Multi-agent Optimal Control of Ball Balancing on a Mobile Robot

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Abstract: Multi-agent systems have origin in computer engineering. However, they have found applications in different fields. One of the newly emerged problems in multi-agent systems is multi-agent control. In multi-agent control it is desired that the control is done in distributed manner. That is the controller of each agent should be implemented based on local feedback. In this paper a mechanism is introduced as a test bed for multi-agent control systems. The introduced mechanism is balancing of a ball on link located on a planar mobile robot. Dynamic equations of the mechanism are derived and the control task is distributed among two agents. For each agent a two loop controller is designed wherein external loop is a LQR controller and inner loop is a simple proportional controller. Regulation and fault tolerance performance of controller scheme is evaluated by simulations.

Keywords: Multi-agent system, Distributed control, Balance mechanism, Optimal control

1. Introduction

Distributed artificial intelligence or theory of multi-agent systems have been initiated in computer engineering (see for [1] and [2]), however in recent years they have been applied in many different fields. Multi-agent systems are composed of several interconnected subsystems. From a viewpoint, these subsystems operate autonomously and are controlled independently; thus each of them can be interpreted as an "agent". From another viewpoint they are components of a large system and they must have interaction and interconnection.

The main characteristic of multi-agent systems is their distributed nature, as they may be spatially or temporally distributed. Due to the advantages in simplicity, fault tolerance and scalability, research on multi-agent systems have been rapidly expanded in recent years and these systems have found interesting applications in multi-robot environments [3]-[6], transportation [7], [8], power networks [9]-[11] and … . Some applications such as multi-robot and transportation systems are innately distributed. However, some other applications such as power networks had been planned and controlled over decades as monolithic systems but in recent years, with some representations, they have been behaved as multi-agent systems.

One of the concepts associated with multi-agent systems is multi-agent control. Due to complexity and bigness of multi-agent systems, it is desired that their controllers is designed and implemented in a distributed manner. That is each controller takes feedback from its local states rather than taking feedback from all states of the system. Here, the main challenge is that the control objectives are essentially defined for the whole system and satisfying global objectives with local information needs innovations in control structure.

In this paper a distributed control scheme is proposed in order to balance a ball on a mobile robot. This work is somehow related to the widespread works done about control of ball and beam system (see [12] and [13] as two of recent ones). The difference is that the typical ball and beam system is not located on a moving platform. Moreover, no distributed control mechanism is applicable for ball and beam system. The ball balance system proposed in this paper is actually a new platform for design and test of multi-agent control systems. A two-loop distributed optimal control strategy is also provided for the proposed system.

The rest of the paper is organized as follows. In the next section the proposed ball balance system is introduced and its dynamic equations are derived. In the third section, the control task is distributed among two agents and a two-loop optimal control scheme is designed for the each agent. In Section 4 simulation results are provided and finally section 5 includes conclusions and discussions.

2. System Modeling

The system under study is shown in Figure 1. The base system is a planar mobile robot. A link with length L
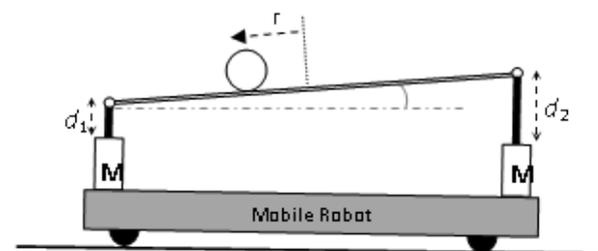
Fig. 1. The proposed ball balance mechanism

is located above the robot that is supported by two prismatic joints with joint variables d_1 and d_2. Prismatic joints are driven by two electric DC motors, shown by **M** in the figure. A ball with radius R is located on the link. Mass and moment of inertia of the link and ball are m_L,

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J_L, m_B and J_B respectively. We denote the distance of the ball from middle point of the link by r and the angle of the link with horizontal line by α. The goal of the control task is to balance the ball on the link and to keep it in the middle point of the link ($r = 0$, $\dot{r} = 0$) while the mobile robot is moving.

The control objective should be achieved by control of prismatic joints. Here, two control strategies are available: centralized control and distributed control. Schematic of these strategies are shown in Figure 2. In centralized control, a feedback from ball configuration is supplied to a central controller who is responsible for generating appropriate values of d_1 and d_2 to achieve control objectives. This problem is a multivariable nonlinear control problem which imposes complexities in design and analysis. In distributed control approach however; two independent controllers are implemented for d_1 and d_2. Each controller, which is considered as an agent, has its own local feedback from configuration of the ball. By using this structure, complexity of design and analysis is reduced and control objective is achieved by simpler controllers.

In order to model the system, as there is no communication between agents d_1 and d_2, it is assumed that each agent self-sufficiently drives the mechanism and it does not take into account the effect of other agent in the motion. That is when we model the system from the viewpoint of d_1, we assume $d_2 = 0$. By this assumption we can write kinetic and potential energy for the link and the ball with respect to a frame fixed on the mobile robot as:

$$K = \frac{1}{2} m_B \dot{r}^2 + \frac{1}{2} J_B \left(\frac{\dot{r}}{R}\right)^2 + \frac{1}{2} m_B \left(\frac{L}{2} - r\right)^2 \dot{\alpha}^2 \quad (1)$$

$$P = m_B g \left(\frac{L}{2} - r\right) \sin \alpha \quad (2)$$

where g is gravity acceleration. Then Lagrangian of the mechanism is obtained as $\mathbf{L} = K - L$. Since there is no direct actuation in r direction, Lagrange equation is written as:

$$\frac{d}{dt}\left(\frac{\partial \mathbf{L}}{\partial \dot{r}}\right) - \frac{\partial \mathbf{L}}{\partial r} = 0 \quad (3)$$

which yields:

$$\left(m_B + \frac{J_B}{R^2}\right)\ddot{r} - m_B\left(\frac{L}{2} - r\right)\dot{\alpha}^2 - m_B g \sin \alpha = 0 \quad (4)$$
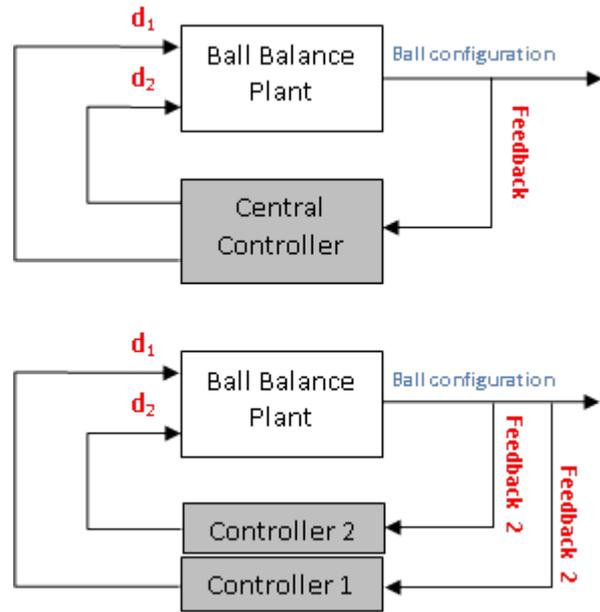
Fig. 2. Schematic diagram of centralized control (top) and distributed control (bottom) schemes available for control of ball balance system

As for a solid ball $J_B = \frac{2}{5} mR^2$, (4) can be written as:

$$\frac{7}{5}\ddot{r} - \left(\frac{L}{2} - r\right)\dot{\alpha}^2 - g \sin \alpha = 0 \quad (5)$$

As explained before, from viewpoint of agent d_1 it is assumed that $d_2 = 0$, then from Figure 1 one can obtain $\sin \alpha = \frac{d_1}{L}$. Assuming small velocities around equilibrium state, (5) is reduced to:

$$\frac{7}{5}\ddot{r} - g\frac{d_1}{L} = 0 \quad (6)$$

and state space representation of motion assuming d_1 as input is:

$$\dot{\mathbf{X}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\mathbf{X} + \begin{bmatrix} 0 \\ \dfrac{5g}{7L} \end{bmatrix} d_1 \quad (7)$$

where $\mathbf{X} = \begin{bmatrix} r & \dot{r} \end{bmatrix}^T$.

Using similar approach, we can derive state space representation of motion assuming d_2 as input as:

$$\dot{\mathbf{X}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\mathbf{X} + \begin{bmatrix} 0 \\ -\dfrac{5g}{7L} \end{bmatrix} d_2 \quad (8)$$

Equations (7) and (8) represent effect of d_1 and d_2 on motion as independent inputs (when each of them is independently treated as an agent). In practice, however, d_1 and d_2 are generated by electric DC motors. To obtain dynamic equations of a DC motor, neglecting armature inductance, equations of motor is given as:

$$V_{in} = I_a R_a + K_m \dot{\theta}_m \tag{9}$$

$$T_m = K_m I_a = \frac{J_m \ddot{\theta}_m + B\dot{\theta}_m}{K_g} \tag{10}$$

where V_{in} is input voltage of motor, R_a and I_a are resistance and current of armature, θ_m and J_m are angle and moment of inertia of motor, K_m is torque constant, K_g is gear ratio and B is friction constant. Assuming that joint variable d_1 is linearly dependent to θ_m with $\theta_m = K\ d_1$, transfer function of d_1 with respect to V_{in} is obtained from (9) and (10) as:

$$\frac{d_1(s)}{V_{in}(s)} = \frac{1}{s(as+b)} \tag{11}$$

where $a = \dfrac{KJ_m R_a}{K_g K_m}$ and $b = K(K_m + \dfrac{BR_a}{K_g K_m})$.

3. Controller Design

As it was explained in the previous section, the objective of control task is to regulate the ball in the middle point of the link. In the proposed control strategy, this objective is achieved in a distributed manner; that is agents d_1 and d_2 independently try to regulate their states to $\mathbf{X} = [r\ \ \dot{r}]^T = [0\ \ 0]^T$. To this aim, we design a linear quadratic regulator for each agent.

For state space equation (7), corresponding to agent d_1,

performance index is defined as:

$$J = \int_0^\infty \left(\mathbf{X}^T \mathbf{X} + {d_1}^2\right) dt \tag{12}$$

By minimizing above performance index we aim at getting fast response as well as consuming low control effort. It is well known from optimal control theory (see [13] for example) that the control law which minimizes (12) is given as:

$$d_1(t) = -\mathbf{B}^T \mathbf{P} \mathbf{X}(t) = -\mathbf{B}^T \mathbf{P}[r(t)\ \ \dot{r}(t)]^T \tag{13}$$

where $\mathbf{B} = \begin{bmatrix} 0 & \dfrac{5g}{7L} \end{bmatrix}^T$ and $\mathbf{P}$ is the positive definite solution of following Riccati equation:

$$\mathbf{A}^T \mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}\mathbf{B}^T \mathbf{P} + \mathbf{I} = \mathbf{0} \tag{14}$$

with $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $\mathbf{I}$ being 2×2 identity matrix. Solving (14) for $\mathbf{P}$ yields:

$$\mathbf{P} = \begin{bmatrix} \sqrt{\dfrac{14L}{5g}+1} & \dfrac{7L}{5g} \\ \dfrac{7L}{5g} & \dfrac{7L}{5g}\sqrt{\dfrac{14L}{5g}+1} \end{bmatrix} \tag{15}$$

substituting in (13), optimal control law is obtained as:

$$d_1(t) = -\begin{bmatrix} 1 & \sqrt{\dfrac{14L}{5g}+1} \end{bmatrix} \mathbf{X}(t) \tag{16}$$

Using similar method, we can obtain optimal control law for agent d_2 as:
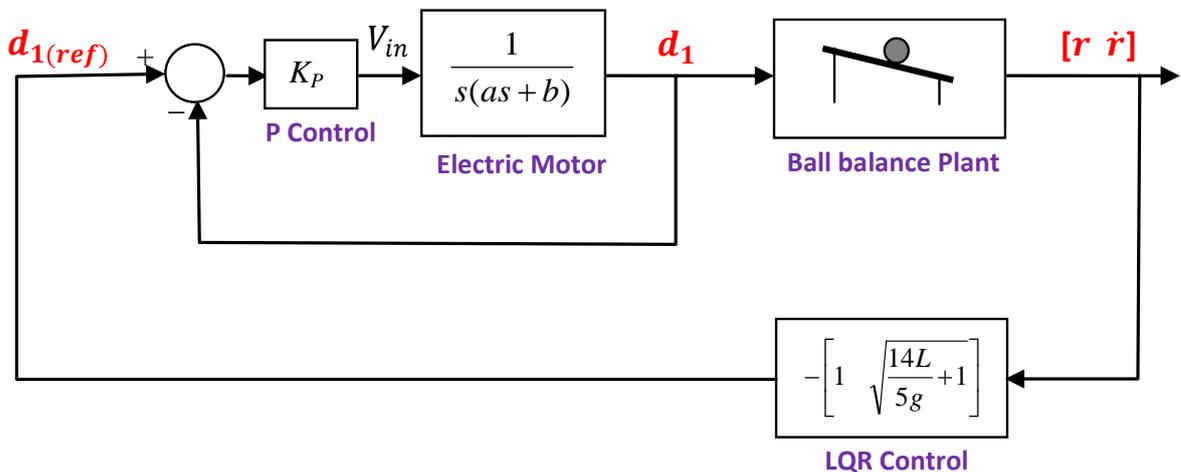
Fig. 3. Two-loop controller for agent d_1 in the proposed distributed control scheme. Inner loop is a P-controller and outer loop is LQR control. Agent d_2 has similar control diagram.

$$d_2(t) = \begin{bmatrix} 1 & \sqrt{\dfrac{14L}{5g}+1} \end{bmatrix} \mathbf{X}(t) \qquad (17)$$

Above LQR controllers provide desired values for d_1 and d_2 in an external control loop. These values are used as references for the controller of the electric motors in internal control loop. This concept is illustrated for agent d_1 in Figure 3. Agent 2 has similar control strategy. In inner loop, a simple proportional controller with parameter K_P is employed. Using this P-controller, transfer function of inner loop becomes:

$$\frac{d_{1(ref)}(s)}{V_{in}(s)} = \frac{\dfrac{K_P}{a}}{s^2 + \dfrac{b}{a}s + \dfrac{K_P}{a}} \qquad (18)$$

We design K_P in such a way that the critically damped response of electric motor is achieved. To this end:

$$K_P = \frac{b^2}{4a} \qquad (19)$$

4. Simulation Results

To evaluate performance of the proposed control scheme, model of mobile robot was built in MSC.ADAMS and, in order to implement controllers, it was linked to MATLAB/SIMULINK. Parameters of the simulated model are given in Table 1. By these parameters open loop transfer function of electric motor (equation (11)) becomes:

$$\frac{d_1(s)}{V_{in}(s)} = \frac{1}{s(22.429s + 321.028)} \qquad (20)$$

and from (16) we obtain $K_P = 1149$ that results in closed loop transfer function for electric motor as:

$$\frac{d_1(s)}{V_{in}(s)} = \frac{51.218}{s^2 + 14.313s + 51.218} \qquad (21)$$

From (14), LQR control law for agent d_1 is given as:

$$d_1 = \begin{bmatrix} 1 & 1.0555 \end{bmatrix} \mathbf{X}(t) \qquad (22)$$

With these controllers, closed loop poles of the entire system will be located at -21.868, -0.967, $-4.98 + j7.4$ which implies on stability of the system with a single dominant pole at -0.967 which makes the behavior of the system similar to first-order stable system. This is expected to provide a satisfactory behavior is for the system.

Table 1. Parameters of simulated model

Parameter	symbol	Value
Ball mass	m_B	0.06 kg
Ball radius	R	0.025 m
Link mass	m_L	0.6 kg
Link Length	L	0.4 m
Torque constant	K_m	0.007 N.m/A
Gear ratio	K_g	24
Armature resistance	R_a	$2\,\Omega$
Moment of Inertia of Motor	J_m	6×10^{-4} Nms^2/rad
Friction constant	B	8×10^{-3} Nms/rad
Rot/Lin transfer ratio	K	3.14×10^3 rad/m
Gravity Acceleration	g	9.81 m/s^2

4.1 Regulation Test

In the first simulation we assumed that the link is initially located in horizontal configuration (i.e. $\alpha = 0$, $d_1 = d_2 = 0$) and the ball is initially at rest and it is located at 0.1m distance from the middle point of the link (i.e. $r = 0.1$, $\dot{r} = 0$). Simulations were carried out in two different cases. In the first case, we assumed that the mobile robot does not move. In the second case, however, it was assumed that the mobile robot moves with the following motion equation:

$$x_{robot}(t) = \begin{cases} -\dfrac{1}{3}t^3 + \dfrac{5}{2}t^2 & 0 < t < 5\text{sec} \\[2mm] \dfrac{125}{6} & t > 5\text{sec} \end{cases} \qquad (23)$$

which imposes acceleration variation of $\begin{bmatrix} -5 & 5 \end{bmatrix}\dfrac{m}{s^2}$ that is relatively large variation. Simulation results for both cases are shown in Figure 4. When the robot does not move, as it is expected, the system can regulate the ball into middle point of the link. When the mobile robot moves and applies an external (disturbance) force to the control mechanism, the system can regulate the ball, as well. The interesting fact is that there is no significant difference between curves in simulations of two cases. This implies that the system can successfully regulate the ball even when the robot moves.

4.2 Fault Tolerance Test

One of the most interesting advantages of multi-agent systems is their fault tolerance property. That is the whole system can undergo a graceful degradation when some agents fail. To evaluate fault tolerance in the proposed mechanism, we repeated the simulation of the system for second case of previous tests. However, here we assumed that the agent d_1 fails at t=2 (because of for example error in its motor or sensor etc.). Simulation

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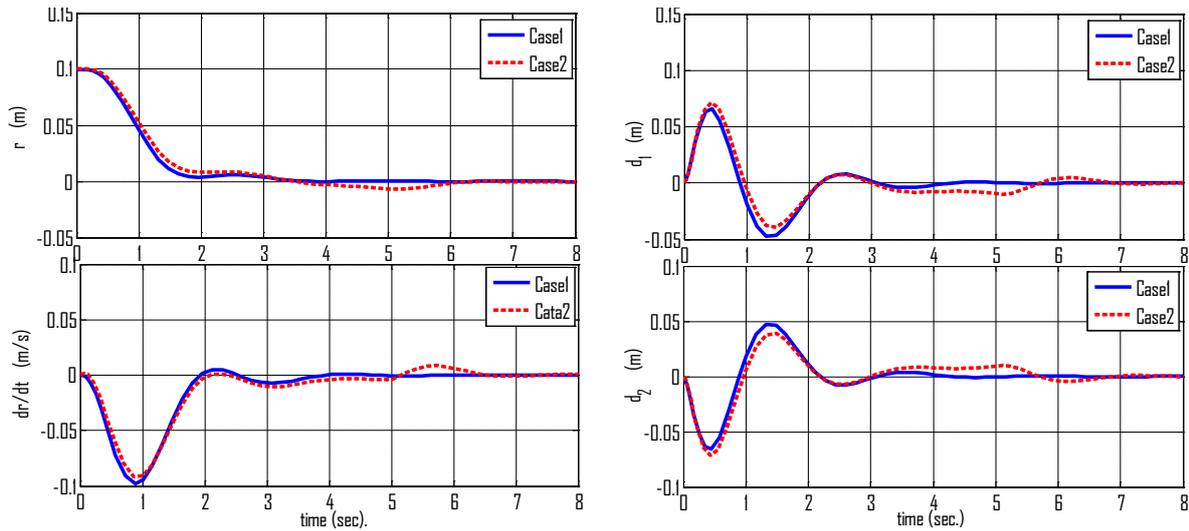
Fig. 4. Simulation results of balancing the ball in two cases.
Case 1: The mobile robot does not move   Case 2: The mobile robot does moves with (23)
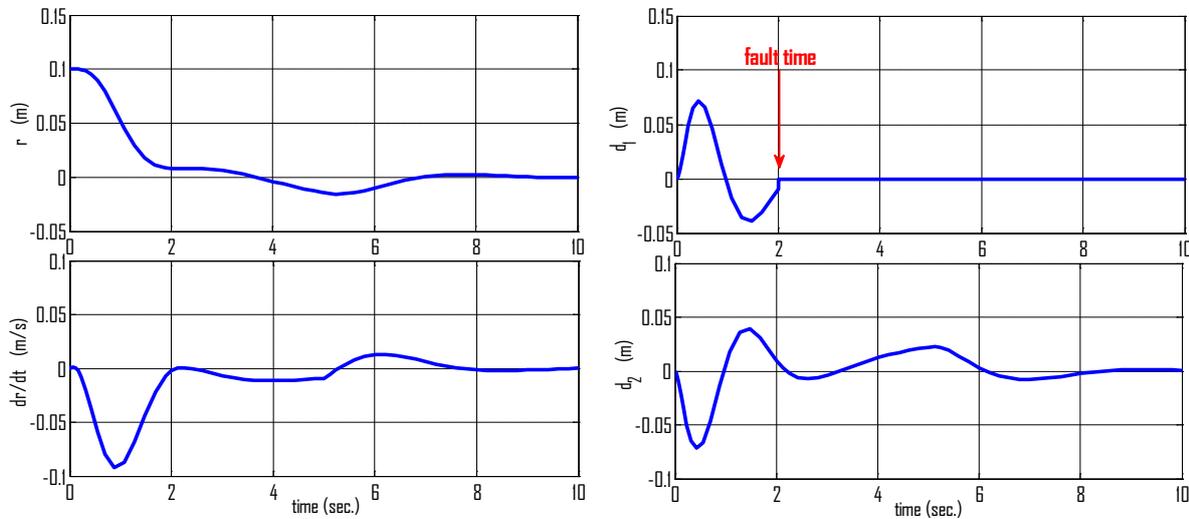
Fig. 5. Simulation results of fault tolerance test. A failure occurs at $t=2$ in d_1, however d_2 can individually complete the control task and regulate the ball.

results are shown in Figure 5. After failure of d_1, agent d_2 is responsible for regulating the ball. By designed control scheme, it could re-plan its motion in order to compensate failure of d_1 and complete the control task successfully. It can be seen in Figure 5 that the mechanism could properly control the ball.

5. Conclusions

A ball balance system was introduced as a platform to design and analyze multi-agent control system. It was illustrated that there is two approaches to control this system. In central control approach a multivariable nonlinear control should be designed for the system that is a complicated task. In distributed control approach the

task is distributed among agents, therefore simpler controllers are expected. Based upon this rationale the system was modeled. The control task was distributed among two agents and for each agent a control scheme was developed. The control scheme includes a LQR control in outer loop and a simple P-controller in inner loop. Appropriate values for control parameters were designed. To evaluate the performance of the controllers, the mechanism was constructed in MSC.ADAMS and linked to MATLAB/SIMULINK. Simulation results showed that the system can balance the ball on the link. Moreover, the system is fault tolerance as one of the agents can individually complete the control task when the other agent fails.

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