1	Type 2	
SD3-1	2	1	1	2.3

SD3-2	2	1	1	7.048
	3	1	2	6.428
	4	2	2	5.448
	5	3	2	14.426
SD3-3	2	1	1	13.908
	3	1	2	13.122
	4	3	1	11.964
	5	3	2	10.5
	6	3	3	9.108

Table 10 illustrates that the numerical example from data set 3 yields N and F^a values that differ. SD3-1 was obtained $N = 2$ with $F^a = 2.3$. SD3-2 was obtained $N = 4$ with$F^a = 5.48$. SD3-3 $N = 6$ with $F^a = 9.108$. Details of size and type in each batch of numerical example data set 3 (SD3) are shown in Table 11.

Table 11 *Batch size and sequence for data sets 3 in scenario 2*

Batch Sequence	SD2-1		SD2-2		SD2-3	
	$Q_{[i]}$	Type	$Q_{[i]}$	Type	$Q_{[i]}$	Type
1	1	1	1	1	1	1
2	1	2	1	1	1	1
3			1	2	1	2
4			1	2	1	2
5					1	1
6					1	2

Based on the numerical example with data set 3 (SD3), it is known that the number of batches, batch size, and composition of each product are not always the same and do not follow a certain pattern.

5. Conclusion

This research focused on the challenge of batch scheduling within a 14-stage flow shop that includes various products and a dedicated machine in both the second and fourth stages. The outcomes of the numerical experiments indicate that the proposed model and algorithm operate efficiently. In this research, the batch size to be produced may vary in the number of unit products. The sequence of batches is established by the index of each production batch, and a particular product type should not be placed consecutively; rather, it may be alternated with a different product type. The proposed model is limited to issues that have a shared (common) due date. Further development is necessary to expand the relevance of this study to more complex issues in flow shop scheduling that involve M machines, including managing several items required by different due dates, the specific placements of dedicated machines, and the accessibility of those machines. Achieving this complexity will require innovative algorithms that can efficiently handle the increased permutations and combinations of tasks. Additionally, incorporating real-time data could enhance the model's adaptability to changing production environments and customer demands.

Acknowledgement

The authors sincerely thank the Universitas Jenderal Soedirman for their invaluable support and resources throughout this research.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of the paper.

Author Contribution

The authors are responsible for the study conception, research design, data collection, data analysis, result interpretation and manuscript drafting.

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