

Fracture Energy Measurement in Different Concrete Grades

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Abstract: Fracture energy is regarded as an intrinsic (material) properties that dominates crack mechanisms and associated crack growth in concrete damage under applied stress. In recent times, significant advancements in computing technology have driven the adoption of finite element analysis (FEA) methodologies that necessitate the integration of constitutive models, including the traction-separation relationship derived from cutting-edge fracture mechanics. A physically-based model requires fracture energy values; therefore, a properly measured fracture energy value is essential to exhibit better structure response within FEA models. There are large arrays of parameters involved during the concrete mixture, such as beam size effect, aggregate size, and concrete grade, that affect the flexural resistance of the concrete. The fracture and failure in concrete ahead of the crack tip are represented by fracture energy values where micro-damage events such as interfacial failure, fiber-bridging, and matrix cracking occurred. This study aims to determine the fracture energy of concrete specimens with combination of notch depth a_0 at mid-span, design concrete strength as specified in the testing series. Independent compression strength, f_c and measured load-displacement profiles under a three-point bending test were used to determine fracture energy by incorporating three available fracture energy expressions such as Bazant, Hillerborg, and CEB-FIP models.

Keywords: Fracture energy, foam concrete, cohesive failures, notched beam, stress concentrations

1. Introduction

Concrete is one of the most widely used materials on Earth, second only to water [1]. It is extensively utilised in housing, bridges, and tall buildings, making it the most utilised man-made material. An excellent characteristic of concrete is its resistance to compression, but it is weak in tension. Therefore, the behaviour of concrete under flexural load is critical. Various factors, such as size effect, aggregate size, and concrete grade, can affect the flexural strength of concrete. Due to its exceptional mechanical properties and cost-effectiveness, concrete holds significant potential for future development. However, concrete is prone to brittle failure, necessitating the consideration of fracture mechanics parameters to address fracture and failure behaviour in concrete materials. While concrete exhibits excellent resistance to compression, it has low tensile strength when subjected to flexural and tension loads. Thus, designing structural elements with resistance to mechanical deformation is crucial. The extensive usage of concrete necessitates the enhancement of its material properties, with a primary focus on excellent compressive, flexural, and tensile strength [2].

The introduction of any irregularities (or discontinuities) in a concrete member can severely reduce its mechanical strength. Additionally, the presence of cracks and notches within their geometrical bodies leads to an interruption of stress flow and demonstrates stress concentration. Due to the high concentration of stress surrounding the irregularities,

deformation or yielding (profound in ductile material) is exhibited for the structural member to keep resisting load as a part of its toughness resistance. In concrete, fracture toughness showed negligible yielding deformation, but damage resistance is measured from the fracture energy value. Determining the fracture energy value is crucial for assessing the energy absorption capacity within a cracked section of concrete. When the applied stress (equivalent to applied energy) surpasses the fracture energy of the specific concrete, it leads to crack propagation within an unreinforced concrete element. This fracture energy value plays a significant role in governing the formation and progression of cracks [3]. An intended introduction of notches in concrete specimens can give a clear picture of how fracture energy affects stress concentration and damage behaviour within a concrete material.

Numerous investigations of fracture energy have been conducted and reported by researchers worldwide, in particular within ductile materials. However, brittle material showed negligible yielding, and not much material softening was exhibited beyond the fracture zone length. There was not much information on fracture energy in concrete material available in the literature. Theoretically, fracture energy drives crack development due to stress concentration, which is of particular interest within brittle engineering materials. Many researchers investigate fracture energy studies in composite materials with the presence of notches or holes, but fewer references to concrete materials were found.

Fracture energy is considered a material property that relies on intrinsic characteristics, including concrete strength, aggregate sizes, and mixture constituents. In order to fully understand and utilise the material to its full extent, even small and likely insignificant parameters needed to be assessed and analysed. The evolution of numerical modelling has led to the incorporation of fracture energy values within constitutive models to predict structures' behaviour and damage behaviour. Traction-separation relationship has been adopted within FEA work by adopting measured fracture energy and associated material strength to investigate strength prediction work. Other constitutive models require fracture energy such as Hashin formulations and smeared concrete plasticity. A good understanding of fracture energy is a feasible solution to comprehend crack formation and propagation processes [4].

Research stated that the fracture energy of conventional concrete is 75 to 100 N/m depending on the concrete grade [5]. Next, the other researchers obtained 185 N/m and 189 N/m as they claimed the true average true fracture energies through Crack Mouth Opening Displacement (CMOD) and stroke control [6]. However, fracture energy is intrinsic which means it is highly dependent on the geometry and dimension of the specimen. Concrete fracture mechanics exhibits a distinctive aspect by incorporating the fracture process zone (FPZ), which emerges ahead of the crack tip and plays a significant role in the initiation and propagation of concrete failure [7].

From the information above, it can be inferred that fracture energy plays an important role in concrete behaviour and contributes to damage growth and material softening. These material properties are not usually reported in the concrete literature; therefore, independently determined experimental set-up are essentially required. Similarly, the tensile properties of concrete can be determined and derived from cohesive models through a three-point bending test [3]. Therefore, conducting a research study on the fracture energy of normal concrete is essential to gather valuable and universally applicable insights into fracture energy in this type of concrete.

The fracture energy of concrete plays a crucial role in determining the damage resistance and crack propagation characteristics of a concrete element. However, there is a scarcity of measured fracture energy data available for various concrete mixtures and design approaches. The Hillerborg, Bazant, and CEB-FIP models are the most commonly used methods to estimate the fracture energy value from concrete or other brittle specimens. Measured fracture energy values can assist engineers in designing and evaluating damage evolutions to achieve optimum concrete size. Additionally, fracture energy value can be incorporated within the available constitutive model to predict structure behaviour and associated strength prediction works.

2. Experimental Methodology

2.1 Testing Series

The main aim of this study is to examine and analyse the fracture energy of normal concrete utilising the RILEM method. Three concrete mixtures with design strengths of 30 N/mm², 40 N/mm² and 50 N/mm² chosen in this study are selected for testing. The compressive and flexural tests were conducted on these concrete mixtures at specified ages of 7 and 28 days. For each concrete grade, a minimum of three concrete cubes with dimensions of 100 × 100 × 100 mm³ were prepared. These cubes underwent compression tests, and the measured strength at the specified age was recorded for subsequent analysis. Alongside the compressive strength tests, the flexural strength of the concrete was determined using 100 × 100 × 400 mm³ concrete prisms. These prisms were subjected to four-point bending tests at 28 days of age, offering insights into the flexural behaviour and strength of the concrete samples. Furthermore, concrete prisms of the same size were produced, but this time notches were introduced. These notched concrete prisms were then subjected to three-point bending tests. The investigation encompassed a total of nine testing series, comprising the three designated concrete strengths in combination with three different notch heights. The notch heights chosen were 10 mm, 30 mm, and 50 mm. To achieve these notch heights, steel bars were securely positioned at the midpoint of the concrete before pouring.

2.2 Specimens Preparation and Mechanical Testing

Cube and beam specimens were prepared to achieve M30, M40, and M50 concrete grades. The concrete constituents used in each calculated concrete grade are given in Table 1.

Table 1 - Concrete constituents required in both concrete grades

Specimen strength design	w/c (%)	Amount of materials (kg/m ³)			
		Cement	Water	Fine aggregate	Coarse aggregate
M30	0.49	435	215	860	895
M40	0.43	440	190	790	1000
M50	0.37	515	190	765	970

The compression tests were conducted at two specific time points: the 7th and 28th days. The time intervals used in this study adhere to the industry-standard practice and comply with the provisions outlined in the ASTM C39/C39M Standard Test Method for compressive strength testing. A total of 27 notched beam specimens were prepared, following the testing series mentioned earlier, to evaluate the fracture energy. These specimens were standardized with dimensions of 100 × 100 × 400 mm³, as depicted in Figure 1. During the testing procedure, the Universal Testing Machine (UTM) was set to a predetermined loading rate of 0.05 mm/min. This loading rate was selected to ensure a controlled and consistent application of the load to the specimens. By maintaining a standardised loading rate across all the tests, the study minimises potential variations and ensures the comparability of the results obtained. The load-displacement profiles obtained from the data logger are extracted and subsequently evaluated. The measured datasets obtained from the above testing will be used to predict fracture energy values from the available fracture model expressions. Three modelling expressions were employed, i.e., Bazant, Hillerborg, and CEB-FIP models. The fracture energy calculated from these expressions was evaluated and discussed accordingly.

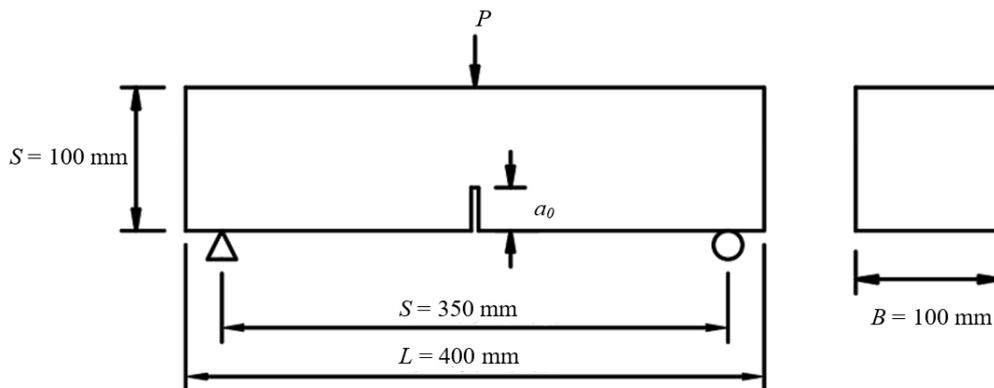


Fig. 1 - Notched beam specimen

3. Calculations of Fracture Energy

The fracture energy values were extracted by using a testing setup method for the load-displacement profile of the notched beam prepared by the Japan Concrete Institute Standard, JCIS-S-002-2003. The specimen was positioned on the support of the testing machine, with the notch located in the concrete tension zone. Subsequently, the specimen was subjected to progressive loading without any sudden impact, with an initial loading rate of 0.05 mm/min. As the peak load was reached and material softening began, the loading rate was reduced to 0.01 mm/min. Throughout the testing process, the testing machine recorded the load and deflection, which were utilized to create the load-deflection profile. The load-deflection profile is then used to determine the fracture energy through Hillerborg's model.

3.1 Bazant Model

Bazant proposed an empirical formula for his model where consideration of compressive strength, size of aggregates, and the water-cement ratio is taken into account. All the available data is inserted into the equation to investigate the fracture energy. As for the shape of the aggregate factor, angular aggregates are used in this experiment. Eq. (1) shows the expression for Bazant's model.

$$G_F = 2.5\alpha_0 \left(\frac{f_c}{0.058} \right)^{0.40} \left(1 + \frac{D_{\max}}{1.94} \right)^{0.43} \left(\frac{w}{c} \right)^{-0.18} \quad (1)$$

where α_0 represent the aggregate shape factor, with $\alpha_0 = 1$ for rounded aggregates and $\alpha_0 = 1.12$ for angular aggregates. The compressive strength of concrete is denoted as f_c , D_{max} represents the maximum aggregate size, and w/c represents the water-to-cement ratio in concrete.

3.2 Hillerborg Model

In the context of fracture energy analysis, Hillerborg introduced a more comprehensive model that considers the size effect of notched specimens and the load-deflection profiles. This model requires a stress-strain curve obtained from experimental data to assess the fracture energy value. Specifically, the stress-strain curve within the softening regions is utilized to determine the fracture energy value. Equation (2) presents the expression proposed by Hillerborg.

$$G_F = \frac{U_0 + m_g d_0}{B(W - a_0)} \quad (2)$$

where U_0 denote the area under the stress-strain curve during the softening phase. B represents the width of the specimen, W represents the depth of the specimen, a_0 represents the depth of the notch, m_g represents the weight of the specimen, and d_0 represents the deflection at the fracture point.

3.3 Construction of References

CEB-FIP proposed a formulation that took similar consideration as in Bazant's where mixture design is taken into account. The fracture energy can be determined by using Eq. (3) below:

$$G_F = 0.0143\alpha_0 (D_{max})^{0.2} - (0.5D_{max} + 26) \times \left(\frac{f_c}{10}\right)^{0.7} \quad (3)$$

where α_0 represents the aggregate shape factor, where α_0 equals 1 for rounded aggregates and 1.12 for angular aggregates. f_c denotes the compressive strength of concrete. D_{max} represents the maximum aggregate size. Lastly, w/c signifies the water-to-cement ratio used in concrete mixing.

4. Results and Discussion

4.1 Load-Displacement Profiles

The universal testing machine is configured to load the specimen at 0.1 mm/min and subsequently reduced to 0.05 mm/min as the crack has been detected as recommended [10]. This loading rate is stable enough to propagate the crack without abrupt shock. Throughout the loading process, cracks that propagate from the notch are marked and labelled according to the applied load for further evaluation. The concrete beam specimens were painted white colour to aid in marking during crack propagation as given in Fig. 2(a). The test stopped as the beam ruptured entirely (beam separation into halves in Fig. 2(b) and the measured peak load from each concrete beam specimen was recorded. Figure 3 illustrates the load-deflection curves for the three concrete strengths with varying notch depths.



Fig. 2 - Mechanical testing set-up; (a) crack propagation marking; (b) a completely ruptured concrete beam specimen

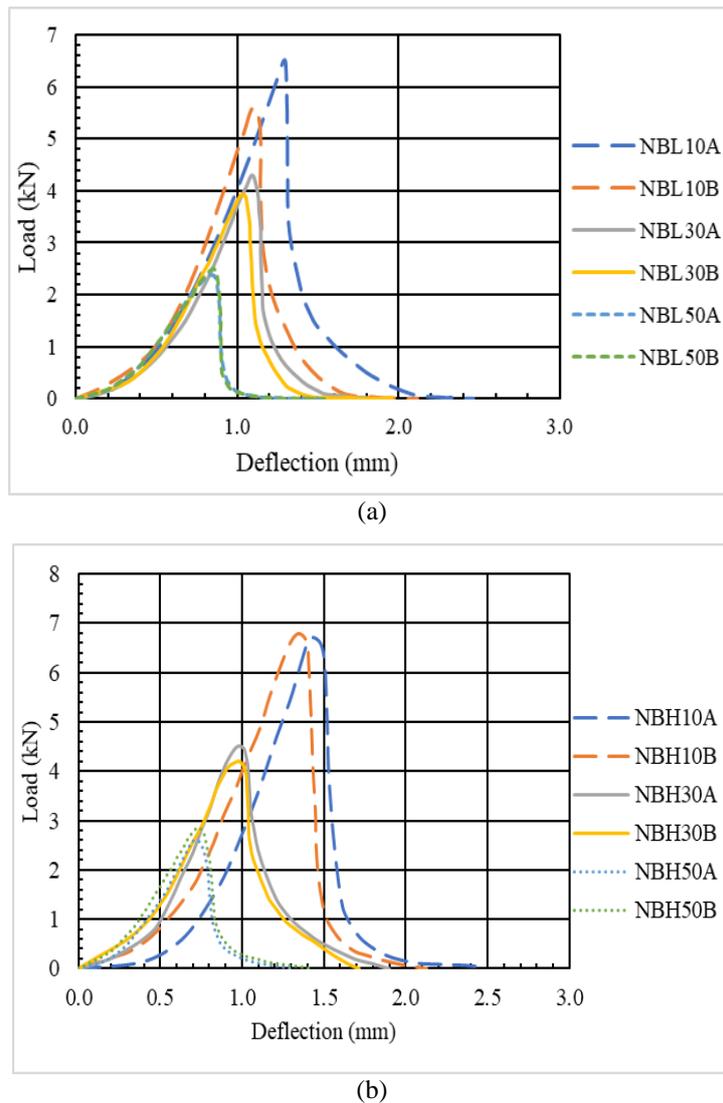


Fig. 3 - Load-deflection profiles at different notch depths with different concrete strengths; (a) M30; (b) M50

In this study, the tested beams exhibited typical brittle failures, characterized by an initial phase of linear elastic deformation until reaching the ultimate load, followed by sudden and catastrophic failure. The load-deflection profiles recorded by the data logger played a crucial role in evaluating the behaviour of the concrete specimens and determining the fracture energy, which was the main focus of this research investigation. By analysing the load-deflection profiles, the results became good evidence that the concrete series with higher compressive strengths exhibited larger ultimate loads and also greater flexural strength, which aligned with the expected mechanical properties of stronger concrete.

Additionally, a significant finding emerged during the investigation of the impact of notch depth on the ultimate load of the tested beams. It was consistently observed across all the examined concrete series that an increase in notch depth corresponded to a decrease in the ultimate load. Specifically, in beams with deeper notches, there was a substantial decrease in the load sustained by the structures after reaching the peak load. This drop in load was accompanied by the slow propagation of cracks, ultimately leading to a sudden failure. This phenomenon suggests that larger notches result in a lower unloading region, which is associated with lower residual strength.

4.2 Measured Fracture Energy

In this experiment, fracture energy is determined using three numerical failure models, i.e., Bazant, CEB-FIP and Hillerborg. The fracture energy obtained from the respective failure model produces different measured fracture energy values, partly due to some models, i.e., the Hillerborg model, which considers size effects and measured strength. On the contrary, Bazant and CEB-FIP models require only measured compressive strength to calculate concrete fracture energy. The latter two fracture models seem to provide a rough idea of estimated concrete fracture energy values and do not take into account the intrinsic properties of the concrete specimen tested. On the other hand, Hillerborg's model used the extracted load-deflection profile as a vital parameter to determine the fracture energy of concrete. The

advantages of the Hillerborg model are the ability to consider the softening effect of tested concrete beams (however, this effect is more pronounced in lightweight concrete beams such as foam concrete as suggested by the researchers [5].

GF, the fracture energy of each concrete specimen was computed using Equations (1), (2), and (3) to determine the fracture energy. The CEB-FIP model predicted an average GF value approximately 25% lower compared to Bazant's model. It is worth noting that both the Bazant and CEB-FIP models incorporate the influence of aggregate size and compressive strength. However, Bazant's model additionally considers the water-cement ratio, w/c, which is recognized as a significant parameter in concrete design strength. Bazant calculation is regarded as more accurate than the CEB-FIP model counterparts. However, both failure models do not provide a correlation between concrete strength and notch depth.

On the contrary, for Hillerborg's model G_f value increased as the concrete strength increased across all notch depths. The G_f decreases as the notch depth increases, a similar observation was revealed [11]. Table 2 presents the comparative analysis of the fracture energy results. The softening region was measured based on integral calculations with a sufficiently small integration interval. It is interesting to note that close fracture energy values were seen from Bazant and Hillerborg fracture models compared to the CEB-FIP model. It is estimated that the effect of notch height is beneficial when considering the notch size effect, as highlighted by the respective researcher [3].

Table 2 - Average fracture energy of normal concrete

Specimen	Measured Concrete strength	Fracture energy (N/m)				
		Hillerborg			Bazant	CEB-FIP
		10 mm	30 mm	50 mm		
NBL	42.2	113.5	84.5	61.4	99.4	70.4
NBM	46.0	120.3	92.8	72.9	102.9	74.8
NBH	62.2	150.8	121.5	94.2	116.0	92.3

Upon comparing the results obtained from the Hillerborg model with the experimental data, it was observed that the average fracture energy (GF) exhibited a variation of approximately 20% to 35%. Interestingly, both the Hillerborg model and another model employed in the study showcased an under-prediction of fracture energy in normal concrete, while accurately predicting fracture energy in foam concrete [3]. To further assess the obtained fracture energy values, Table 3 presents a compilation of fracture energy values reported in the existing literature. This table includes data from various concrete grades and incorporates the incorporation of different admixtures. Upon analysing the table, it becomes evident that the overall fracture energy values reported exhibit a reasonable agreement with the outcomes of previous research experiments. This discovery reinforces the credibility and coherence of the fracture energy values acquired in the present study. Through a comparison between the experimental findings and the existing literature, it becomes apparent that the fracture energy of concrete can fluctuate based on factors like concrete grade and the inclusion of admixtures. These variations highlight the importance of considering specific concrete compositions and conditions when predicting fracture energy values. It also emphasises the need for comprehensive research and analysis to develop accurate models that can reliably estimate fracture energy across a wide range of concrete types and scenarios.

Table 3 - G_f value obtained by previous researchers

References	Compressive strength	Tensile Strength	Admixture	a_0 (m)	Fracture energy (N/m)	
	(N/mm ²)	(N/mm ²)				
NBL	[12]	41.0	3.4	GGBS	0.10	122.0
	[13]	40.4	3.16	None	0.18	131.0
	[14]	40.0	2.39	None	0.50	76.6
NBM	[13]	49.1	3.27	Silica fume	0.18	132.0
	[12]	50.0	4.2	GGBS	0.10	160.0
	[15]	46.1	2.96	Fly ash	0.30	112.0
NBH	[12]	57.0	3.9	GGBS	0.10	145.0
	[13]	58.4	3.74	Silica fume	0.18	148.0
	[14]	61.0	3.06	None	0.50	142.0

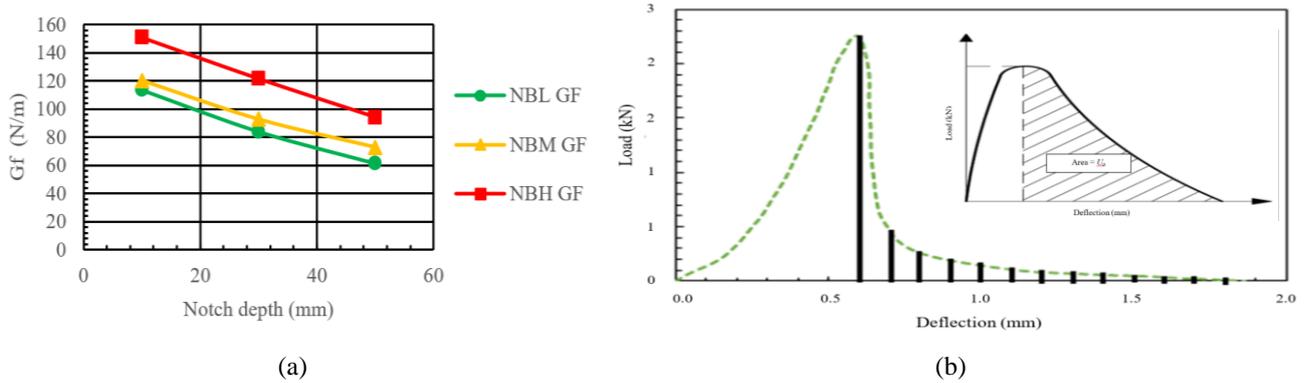


Fig. 4 - Fracture energy values determined from Hillerborg expression; (a) the correlation of concrete strength and notch depth to fracture energy value; (b) softening curve area used in Eq. (2)

The utilization of the Hillerborg model in this experiment incorporates the softening phase area, which is evident in the load-deflection curve as the declining section. In Figure 4(a), it is observed that the fracture energy (GF) increases as the compressive strength rises, while it decreases with an increase in the notch depth. Additionally, the Hillerborg model specifically requires the incorporation of the softening phase area, as shown in Figure 4(b), which corresponds to the declining section of the load-deflection curve. To calculate the area of the softening phase, the trapezoidal approximation technique is employed. For simplification purposes, the load on the y-axis is denoted as P , and the deflection on the x-axis is denoted as δ . Each trapezoid's area is calculated using the simple trapezium formula $A = 1/2h(a - b)$. In the context of this curve, the formula is expressed as $U_o = (P1 - P2/2)(\delta 1 - \delta 2)$. The area, U_o , represents the summation of all the trapezoid areas under the softening phase of the graph. It is worth noting that each concrete series exhibits dissimilarities due to variations in parameters. Therefore, the Hillerborg model proves to be more relevant in predicting the fracture energy in this particular experiment.

5. Conclusions

The experiment revealed a dependency of fracture energy and flexural strength on the compressive strength of the tested concrete. Generally, it was observed that fracture energy and flexural strength increased with larger concrete compressive strengths. This correlation suggests that higher-strength concrete possesses greater resistance to crack propagation and offers improved flexural performance. Furthermore, the fracture energy was found to decrease with deeper notch heights. The models proposed by Bazant and CEB-fib are widely used to analyse fracture behaviour in concrete. These models incorporate parameters such as aggregate size, measured compressive strength, and in the case of the CEB-fib model, the water-to-cement (w/c) ratio as well. On the other hand, Hillerborg's model introduces the requirement of a load-deflection profile to assess the concrete's softening region after cracking occurs. Interestingly, both the Bazant and Hillerborg failure models demonstrated better correlations compared to the CEB-fib model. In order to ensure the validity and reliability of the obtained results, a comparison was conducted between the findings of this experiment and those of other researchers in the field. This comparison aimed to validate the consistency of the results and maintain their legitimacy. Encouragingly, good agreements were found in terms of fracture energy values, as reported in the available literature.

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