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Using Historical Return Data in the Black-Litterman Model for Optimal Portfolio Decision

Stephanie See Weng Su[1*], Sie Long Kek[1], Mohd Asrul Affendi Abdullah[1]

[1]Department of Mathematics and Statistics, Universiti Tun Hussein Onn Malaysia, Pagoh Edu Hub, 84600 Johor, MALAYSIA

*Corresponding author

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Abstract: In this paper, the Black-Litterman model which is the improved mean-variance optimization model, is discussed. Basically, the views given by the investors were incorporated into this model so that their views on risk and return, and risk tolerance could be quantified. For doing so, the market rates of return for the assets were calculated from the geometric mean. Moreover, the views of the investors were expressed in the matrix form. Then, the covariance matrix and the diagonal covariance matrix of the assets return were calculated. Accordingly, the mean rate of the asset return was computed. On this basis, the Black-Litterman optimization model was constructed. This model formulation was done by taking a set of possible rates of return for the assets. Particularly, the corresponding optimal portfolios of the assets with lower risk and higher expected return were further determined. For illustration, the historical return data for S&P 500, 3-month Treasury bill, and 10-year Treasury bond from 1928 to 2016 were employed to demonstrate the formulation of the ideal investment portfolio model. As a result, the efficient frontier of the portfolio is shown and the discussion is made. In conclusion, the Black-Litterman model could provide the optimal investment decision practically.

Keywords: Black-Litterman Model, Mean-Variance Optimization, Risk and Return, Efficient Portfolio, Investment Decision

1. Introduction

The Black-Litterman model was suggested by Black and Litterman in 1991. This model considers the views of investors in making an investment decision. As such, a portfolio optimization model, which contains the equilibrium investor's market views and the expected return, could be created. Its return equilibrium is derived from the Capital Asset Pricing Model (CAPM) and the Markowitz's mean-variance optimization model. On this point of view, the Black-Litterman model provides a sensible financial decision for investors (Alexander, Wai, & Bobby, 2009).

Basically, there are two types of market views, which are the absolute view and the relative view (Black & Litterman, 1992). The absolute view indicates the percentage of the return that is believed by investors to be provided by a certain asset, for example, "It is predicted that the asset *A* will give a return of *a*%". The relative view shows the percentage of the return of an asset is compared by investors to another asset, for example, "It is predicted that the return of the asset *A* would outperform the return of the asset *B* by *b*%". Thus, for each views, investors specify a confidence level that shows a certain view on the return of the asset. Indeed, these views are a subjective reaction to the asset portfolio given by investors.

**Corresponding author: stephanie.su@live.com.my*

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In fact, the Black-Litterman model has two major problems (Meucci, 2005). The first problem is the assumption for the multivariate normality on the market prior and the investor views on the asset return. The second problem is the estimation of the parameters on the market prior to the Bayesian framework from the non-normal distribution. It is clearly revealed that the difficulty to fulfil the assumption of normality is the main problem of the Black-Litterman model formulation. Thus, an alternative solution for the Black-Litterman model, where the prior market and the views of the investor are not normally distributed, is proposed (Meucci, 2005). In addition to this, the time-series approach can be used in the Black-Litterman model to form the views of investors.

In the application of the Black-Litterman model in finding the optimal investment portfolio, firstly, the market rate of return is calculated by using the geometric mean of the rate of return of the assets. Moreover, the covariance and the diagonal covariance matrices of the assets are computed such that the investor views are quantified in the matrix form. With the past historical return data, the mean of the expected return can be calculated certainly. Then, the Black-Litterman model is formulated as a mean-variance portfolio optimization model. For illustration, an empirical example is discussed, where the historical return data for the S&P 500, 3-month Treasury bill, and 10-year Treasury bond from 1928 to 2016 were taken into consideration. The portfolio optimization model for these assets was formulated and solved. As a result, the optimal investment portfolios were obtained, and the efficient frontier was presented. Therefore, it is highly recommended that the result can assist the investor in making the optimal investment decision definitely.

2. Problem Description

Here we consider the general portfolio optimization problem, given by

$$\begin{aligned} \text{Minimize} \quad & \omega_{ii}x_i^2 + \omega_{jj}x_j^2 + 2\left(\omega_{ij}x_i x_j\right) \\ \text{subject to} \quad & \bar{\mu}_i x_i^2 + \bar{\mu}_j x_j^2 \geq R \\ & x_i + x_j = 1 \\ & x_i, x_j \geq 0 \end{aligned} \tag{1}$$

where x_i and x_j are the weightings of assets i and j in the investment portfolio, whereas R is the rate of return. Here, $\mathbf{W} = [\omega_{ij}]$, $i = 1, 2, \cdots, n,$ and $j = 1, 2, \cdots, n,$ is the covariance matrix of the asset return. The aim of this problem is to determine the value of the weightings of assets such that the optimal portfolio decision can be obtained in the minimum variance sense.

Notice that this problem, which is also known as the Markowitz mean-variance optimization problem, assumes that the asset returns are normally distributed. In the Markowitz mean-variance optimization problem, there exists some issues, such as the investor's view is less regarded and the small changes in the historical return data are extremely sensitive. By virtue of this, the Black-Litterman model is created to mitigate these issues. In fact, the Black-Litterman model incorporates the investor's view and uses the estimated parameters to solve the Markowitz mean-variance optimization problem in order to provide an efficient portfolio (He & Litterman, 1999).

3. Methodology for the Black-Litterman Model

Let $I_{i,t}$ be the total return for the asset i, $i = 1, 2, \cdots, n,$ and the time $t = 1, \ldots, T.$ The rate of return $r_{i,t}$ is given by

$$r_{i,t} = \frac{I_{i,t} - I_{i,t-1}}{I_{i,t-1}} \tag{2}$$

The geometric mean for the rate of return for each asset is calculated from

$$\pi_i = \left(\prod_{t=1}^{T} \left(1 + r_{i,t}\right) \right)^{\frac{1}{T}} - 1 \tag{3}$$

Here, the covariance matrix of return, which measures the directional relationship between two assets but does not show the strength of the relationship, is given by

$$\mathbf{W} = \frac{1}{T} \sum_{t=1}^{T} \left(r_{i,t} - \bar{r}_i\right)\left(r_{j,t} - \bar{r}_j\right) \tag{4}$$

where $r_{i,t}$ and $r_{j,t}$ are the annual rates of return from the historical return for the assets i and j considered in the data set, whereas $\bar{r}_i$ and $\bar{r}_j$ are the means of the annual rates of return $r_{i,t}$ and $r_{j,t}$, respectively.

Assume that the expected return vector of the portfolio $\boldsymbol{\mu}$ has a probability distribution, which is the product of two multivariate normal distributions. The first multivariate normal distribution denotes the returns at the market equilibrium with the mean $\boldsymbol{\pi}$ and the covariance matrix $\tau \cdot \mathbf{W}$, where τ is a small constant and $\mathbf{W}$ represents the covariance matrix of asset returns. Meanwhile, the second multivariate normal distribution represents the views of investor towards the expected return of the portfolio $\boldsymbol{\mu}$. Hence, the views of the investor are expressed as

$$\mathbf{P} \times \boldsymbol{\mu} = \mathbf{q} + \boldsymbol{\varepsilon} \qquad (5)$$

where $\mathbf{P}$ is a $k \times n$ matrix and $\mathbf{q}$ is a k-dimensional vector, which is responded from the investor's view, whereas $\boldsymbol{\varepsilon}$ is a normally distributed random vector with zero mean and diagonal covariance matrix $\boldsymbol{\Omega}$, which is denoted by $\boldsymbol{\varepsilon} \sim \mathrm{N}(0, \boldsymbol{\Omega})$. Here, the diagonal covariance matrix $\boldsymbol{\Omega}$ is given by

$$\boldsymbol{\Omega} = \begin{bmatrix} \left(p_1 \mathbf{W} p_1^{\mathrm{T}}\right) \cdot \tau & 0 & \cdots & 0 \\ 0 & \left(p_2 \mathbf{W} p_2^{T}\right) \cdot \tau & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \left(p_k \mathbf{W} p_k^{T}\right) \cdot \tau \end{bmatrix} \qquad (6)$$

where p_i, $i = 1, \ldots, k$, is the vector corresponds to the kth view of the investor in the matrix $\mathbf{P}$ with a particular scalar τ. Once the matrix $\mathbf{P}$ is defined, the variance of each individual view on the portfolio can be computed from $p_k \mathbf{W} p_k^{\mathrm{T}}$ as expressed in (6) (Idzorek, 2007). Notice that if the investor's view is strong, then the value in the diagonal covariance matrix $\boldsymbol{\Omega}$ will be small (Cornuejols & Tütüncü, 2006). This situation reveals the confidence of the investor's view on the portfolio concerned.

Therefore, the probability distribution of the expected return of the portfolio $\boldsymbol{\mu}$ is a multivariate normal distribution with the mean given by

$$\bar{\boldsymbol{\mu}} = \left[\left(\tau \cdot \mathbf{W}\right)^{-1} + \mathbf{P}^{\mathbf{T}} \boldsymbol{\Omega}^{\mathbf{-1}} \mathbf{P}\right]^{-1} \left[\left(\tau \cdot \mathbf{W}\right)^{-1} \boldsymbol{\pi} + \mathbf{P}^{\mathbf{T}} \boldsymbol{\Omega}^{\mathbf{-1}} \mathbf{q}\right] \qquad (7)$$

where $\bar{\boldsymbol{\mu}}$ is the $n \times 1$ vector of the expected return, which is a combined return vector, and $\boldsymbol{\pi}$ is the $n \times 1$ vector of the market rate of return defined in (3), which is an implied equilibrium return vector. Here, (7) is the standard Black-Litterman equation, where the first term at the right-hand side is a normalization factor and the second term at the right-hand side is a vector involving the equilibrium returns $\boldsymbol{\pi}$ and the estimate $\mathbf{q}$. Note that both $(\tau \cdot \mathbf{W})^{-1}$ and $\mathbf{P}^{\mathrm{T}} \boldsymbol{\Omega}^{-1}$ in the second term are the weighting factors.

Moreover, the expected return

$$\bar{\boldsymbol{\mu}} = \left[\left(\tau \cdot \mathbf{W}\right)^{-1}\right]^{-1} \left[\left(\tau \cdot \mathbf{W}\right)^{-1} \boldsymbol{\pi}\right] = \boldsymbol{\pi} \qquad (8)$$

denotes the equilibrium returns if there are no views from investors and the expected return

$$\bar{\boldsymbol{\mu}} = \left[\mathbf{P}^{\mathrm{T}} \boldsymbol{\Omega}^{-1} \mathbf{P}\right]^{-1} \left[\mathbf{P}^{\mathrm{T}} \boldsymbol{\Omega}^{-1} \mathbf{q}\right] = \mathbf{P}^{-1} \mathbf{q} \qquad (9)$$

represents the view return if there is no estimation error (Martin, 2016; Tetyana, 2017).

4. Empirical Example

Now, let us illustrate the formulation of the Black-Litterman model by generating a portfolio containing S&P 500, 3-month Treasury bill, and 10-year Treasury bond. The historical return data from 1928 to 2016 were used to estimate their future expected returns (Aswath, 2019). The rates of return of these assets are shown in Table 1.

Table 1 - Annual rate of return for different investment.

Year	S&P 500	3-month T. Bill	10-year T. Bond
1928	43.81%	3.08%	0.84%
1929	-8.30%	3.16%	4.20%
1930	-25.12%	4.55%	4.54%
1931	-43.84%	2.31%	-2.56%
1932	-8.64%	1.07%	8.79%
1933	49.98%	0.96%	1.86%
1934	-1.19%	0.32%	7.96%
1935	46.74%	0.18%	4.47%
1936	31.94%	0.17%	5.02%
1937	-35.34%	0.30%	1.38%
1938	29.28%	0.08%	4.21%
1939	-1.10%	0.04%	4.41%
1940	-10.67%	0.03%	5.40%
1941	-12.77%	0.08%	-2.02%
1942	19.17%	0.34%	2.29%
1943	25.06%	0.38%	2.49%
1944	19.03%	0.38%	2.58%
1945	35.82%	0.38%	3.80%
1946	-8.43%	0.38%	3.13%
1947	5.20%	0.57%	0.92%
1948	5.70%	1.02%	1.95%
1949	18.30%	1.10%	4.66%
1950	30.81%	1.17%	0.43%
1951	23.68%	1.48%	-0.30%
1952	18.15%	1.67%	2.27%
1953	-1.21%	1.89%	4.14%
1954	52.56%	0.96%	3.29%
1955	32.60%	1.66%	-1.34%
1956	7.44%	2.56%	-2.26%
1957	-10.46%	3.23%	6.80%
1958	43.72%	1.78%	-2.10%
1959	12.06%	3.26%	-2.65%
1960	0.34%	3.05%	11.64%
1961	26.64%	2.27%	2.06%
1962	-8.81%	2.78%	5.69%
1963	22.61%	3.11%	1.68%
1964	16.42%	3.51%	3.73%
1965	12.40%	3.90%	0.72%
1966	-9.97%	4.84%	2.91%
1967	23.80%	4.33%	-1.58%
1968	10.81%	5.26%	3.27%

1969	-8.24%	6.56%	-5.01%
1970	3.56%	6.69%	16.75%
1971	14.22%	4.54%	9.79%
1972	18.76%	3.95%	2.82%
1973	-14.31%	6.73%	3.66%
1974	-25.90%	7.78%	1.99%
1975	37.00%	5.99%	3.61%
1976	23.83%	4.97%	15.98%
1977	-6.98%	5.13%	1.29%
1978	6.51%	6.93%	-0.78%
1979	18.52%	9.94%	0.67%
1980	31.74%	11.22%	-2.99%
1981	-4.70%	14.30%	8.20%
1982	20.42%	11.01%	32.81%
1983	22.34%	8.45%	3.20%
1984	6.15%	9.61%	13.73%
1985	31.24%	7.49%	25.71%
1986	18.49%	6.04%	24.28%
1987	5.81%	5.72%	-4.96%
1988	16.54%	6.45%	8.22%
1989	31.48%	8.11%	17.69%
1990	-3.06%	7.55%	6.24%
1991	30.23%	5.61%	15.00%
1992	7.49%	3.41%	9.36%
1993	9.97%	2.98%	14.21%
1994	1.33%	3.99%	-8.04%
1995	37.20%	5.52%	23.48%
1996	22.68%	5.02%	1.43%
1997	33.10%	5.05%	9.94%
1998	28.34%	4.73%	14.92%
1999	20.89%	4.51%	-8.25%
2000	-9.03%	5.76%	16.66%
2001	-11.85%	3.67%	5.57%
2002	-21.97%	1.66%	15.12%
2003	28.36%	1.03%	0.38%
2004	10.74%	1.23%	4.49%
2005	4.83%	3.01%	2.87%
2006	15.61%	4.68%	1.96%
2007	5.48%	4.64%	10.21%
2008	-36.55%	1.59%	20.10%
2009	25.94%	0.14%	-11.12%
2010	14.82%	0.13%	8.46%
2011	2.10%	0.03%	16.04%
2012	15.89%	0.05%	2.97%

2013	32.15%	0.07%	-9.10%
2014	13.52%	0.05%	10.75%
2015	1.36%	0.21%	1.28%
2016	11.74%	0.51%	0.69%

Two views were considered to be incorporated into the Black-Litterman model. First, the strong view that is held for the 10-year Treasury bond will be 2% next year. Second, the weak view for the S&P 500 will be outperformed than 3-month Treasury bill by 5%. The information is summarized as follows:

(a) The expected return of the portfolio in the vector form is defined by

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_{stock} \\ \mu_{bill} \\ \mu_{bond} \end{bmatrix}$$

(b) The expected returns for the views are given by

$$\mu_{bond} = 0.02\,, \quad \mu_{stock} - \mu_{bill} = 0.05$$

(c) The coefficient matrix for the views is represented by

$$\mathbf{P} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

where the first row of the $\mathbf{P}$ matrix represents the first view, which is the absolute view that only involves the 10-year Treasury bond. Since the bond is the third asset in this example, it has corresponded with the "1" in the third column of row one of the $\mathbf{P}$ matrix. The second view is represented in the second row. In the case of relative views, each row has the sum equal to zero. In general, the outperforming assets receive positive weightings, while the underperforming assets receive negative weightings. For this example, the S&P 500 outperformed the 3-month Treasury bill by 5% reveals that the S&P 500 has the positive weighting "1", while the 3-month Treasury bill shows the negative weighting of "–1".

(d) The vector for the views is

$$\mathbf{q} = \begin{bmatrix} 0.02 \\ 0.05 \end{bmatrix}$$

where the first row is referred to the first view, which the 10-year Treasury bond is 2% next year, while the second row denotes the second view, which the S&P 500 outperform than the 3-month Treasury bill by 5%.

(e) The market rate of return, which is calculated from (3), is

$$\boldsymbol{\pi} = \begin{bmatrix} 9.53\% \\ 3.42\% \\ 4.91\% \end{bmatrix}$$

(f) The covariance matrix of the asset return, which is computed from (4), is

$$\mathbf{W} = \begin{bmatrix} 0.038384 & -0.00015490 & -0.00039144 \\ -0.00015 & 0.000928476 & 0.000692674 \\ -0.00039 & 0.000692674 & 0.005957845 \end{bmatrix}$$

(g) The diagonal covariance matrix, which is calculated from (6) with assuming $\tau = 0.1$, is

$$\mathbf{\Omega} = \begin{bmatrix} 0.000595785 & 0 \\ 0 & 0.003962243 \end{bmatrix}$$

(h) The weighting of the portfolio in the vector form is defined by

$$\mathbf{x} = \begin{bmatrix} x_{stock} \\ x_{bill} \\ x_{bond} \end{bmatrix}$$

From the information discussed above, the mean for the probability distribution of the expected return of the portfolio $\boldsymbol{\mu}$, which is known as the mean rate of return $\bar{\boldsymbol{\mu}}$, was calculated from (7) and the result is shown in Table 2.

Table 2 - Mean rate of return.

	S&P 500	3-month Treasury Bill	10-year Treasury Bond
Mean Rate of Return $\bar{\boldsymbol{\mu}}$	8.95%	3.27%	3.46%

Following this, the Black-Litterman model would be further formulated. The objective function of this model is to minimize the variance of portfolio σ^2 and yields at least a target value of the expected return *R*. Mathematically, this formulation generates a quadratic programming problem with constraints given by the sum of weighting of all assets are equal to one and the weighting of each asset should be more than or equal to zero. Thus, refer to (1), the Black-Litterman portfolio optimization model is constructed as follows:

$$\begin{aligned} \text{minimize} \quad & 0.38384 \cdot x_{\text{Stock}}^2 - 2(0.00015) x_{\text{Stock}} \cdot x_{\text{Bill}} \\ & - 2(0.00039) \cdot x_{\text{Stock}} \cdot x_{\text{Bond}} + 0.000928 \cdot x_{\text{Bill}}^2 \\ & + 2(0.000693) x_{\text{Bill}} \cdot x_{\text{Bond}} + 0.005958 \cdot x_{\text{Bond}}^2 \\ \text{subject to} \quad & \\ & 0.0895 x_{Stock} + 0.0327 x_{Bill} + 0.0346 x_{Bond} \geq R \\ & x_{Stock} + x_{Bill} + x_{Bond} = 1 \\ & x_{Stock}, x_{Bill}, x_{Bond} \geq 0 \end{aligned} \tag{10}$$

The quadratic optimization model defined in (10) could be solved by setting the rate of return *R* = 1% to *R* = 9% with the increment 1% for the corresponding portfolios, and the Spreadsheet solver was used to obtain the optimal solution of the Black-Litterman model given in (10). The result is shown in Table 3.

Table 3 - Black-Litterman efficient portfolios.

	Return R	Var σ^2	Stdev σ	Stock x_{Stock}	Bill x_{Bill}	Bond x_{Bond}
1	0.01	0.0009	0.0298	0.0273	0.9298	0.0429
2	0.02	0.0009	0.0298	0.0273	0.9298	0.0429
3	0.03	0.0009	0.0298	0.0273	0.9298	0.0429
4	0.04	0.0013	0.0358	0.1268	0.8055	0.0677
5	0.05	0.0039	0.0623	0.3011	0.5875	0.1114
6	0.06	0.0089	0.0944	0.4754	0.3696	0.1550
7	0.07	0.0164	0.1279	0.6497	0.1517	0.1986
8	0.08	0.0263	0.1621	0.8264	0	0.1736
9	0.09	0.0384	0.1959	1	0	0

Refer to Table 3, a line graph of the rate of return against the portfolio standard deviation as shown in Figure 1 was plotted in order to present the efficient frontier of the portfolio. It is indicated that when the rate of return is increased from R = 1% to 3%, the standard deviation remains constant at R = 3%. However, the efficient frontier shows an increasing trend started from R = 3% as the standard deviation become larger.

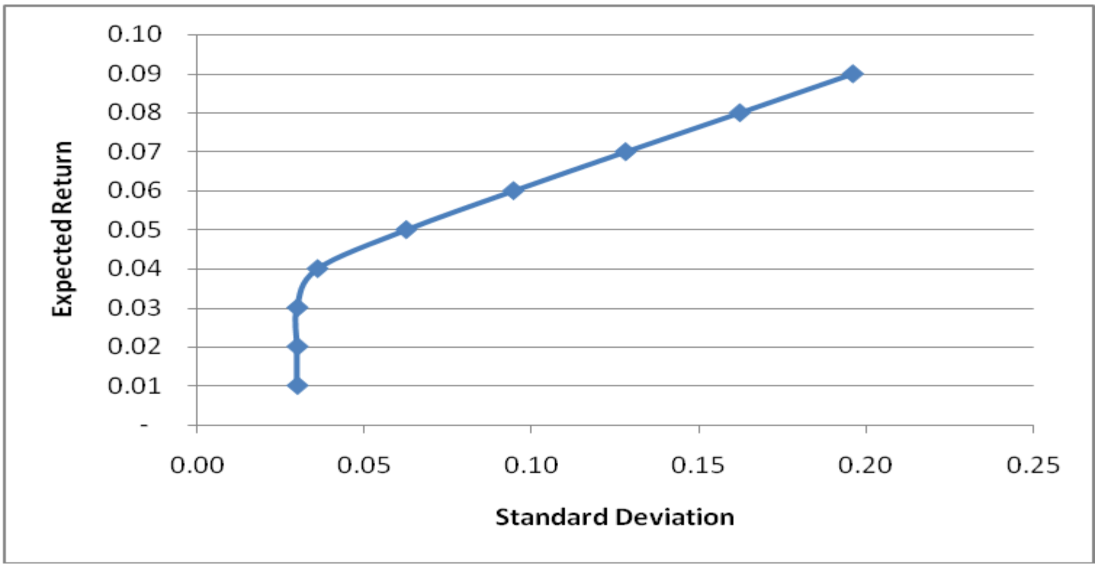

Fig. 1 - Efficient frontier of portfolios.

On the other hand, Figure 2 shows the composition of the portfolio at each level of the rate of return. For the S&P 500, it shows a constant composition until R = 3% and a soaring trend after R = 3% is presented. Furthermore, the 3-month Treasury bill keeps constant before R = 3% and decreases its composition after that. It shows no composition at R = 8% and 9%. Additionally, the 10-year Treasury bond recorded a constant value at first then it started to increase its composition at R = 4% and bottomed out at R = 8%. It has no composition when R = 9%.

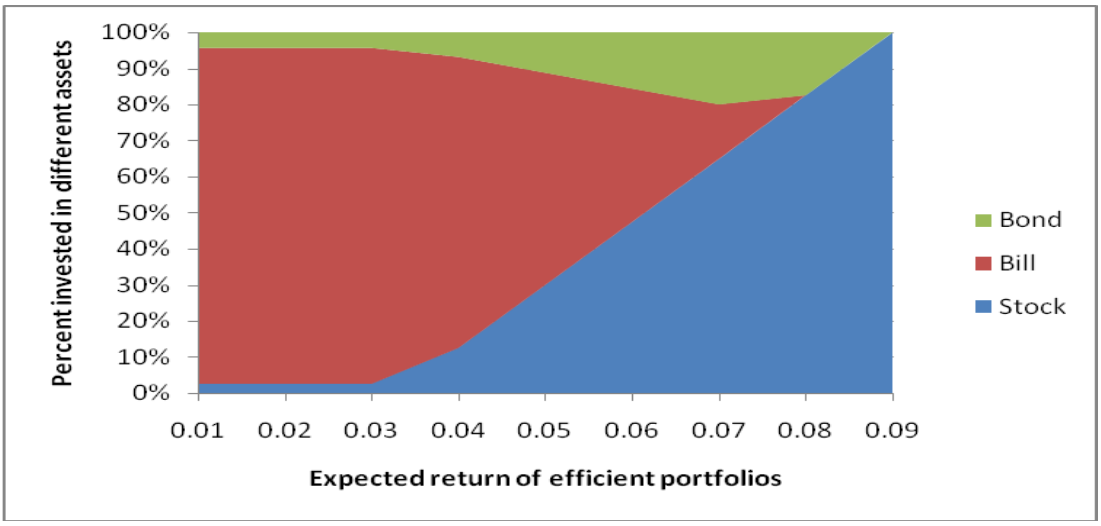

Fig. 2 - Composition of efficient portfolios.

Hence, the efficient portfolio consists of 2.7% S&P 500, 93% 3-month Treasury bill, and 4.3% 10-year Treasury bond at the rate of return 3%, where the variance is 0.0009. This result provides the optimal investment decision on the portfolio, where the views of the investor were also taken into consideration.

5. Conclusion

In a nutshell, for the risk-averse investor, the portfolio with the minimum variance and the maximum expected return is the most ideal portfolio. In this case, the optimal portfolio with composition 2.7% of the S&P 500, 93% of the 3-month Treasury bill, and 4.3% of the 10-year Treasury bond is regarded as the efficient portfolio. This is because it has the highest rate of return among the portfolios with the same lowest variance, which represents the risk. The results obtained reflect that the Black-Litterman model is quite useful in making the decision for investment portfolio. In conclusion, this model is not only providing efficient allocation, but also taking into account the investor's view, which means this model can customize the investment portfolio for every different investor.

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