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Modeling of Mass Transfer Process of Prolate Spheroidal Drops in Rotating Disc Contractor Column

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Abstract

Several models have been developed for the modeling of Rotating Disc Contactor (RDC) columns. The modeling shows that the drop size distribution and the mass transfer processes are important factors for the column performances. Since the behavior of the drop breakage and the mass transfer process involve complex interactions between relevant parameters, the need to get as close as possible to the reality of the processes is evident. Several researchers have been working in this area. Most of these models have been studied based on the assumption of spherical droplets. The problem of spherical drop or bubble is known as the simplest and ideal case in which the problem can be considered in spherical coordinate system. However there are many physical situations the shape of the drops contained in liquid is not perfectly spherical, and may be classified as prolate or oblate spheroids. For most industrial applications particles encountered are irregular or non-spherical. In this research, the diffusion equation given in the prolate spheroidal coordinate system is used for a two-dimensional case. An analytical solution of the unsteady diffusion equation describing mass transfer for prolate spheroidal drops, considering a constant diffusion coefficient is presented. The resulting equations are analytically solved by using the Laplace transform method.

Keywords: mass transfer; RDC column; prolate spheroidal coordinates; laplace transform method

1. INTRODUCTION

Several models have been developed for the modeling of RDC columns. The modeling shows that the drop size distribution and the mass transfer processes are important factors for the column performances. Several researchers namely Korchinsky and Azimzadeh [8], Talib [14], Ghalehchian [6], Maan [10] and Bahmanyar [1] had been working in this area. Most of these models have been studied based on the assumption of spherical drops. The problem of mass transfer of spherical drop is known as the simplest and ideal case in which the problem can be considered in spherical coordinates system. However there are many physical situations the shape of the drops contained in liquid is not perfectly spherical, and may be classified as prolate or oblate spheroids [3]. For most industrial applications particles encountered are irregular or non-spherical [11]. Furthermore According to [8] the drops or particles have the shapes that are closer to the spheroidal than to the spherical. In this paper, the model will be approximated by prolate spheroidal coordinates. The resulting of equations are analytically solved by using the Laplace Transform method.

2. PROLATE SPHEROIDAL COORDINATES

A prolate spheroid is generated by rotating an ellipse about its major axis contrasted with oblate spheroid. The prolate spheroidal coordinate related to the Cartesian coordinate was presented by [5], [12] and [15] through the transformation equations

$$x = L\sinh\mu\sin\phi\cos\omega \tag{1a}$$

$$y = L\sinh\mu\sin\phi\sin\omega \tag{1b}$$

$$z = L\cosh\mu\cos\phi \tag{1c}$$

where $L = \sqrt{L_2{}^2 - L_1{}^2}$ is the focal distance of the prolate spheroidal drop measured from the coordinate origin, and L_1 and L_2 are the major and minor axes, respectively.

An ellipsoid of revolution scheme is shown in figure 1.

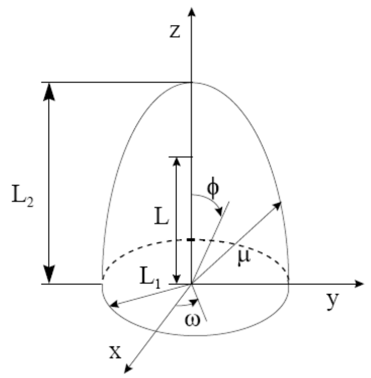

Figure 1: Characteristics of a prolate spheroidal drop

Acording to Elkamel [4] there are other equivalent transformations obtained from (1) by defining $\xi = \cosh \mu$ and $\eta = \cos \phi$

$$x = L\sqrt{(\xi^2 - 1)(1-\eta^2)}\cos\omega \tag{2a}$$

$$y = L\sqrt{(\xi^2 - 1)(1-\eta^2)}\sin\omega \tag{2b}$$

$$z = L\xi\eta \tag{2c}$$

Where $\xi \geq 1, \quad -1 \leq \eta \leq 1, \quad 0 \leq \omega \leq 2\pi, \quad \mu, \phi$ are called the radial and angular variables, respectively. The Laplacian operator ∇^2 in the prolate spheroidal coordinate can be written as

$$\nabla^2 = \frac{1}{L^2(\xi^2 - \eta^2)}\left\{\frac{\partial}{\partial \xi}\left[(\xi^2 - 1)\frac{\partial}{\partial \xi}\right] + \frac{\partial}{\partial \eta}\left[(1-\eta^2)\frac{\partial}{\partial \eta}\right] + \frac{\xi^2 - \eta^2}{(\xi^2 - 1)(1-\eta^2)}\frac{\partial^2}{\partial \phi^2}\right\} \tag{3}$$

Eq. (3) will be used later to derive the diffusion equation in prolate spheroidal coordinates.

3. GOVERNING EQUATIONS AND SOLUTION MODEL

The governing equation for the diffusion process based on Fick's second law of diffusion can be written in simplified notation as:

$$\frac{\partial u}{\partial t} = D(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) = D\nabla^2 u \tag{4}$$

where ∇^2 is known as the standard Laplacian operator which is the second order partial derivative. Eq. (4) is an appropriate equation to predict mass diffusion in bodies with a rectangular shape [2] and [9]. To predict the phenomenon in ellipsoidal drops, it is necessary to transform this equation into an appropriate coordinate system, in this case , the prolate spheroid coordinate system. By using (3) and considering the constant diffusion coefficient, (8) can be written in prolate spheroidal coordinates as follows:

$$\frac{\partial u}{\partial t}=\frac{D}{L^2(\xi^2-\eta^2)}\left\{\frac{\partial}{\partial\xi}\left[(\xi^2-1)\frac{\partial u}{\partial\xi}\right]+\frac{\partial}{\partial\eta}\left[(1-\eta^2)\frac{\partial u}{\partial\eta}\right]+\frac{\xi^2-\eta^2}{(\xi^2-1)(1-\eta^2)}\frac{\partial^2 u}{\partial\phi^2}\right\} \quad (5)$$

The model to predict mass transfer in prolate spheroidal coordinates, for a situation with symmetry around the z axes is given by

$$\frac{\partial u}{\partial t}=\frac{D}{L^2(\xi^2-\eta^2)}\left\{\frac{\partial}{\partial\xi}\left[(\xi^2-1)\frac{\partial u}{\partial\xi}\right]+\frac{\partial}{\partial\eta}\left[(1-\eta^2)\frac{\partial u}{\partial\eta}\right]\right\},$$

$$\xi\geq 1,-1\leq\eta\leq 1 \quad (6)$$

Eq. (6) With Initial condition

$$u(\xi,\eta,0)=0,\xi\geq 1,-1\leq\eta\leq 1 \quad (7)$$

and boundary condition

$$u(\xi_0,\eta,t)=u_0,t\geq 0,\xi=\xi_0,-1\leq\eta\leq 1 \quad (8)$$

$$\frac{\partial u}{\partial\eta}=0,\eta=0 \quad (9)$$

Where u is the concentration, t is the time, D is the coefficient of diffusion and ξ,η are the radial and angular coordinates in prolate spheroidal coordinates. This model is solved by using the method of Laplace Transform [13]. Taking the Laplace transform of the concentration with respect to time, we can obtain

$$\frac{\partial}{\partial\xi}\left[(\xi^2-1)\frac{\partial U}{\partial\xi}\right]+\frac{\partial}{\partial\eta}\left[(1-\eta^2)\frac{\partial U}{\partial\eta}\right]=L^2(\xi^2-\eta^2)q^2U \quad (10)$$

Where $U(\xi,\eta,p)=\ell_t u(\xi,\eta,t)$ and $q=\left(\frac{p}{D}\right)^{\frac{1}{2}}$

$$\xi=\xi_0,U=\frac{u_0}{p},-1\leq\eta\leq 1 \quad (11)$$

$$\eta=0,\frac{\partial U}{\partial\eta}=0,\xi_0\leq\xi\leq\infty \quad (12)$$

$$u(\xi,\eta,t)=u_0\frac{Q_0(\xi)}{Q_0(\xi_0)}\left\{\begin{array}{l} erfc\left[\frac{\xi-\xi_0}{2}T^{-\frac{1}{2}}\right]+A_0(\xi,\xi_0)(\xi-\xi_0)(\pi T)^{-\frac{1}{2}}\exp\left[-\frac{\xi-\xi_0}{4T}\right] \\ +\left(B_0(\xi,\xi_0)+5P_2(\eta)B_2(\xi,\xi_0)\right) \\ \frac{(\xi-\xi_0)^3}{2}(\pi T^3)^{-\frac{1}{2}}\exp\left[-\frac{(\xi-\xi_0)^2}{4T}\right]+\ldots \end{array}\right\}$$

(13)

Where

$$A_0(\xi,\xi_0) = 1 + \frac{Q_0(\xi) - Q_0(\xi_0)}{(\xi-\xi_0)Q_0(\xi)Q_0(\xi_0)} \tag{14}$$

$$B_0(\xi,\xi_0) = \left\{ \begin{array}{l} \dfrac{2\xi-\xi_0}{3(\xi-\xi_0)} + \dfrac{3\xi-4\xi_0}{3(\xi-\xi_0)^2 Q_0(\xi_0)} + \dfrac{1}{(\xi-\xi_0)^2 {Q_0}^2(\xi_0)} + \\ \dfrac{3\xi_0-\xi}{3(\xi-\xi_0)^2 Q_0(\xi)} - \dfrac{1}{(\xi-\xi_0)^2 Q_0(\xi) Q_0(\xi_0)} \end{array} \right\} \tag{15}$$

$$Q_0(z) = \frac{1}{2}\ln\left(\frac{z+1}{z-1}\right) \tag{16}$$

$$T = \frac{Dt}{L^2} \tag{17}$$

Figure 2 shows profile of the concentration (u) of the drop as a function of the Fourier number (T) for different values of the shape. For a prolate spheroidal drop, as the shape decreases the concentration of the drop increases.

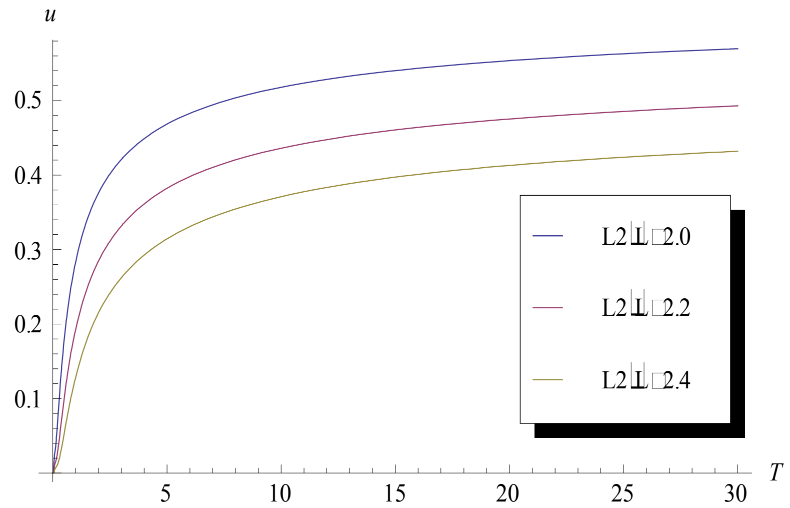

Figure 2: Profile the concentration as a function of Fourier number for several different of shapes

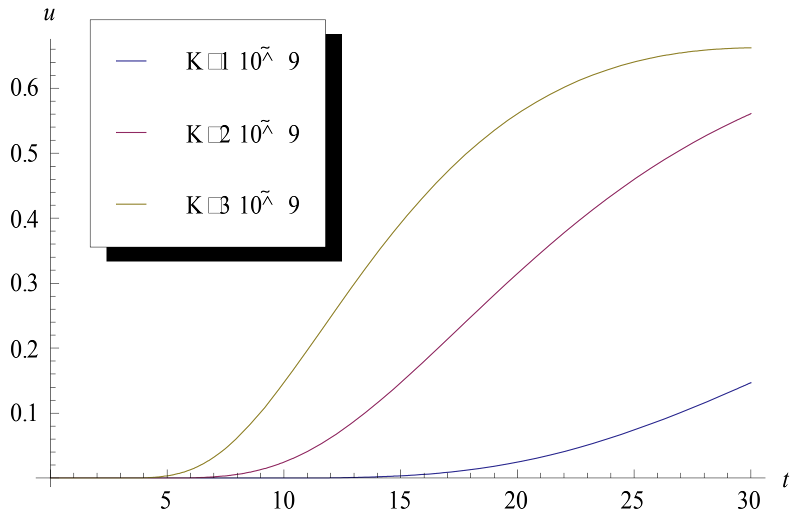

Figure 3: Profile the concentration as a function of time for several different of diffusivities

On the other hand in Figure 3 shows profile of the concentration (u) of the drop as a function time (t) for different values of the diffusivity. These results agreed to the previous results presented in [2] and [14].

4. CONCLUSIONS

It has been presented the analytical solution for two dimensional diffusion equations in prolate spheroidal coordinates by using the Laplace transform method. This model presents a good agreement to the previous results and can be used to describe mass transfer for prolate spheroidal drops in RDC column.

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