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A Comparison of Beat Frequency Estimation Methods for Large Ring Laser

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Abstract: Autoregressive (AR2) technique has always been used to estimate frequency of the output signal from Large ring laser. However, the acquisition rate is not at near real time which is the requirement and noise level still challenge the process resulting to errors in the final estimation. A research was done to compare the Autoregressive (AR2) with the counterparts such as Pisarenko, Quinn, Hilbert and Phase looking for a better technique that will estimate the frequency at near real time to minimize errors. Secondary data from G and C – II ring laser were used during the comparison between the techniques and Autoregressive (AR2). Results shows that, the output characteristics from the counterpart does not depict the oscillations of the Earth rotation as expected contrast to that of Autoregressive (AR2) which does. Moreover, there were much deviation from the expected true value for the techniques contrast to that of AR2 which is very minimum. On the other hand, when the C – II data were used, it was observed that both techniques resemble on their output characteristics though AR2 was still better in the acquisition rate expect for Hilbert transform which does not resemble with others. Following the scope of this paper, Autoregressive (AR2) technique still emerge as a favorite frequency estimation technique contrast to the four counterparts due to its robustness, high acquisition rate as well as low noise level.

Keywords: Large Laser, C – II Ring Laser, Frequency Estimation, Earth Rotation Rate, Acquisition rate

1. Introduction

Monitoring the Earth rotation rate is usually done by the use of different instruments which differs in their resolution following the number of factors leading to its variation (Iranfar, 2015). Among many, large ring laser is the prominent instrument used for the purpose due to the fact that it is capable of being used in both seismology as well as geodesy (Virgilio et al., 2010). The instruments have demonstrated to currently the most sensitive when it comes to test the rotation motion of the Earth with respect to inertia frame (Beverini et.al, 2016). Other instruments are limited to only one application like Very-Long-Baseline-Interferometry (VLBI) which is only applicable for geodesy while seismometer array and fiber optic gyroscope (FOG) are not as good as Large Ring Laser in terms of accuracy. Monitoring Earth rotation is of importance as the data obtained are used in various application that includes disaster prediction, monitoring and mitigation (Zuzek, 2019).

Monitoring Earth rotation rate by Large Ring Laser is done in an indirect way such that the frequency of the photodiode's output is to be estimated in a very short time for accurate results. Frequency estimation is mostly done by the use of Autoregressive technique (AR2) as applied by Clive et al. (1999) which was implemented by B. Tom King for the purpose of estimating the beat frequency of a Ring Laser Gyroscope (RLG) and its spectral line width. During estimation, the expected frequency to be detected are in the range of $10-14$ rad/sec $\leq \Omega s \leq 1$ rad/sec and frequency of the signal for seismic wave lies in the range of 0.03 Hz $\leq fs \leq 10$ Hz. The standard acquisition rate of seismic signal which covers most of the frequency contents associated with seismic activities is 20 Hz which is equivalent to 0.05 sec.

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This means that in every 0.05 sec, the Sagnac frequency is estimated. However, the acquisition rate may vary depending on the frequency of the waves to be detected.

To estimate the frequency, the second order autoregressive technique (AR2) employ two parameters such as a_1 and a_2 as given in equation (1) below:

$$\epsilon(t) = y_t + a_1 y_{(t-1)} + a_2 y_{(t-2)} \tag{1}$$

Where $\epsilon(t) - \text{noise imposed}, y_t - \text{descrete time sample of the data set}$

The frequency is then calculated from the equation (2);

$$\mathrm{f} = \cos^{-1}\left(-\frac{\beta_1}{2\sqrt{\beta_2}}\right) \tag{2}$$

Frequency estimation can be done using MATLAB or LABVIEW whereby the values obtained are plotted on a graph to observe frequency variations and eventually its' efficiency.

The technique is well suited in the dynamic situations such as seismic event events on monitoring the variations of Earth rotation (Jason, 2011), it is also stable for short segment of signal as well as the estimation is done in an online at reasonable rate. The technique also assumes that each observation in the time series is a noisy linear combination of some previous observations along with a constant shift (Chen et al., 2019).

The prevailing challenge associated with Autoregressive technique (AR2) during frequency estimation is that the acquisition rate is not high enough i.e., at near real time as required, thus leading to error accumulation during final estimation. Following this, effective and efficient technique that will be able to estimate frequency at short period of time (near real time) contrast to AR2 needs to be identified and used during estimation for error minimization.

Frequency estimation has been done in various fields using different techniques other than autoregressive. Among many fields that use an estimated frequency for further analysis includes medicine for monitoring heart rate (Johnson et al. 2011), determination of Earth rotational rate, carrier recovery in communication systems (Scholnik, 1980), determination of object position in radar and sonar systems (Richard, 2010), vibration analysis in machines and buildings etc. For Earth rotation rate determination, in addition to autoregressive technique, other four techniques can also be used for the purpose provided their efficiency surpass that of autoregressive technique in terms of acquisition rate, output characteristic as well as standard deviation.

One of the four is Quine-Fernandes frequency estimator which belongs to an adaptive notch filters class with the general idea of introducing a filter that removes a particular frequency from the signal, the "notch" frequency Kootsooks (1959). The accuracy of the technique depends on the noise level which increases as the noise level is lowered and the perfect initial estimate. Quinn and Fernandes also employ two parameters in the estimation which are α and β found in equation (3).

$$y_t - \beta y_{t-1} - y_{t-2} = \varepsilon_t - \alpha\varepsilon_{t-1} + \varepsilon_{t-2} \tag{3}$$

Subject to $\alpha = \beta$.

In MATLAB, the estimation is done based on the number of samples (N) for each estimate in which a function with only one output was developed by Quinn and Fernandes and the input were a time series signal from ring laser. Pisarenko Harmonic Decomposition is also capable of estimating frequency from large ring laser whereby it assumes that the signal is composed of a complex exponential in the presence of a white noise. It also assumes that the values of the correlation matrix are either known or estimated (Pisarenko, 1973). The frequency estimated can be done mathematically using equation (4) which represents the angles of the roots of eigen filter.

$$V_{min}(z) = \sum_{k=0}^{p} V_{min}(k) Z^{-k} \tag{4}$$

Using MATLAB, Pisarenko generate a function called "omegahat" which finds the Pisarenko frequency estimates of the signal in each column. The input data should be filtered first to get rid of other frequencies or large variation due to noise and the data are executed to obtain frequency estimates.

Moreover, Kay (1988) developed a window estimator used to analyse the output signal from ring laser. The window estimator is used in the weighted linear predictor or phase frequency estimation technique to estimate the Sagnac frequency using equation (5).

$$\hat{w}_0 = arg\left(\sum_{t=1}^{T-1} w_t\, z_t z_{t-1}^{*}\right), \tag{5}$$

Where w_t-is a given window function, Arg (z) – phase of complex-valued z

Hilbert transform is also a technique is used to estimate the frequency of the output signal from ring laser as it is used in MATLAB to calculate the instantaneous attributes of the time series signal. It is capable of calculating the unknown parameters with a high precision as well as delivering results on a real time basis (Petrovic & Nada, 2017). It first calculates the Fast Fourier Transform (FFT), replacing other frequencies that do not match with the intended one by zeros and finally the inverse of FFT (Ditchborn, 1959). For the sake of effective technique during estimation, the four techniques are to be compared with the Autoregressive (AR2) technique especially on their acquisition rate, output characteristic as well as standard deviation.

2. Methodology

The study is descriptive in nature whereby the collected data from the experiment were used to examine and compare the discussed frequency estimation techniques in the previous sections. Secondary data from Large Ring Laser (G-ring laser) and C-II ring laser were used during the comparison.

For the case of G-ring laser two data sets with different sampling rate collected during the Ring laser operation in the site were used. The sampling rate for the first data set was 10 kHz, expected Sagnac frequency was 348.5 Hz, the total acquisition time was 600 sec and the acquisition rate was 1 Hz. For the second data set from G-ring laser, sampling rate was changed to 2 kHz and the volume of the data were large with some Earth quake and other disturbances carrier frequency. The acquisition rate, total acquisition time as well as the expected Sagnac frequency were kept constant due to the fact that the same instrument was used during the process.

The two data sets from G-ring laser were used by a researcher contrast to C-II ring laser data due to the fact that, the instrument is currently used (active) for Earth rotation rate monitoring, therefore it will provide reliable results during the comparison.

On the other hand, single secondary data set from C-II ring laser was used for the analysis, due to the fact that the instrument is no longer used (inactive) for the purpose thereof. The C-II ring laser data are significant during comparison in the sense that they will ascertain the performance of the techniques in addition to what has been observed in the G-ring laser which will subsequently provide reliable results from the comparison.

For the C-II ring laser, the expected Sagnac frequency was 79.76 Hz, the sampling rate of 1000 Hz was used, the acquisition rate was 1 Hz while the total acquisition time was 600 sec. The expected Sagnac frequency for the two ring lasers observed to be different because the instruments were designed differently and so their operation as well as Sagnac frequency.

The analysis of the mentioned techniques was done through the use of MATLAB software. The software was mainly used during frequency estimation after input the secondary data from the instruments. The two different types of data set from G-ring laser were used during analysis or comparison of the techniques with respect to Autoregressive (AR2). The analysis was done separately for each of the data and results were obtained.

Nevertheless, data from C – II ring laser were also used in the analysis though the instruments is currently inactive at the station. This is done for the purpose of comparison due to the fact that both ring laser were designed for the same function i.e., monitoring the Earth rotation rate with different Sagnac frequency. The results were presented graphically indicating the elapsed time (ET), standard deviation (SD) as well as the expected output characteristics. In addition to that, the nearly correlated techniques were presented on the same graph representing their deviation from mean value to observe how do they relates on their output's characteristics.

3. Results and Discussion

Frequency estimation algorithms are used to estimate frequency of the varying output signal on the photodetector resulted from the Earth rotation with the variations imposed by seismic activities. Secondary data that has been collected from large, G- and C-II ring laser at a specified sampling rate were used as inputs to the frequency estimation algorithm and the end result will be the estimated frequency of the Earth rotation rate.

As mentioned before, when comparing the algorithms, Autoregressive is the reference because the aim of the comparison is to find out the algorithm that will surpass the performance of AR (2).

For G–ring laser, two data sets with different sampling rate were used to analyse results. The sampling rate of the first secondary data from the ring laser was 10 kHz collected for 600 sec, 348.5 Hz as the Sagnac frequency and acquisition rate was chosen to be 1 Hz (1sec).

The outputs from MATLAB for each algorithm were presented in Fig. 1 with the same length of time.

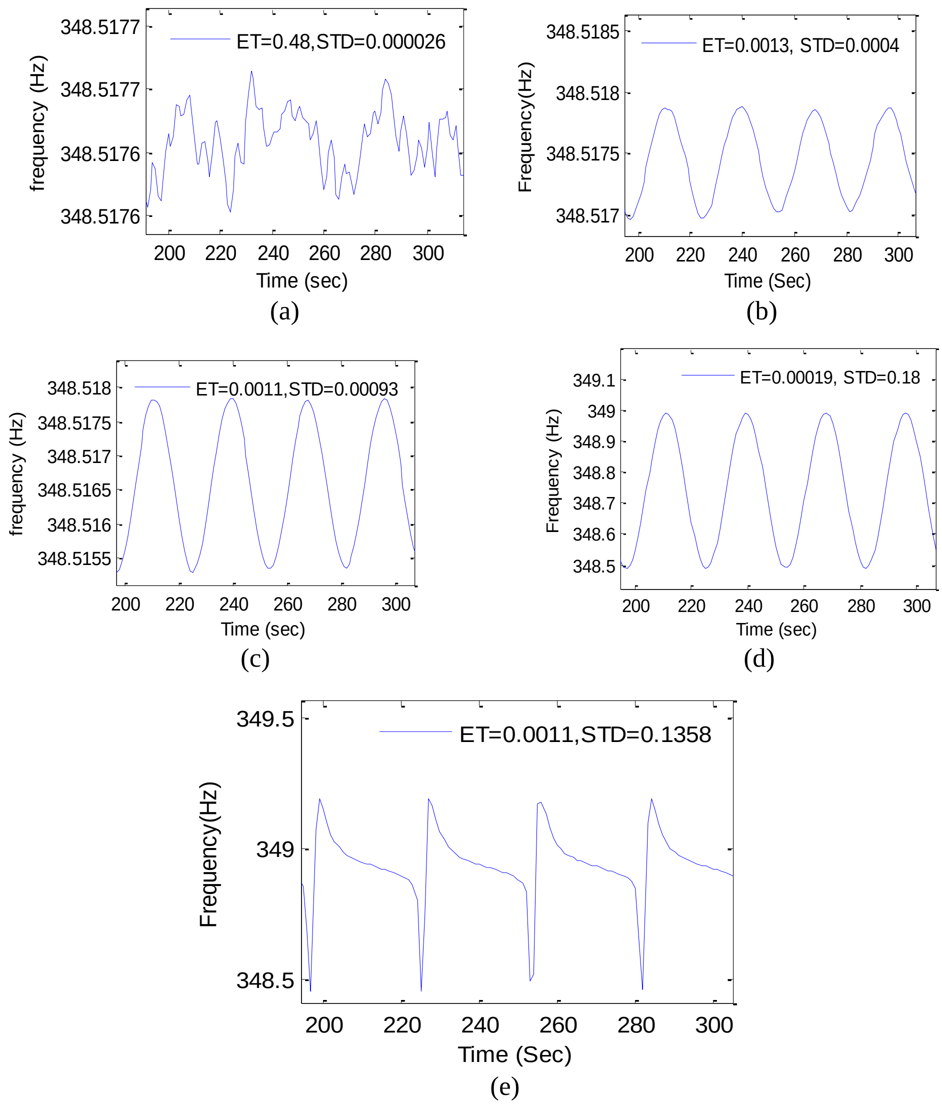

Fig. 1 - (a) AR (2); (b) Quinn & Fernandes; (c) Weighted linear phase predictor; (d) Pisarenko; (e) Hilbert transform

From Fig. 2, both of the techniques do not match on their outputs with that of AR (2) as they contain oscillations on their outputs except for Hilbert transform which varies in a different way not resemble to either of the techniques. Moreover, the deviations of Pisarenko and Hilbert transform are quite large as compared to the three techniques which seems at least to be equal. Because the deviation of AR (2), Phase and Quinn & Fernandes are nearly equal, they can be plotted in the same graph to see their deviations as shown in Fig. 2.

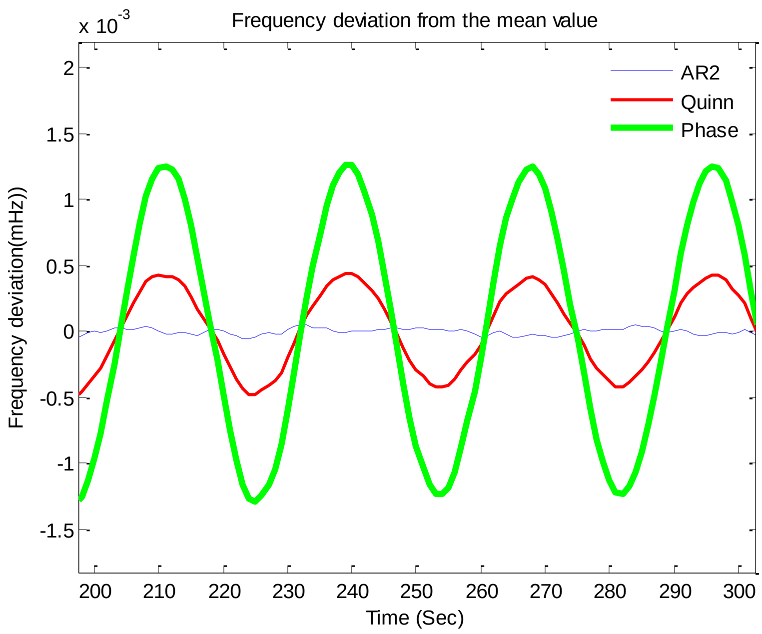

Fig. 2 - AR (2), Quinn and Phase technique on the same plot

Fig. 2, indicates that Quinn and Phase technique has the same oscillations. They are still not resembling to AR (2) due to oscillations. Hence to be able to compare the techniques especially on the acquisition rate and standard deviation, one of the prerequisites is that their outputs should first resemble to that of AR (2) i.e., there should be no any oscillations as observed. When the acquisition or sampling rate was changed to 2000 HZ, using the other data set from large ring laser containing seismic induced waves, the AR (2) still perform better than the rest as they still contain oscillations on the outputs as seen in Fig. 3.

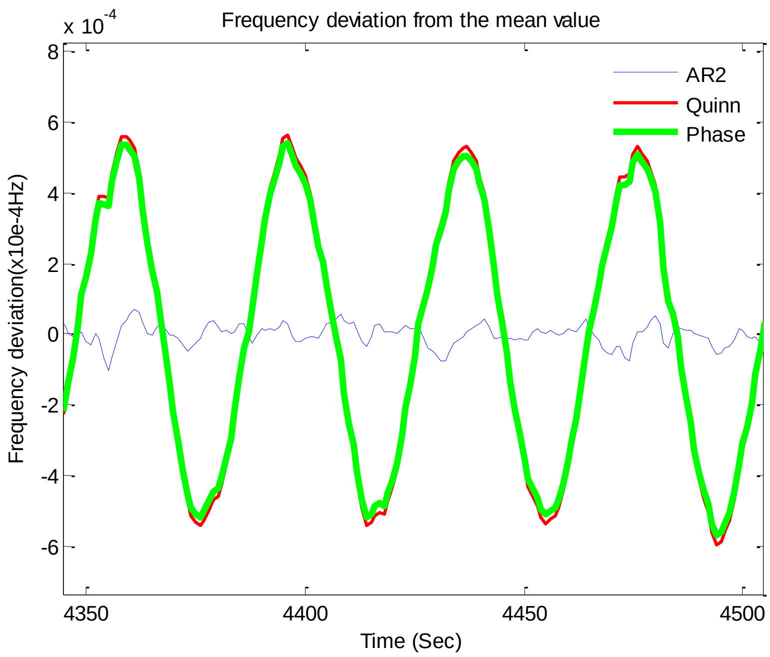

Fig. 3 - AR (2), Quinn and Phase technique on the same plot for second G-ring laser data

Likewise for the C-II ring laser data, the predetermined Sagnac frequency was 79.76 Hz, with the sampling frequency of 1000 Hz and the acquisition rate was primarily considered to be 1Hz during the estimation. The output from each algorithm is obtained using MATLAB software and their trends are also observed in comparison with AR (2) as shown in Fig. 4.

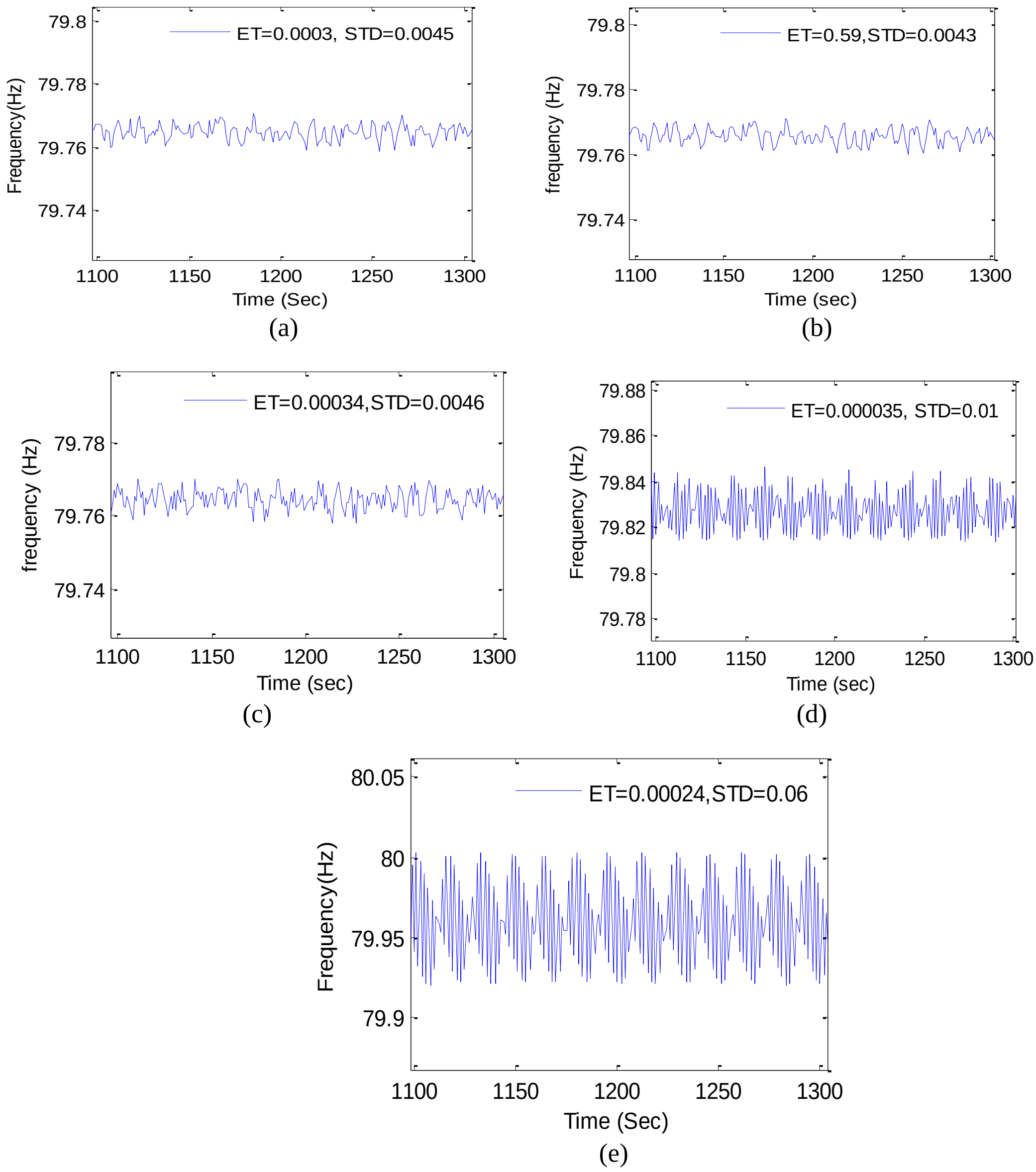

Fig. 4 - (a) AR (2); (b) Quinn & Fernandes; (c) Weighted linear phase predictor; (d) Pisarenko; (e) Hilbert transform

The first three graphs are nearly equal in their deviations, and in comparisons they can be plotted on the same graph and observe their trend. Fig. 5 presents the graphs for Autoregressive technique (AR (2)), Quinn and Fernandes as well as Weighted phase technique both on the same plot.

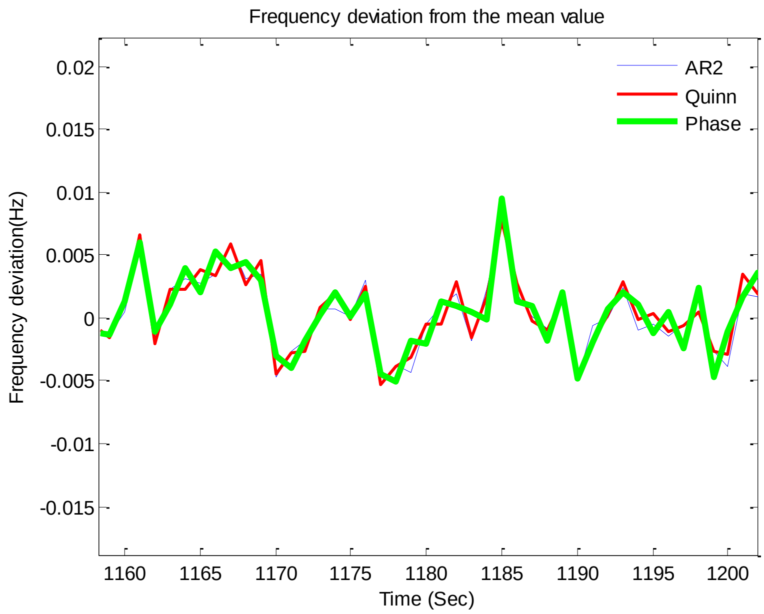

Fig. 5 - AR (2), Quinn & Fernandes and Phase technique for C-II ring laser

The three techniques in Fig. 5 above seems to produce resemble outputs. The outputs from AR (2), Quinn as well as Phase technique produce the same output that depicts the rotation of the Earth. The remaining two techniques i.e., Pisarenko and Hilbert (Fig. 4 (d & e)) still outputs different characteristic which does not depict the rotation of the Earth; thus, they are not comparable to AR (2).

Following the comparable outputs of Quinn and Phase technique to AR (2), they are qualified for further comparison on the other factors such as processing time as well as standard deviation during frequency estimation. Even though the Sagnac frequency for the two instruments are different following their designation, processing time and standard deviation for AR (2) still perform better than the counterparts which make it to suitable for the purpose. Unfortunately, the comparison would not be productive following the inactiveness of the instruments in the site as it is no longer used for monitoring the Earth rotation.

4. Conclusion

From the analysis of the outputs from Large Ring laser based on the two data sets it is clearly that despite the fact that the sampling rate were different but the AR (2) technique still appear to produce the expected output characteristic of the Earth rotation rate. The other techniques seem to be out of the expected outputs characteristics which is the pre-requisite factor for further comparison with AR (2) based on other criteria such as processing time as well as standard deviation when estimating Sagnac frequency. Therefore, the comparison between AR (2) techniques with respect to other techniques was feasible for only one criterion i.e., output characteristics whereby the technique still prevail. The comparison based on acquisition rate and standard deviation was not possible following the discrepancies of the output characteristic with that of AR (2). On the other hand, both AR (2), Quinn and Phase technique produce the same output characteristics for C – II ring laser as observed. On the case of processing time and standard deviation AR (2) still emerge victorious as compared to the two techniques whereby the processing time and standard deviation for AR (2) are less compared to the two counterparts. Therefore, following the results from the two instruments, the Autoregressive (AR2) technique still remain to be robust as far as estimation of Sagnac frequency from large ring laser is concerned. This is due to the fact that acquisition rate is much better compared to the rest of the technique though it is not at near real time, the standard deviation is less contrast to other techniques.

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