Complex Permittivity of Materials at Broadband Frequency using Transmission Phase Shift Method

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Abstract: This paper reports determination of complex relative permittivity $\left(\varepsilon_r = \varepsilon_r^{'} - \varepsilon_r^{''}\right)$ of non-magnetic samples $\left(\mu_r = 1\right)$ Teflon, Nylon and Epoxy via transmission phase shift method. The transmission phase shift method offers calibration independent and invariant position features. In this work, a Coaxial Cavity Test Fixture (CCTF) with dimension of 1.8 and 4.1 mm for inner and outer conductor, respectively was customized based on impedance matching theory to study broadband scattering in the range of frequency 1-18 GHz. Impedance matching and cable calibration were performed to reduce the measurement error of scattering parameters; reflection $\left(S_{11}\right)$ and transmission $\left(S_{21}\right)$. Magnitude and phase of S_{11} and S_{21} were measured by vector network analyzer (VNA). The measured phase of S_{21} ($j\phi_{21}$) for all samples under test were linearized using the MATLAB "unwrap" command for further calculation of ε_r. The phase shift in S_{21} between with and without sample inside the cavity was detected and it was used to determine ε_r for each material under test. The average value of dielectric constants $\left(\varepsilon_r^{'}\right)$ for Teflon, Nylon and Epoxy were 2.11, 2.42 and 3.05, correspondingly for the selected frequency 2, 4, 6, 8, 10, 12 and 14 GHz. While, the average of dielectric losses $\left(\varepsilon_r^{''}\right)$ for Teflon and Epoxy were almost overlapped which the values were 0.001 and 0.002, respectively that were higher than that of Nylon at 0.0006. The calculated values are comparable with the theoretical values with small absolute errors. These results suggested that this transmission phase shift method using CCTF was equally consistent when compared to the other techniques used for the determination of $\varepsilon_r^{'}$ and $\varepsilon_r^{''}$.

Keyword: Coaxial Cavity Test Fixture; Scattering Parameters; Transmission Phase Shift; Complex Relative Permittivity, Non-magnetic samples.

1. Introduction

There are several methods to determine complex relative permittivity $\left(\varepsilon_r = \varepsilon_r^{'} - \varepsilon_r^{''}\right)$ of materials such as resonator, free space, rectangular waveguide and coaxial cavity. However, some of them have drawbacks in material characterization. Resonator offers accurate data but it is limited for narrow band. Free space is a non- destructive test however it yields a less accurate data due to substantial multiple noises. Rectangular waveguide requires a time consuming calibration in aperture and transmission line. Furthermore, different dimensions of the rectangular waveguide are needed since each of the rectangular waveguide offers different cut-off frequency [1]. Thus, a Coaxial Cavity Test Fixture (CCTF) is introduced, which used transmission measurement principle. Basically, the determination of ε_r is based on measured scattering parameters (S-parameters) using vector network analyser (VNA). The S-parameters involved in the transmission measurement are reflection, S_{11} and transmission, S_{21} coefficient.

On the other hand, calculation of ε_r and μ_r for transmission measurement technique are commonly by Nicholson-Ross-Weir

(NRW) conversion routine [2-6]. Nevertheless, the NRW routine needs phase shift of measured S_{11} and S_{21} to be on the calibration plane and consistent position of sample in the CCTF [7]. In this study, error due to the phase shift of measured S_{11} and S_{21} affect the calculation of ε_r of sample can be eliminated by using transmission phase shift method.

The used CCTF is made up of inner and outer diameter of 1.8 and 4.1 mm, respectively in order to get as close as possible to 50 Ω line matching with coaxial cable. Therefore, error occurs due to mismatch impedance between CCTF and coaxial cable could be reduced. Dielectric material under test (MUT) Teflon, Nylon and Epoxy were prepared in toroidal shaped in order to fit into CCTF for S-parameters measurement. Complex relative permittivity of these non-magnetic samples $(\mu_r = 1)$ can be computed by difference in the phase shifts of measured S_{21} between air (without sample) and with sample

2. Theoretical analysis

Cut-off frequency of CCTF [1] can be expressed as below:

$$f_c = \frac{2c}{\pi(D+d)\sqrt{\varepsilon_r \mu_r}}. \qquad (1)$$

where f_c is cuf-off frequency of CCTF, D and d are outer diameter and inner diameter of conductor, ε_r and μ_r are relative permittivity and permeability of air. The designed CCTF with outer and inner diameter 4.1 mm and 1.8 mm, respectively can operate up to 32 GHz. However, our measurement using this CCTF is only up to 18 GHz since the used VNA can only operate up to 20 GHz.

Mismatch of airlines occurs due to the connection of different transmission lines. Uncertainty arises from the mismatch affect the measurement data of S-parameters (S_{11} and S_{21}). Characteristic impedance of airlines Z_0 can be calculated [8]:

$$Z_0 = \frac{59.9585}{\sqrt{\varepsilon_r}} \ln \frac{D}{d}. \qquad (2)$$

where D and d are outer diameter and inner diameter of conductor, ε_r is relative dielectric constant of air.
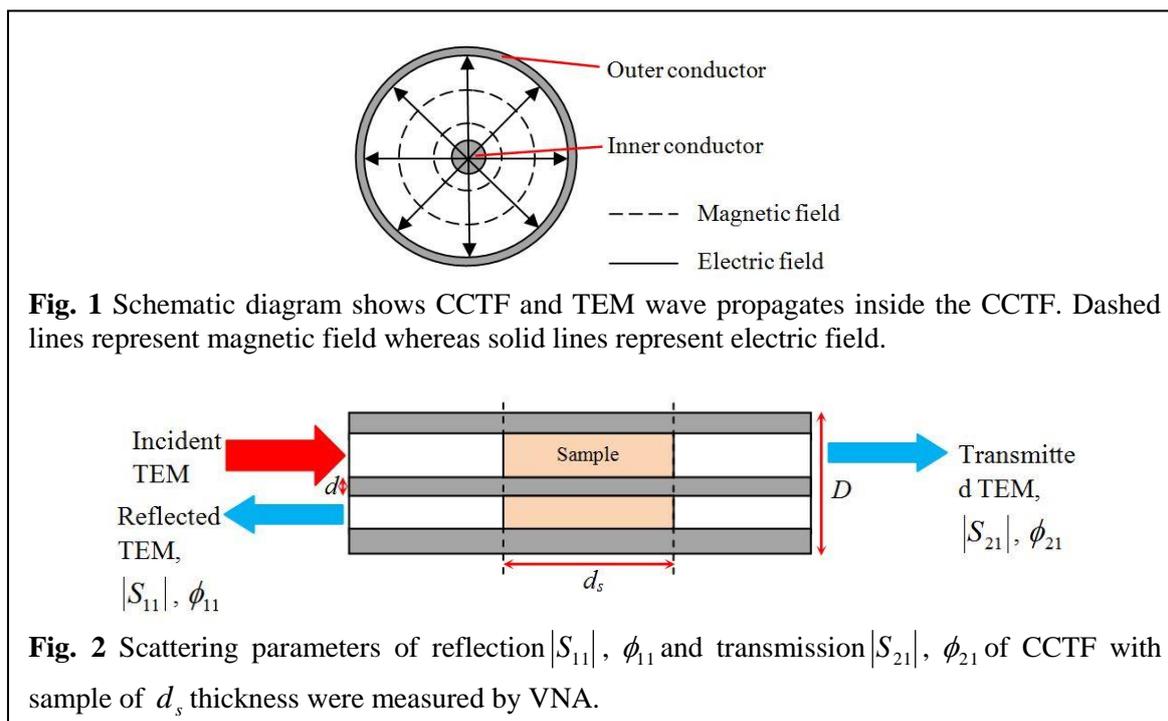
A transverse electromagnetic (TEM) mode electromagnetic wave transmits through the CCTF as shown in Fig. 1. In this mode, electric and magnetic field are perpendicular to each other and to the propagation direction of the wave. Fig. 2 shows the measurement of S-parameters of S_{11} and S_{21} of a homogenous and isotropic toroidal shaped sample material with thickness d_s is partly placed in the CCTF. When incident TEM wave touches the sample, part of the incident TEM wave is reflected and transmitted through sample as shown in Fig. 2. S_{11} is the ratio of reflected TEM wave to the incident TEM wave whereas S_{21} is the ratio transmitted TEM wave to the incident TEM wave. Both of S_{11} and S_{21} consist of magnitude ($|S_{11}|, |S_{21}|$) and phase (ϕ_{11}, ϕ_{21}). The phase shift of air ($j\phi_{21}^A$) and sample ($j\phi_{21}^S$) can be expressed explicitly as [9]:

$$j\phi_{21}^A = -\alpha_c d_S - j\gamma_o d_S. \qquad (3)$$

$$j\phi_{21}^S = -\alpha_c d_S - \alpha_s d_S - j\gamma_s d_S. \qquad (4)$$

$$\gamma_o = \sqrt{k_o{}^2} = \frac{2\pi f}{c}. \qquad (5)$$

where α_c (nepers/meter) is the conductor attenuation constant, γ_o is the propagation constant of electromagnetic wave in free space, j is the unit of imaginary part, π is 3.14159, c is the speed of light (2.99792458 ms^{-1}).

Fig. 1 Schematic diagram shows CCTF and TEM wave propagates inside the CCTF. Dashed lines represent magnetic field whereas solid lines represent electric field.

Fig. 2 Scattering parameters of reflection $|S_{11}|$, ϕ_{11} and transmission $|S_{21}|$, ϕ_{21} of CCTF with sample of d_s thickness were measured by VNA.

Meanwhile,

$$\gamma_S = \sqrt{k_0^2 \mu_r \varepsilon_r}. \qquad (6)$$

where γ_S is the propagation constant of electromagnetic wave in sample, $\left(\varepsilon_r = \varepsilon_r^{'} - \varepsilon_r^{''}\right)$ and $\left(\mu_r = \mu_r^{'} - \mu_r^{''}\right)$ are the complex relative permittivity and permeability of the sample respectively and symbol α_S (nepers/meter) is the dielectric attenuation constant of the sample. The value of α_S is calculated by [9]

$$\alpha_s \approx -1.15129254 \left[\begin{array}{c} \log_{10}\left(\left|S_{11}^S\right|^2 + \left|S_{21}^S\right|^2\right) \\ -\log_{10}\left(\left|S_{11}^A\right|^2 + \left|S_{21}^A\right|^2\right) \end{array} \right] \qquad (7)$$

where $\left|S_{11}^A\right|$, $\left|S_{21}^A\right|$, $\left|S_{11}^S\right|$ and $\left|S_{21}^S\right|$ are the measured linear magnitudes of S-parameter reflection and transmission coefficients of air and sample, respectively. The differences between the phase shift of air (1) and sample (2) can be expressed as:

$$\alpha_S d_S + j\left(\phi_{21}^A - \phi_{21}^S\right) = -j\left(\gamma_o - \gamma_S\right)d_S \qquad (8)$$

Substitute Eq. 5 and Eq. 6 into Eq. 8, the complex relative permittivity ε_r and permeability μ_r of the sample can be expressed as Eq. 9: (where μ_r is assumed to be 1 in this study).

$$\varepsilon_r \mu_r = \frac{1}{k_o^2}\left[k_o - \frac{j\alpha_S d_S - j\left(\phi_{21}^A - \phi_{21}^S\right)}{d_S}\right]^2 \qquad (9)$$
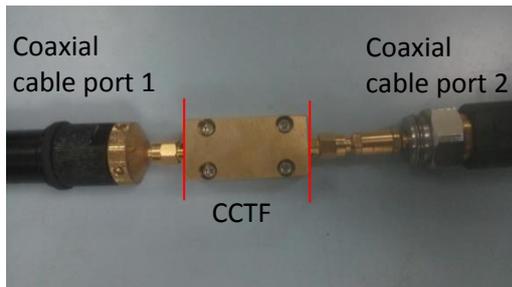
3. Experimental setup

The CCTF dimension is 1.8, 4.1 and 40 mm in inner, outer conductor and thickness, respectively. Toroidal samples Teflon, Nylon and Epoxy were placed inside the CCTF which it served as a sample holder for S-parameter measurement of reflection and transmission coefficients. Linear magnitudes $|S_{11}|$, $|S_{21}|$, and phase shifts $j\phi_{11}$, $j\phi_{21}$ of S-parameters of air and sample were measured using Agilent E5071C VNA which operated at frequency range of 1 – 18 GHz.

4. Calibration of experimental setup

Calibration of experiment setup was performed for coaxial cable and CCTF in order to obtain accurate measurement S-parameter. The calibrations of coaxial cable

involve open, short and load (OSL) whereas calibrations of CCTF involves thru-reflect-line (TRL). The TRL calibration was carried out within CCTF as shown in Fig. 3. The CCTF is invariant sample position to the calibration plane. Thus, the accuracy of the measured S-parameter was not affected by the thickness of sample, d_s hence it could be placed in any arbitrary position of CCTF.


Fig. 3 S-parameters experimental setup where red lines are the OSL calibration planes.

5. Results and discussion

A TEM wave transmits through CCTF that filled with toroidal shaped samples for measurement of linear magnitudes $|S_{11}|$, $|S_{21}|$ and phase shifts $j\phi_{11}$, $j\phi_{21}$ in the operating frequency range of 1 – 18 GHz. The variation of phase shift, ϕ_{21} between air and samples caused phase ambiguity regions as shown in Fig. 4 (a). These uncalibrated ambiguity regions of phase shift air, $j\phi_{21}^A$ and samples, $j\phi_{21}^s$ reduced the accuracy of extracted values of complex relative permittivity $\left(\varepsilon_r = \varepsilon_r^{'} - \varepsilon_r^{''}\right)$ using Eq. 9. The phase ambiguity could be eliminated by using MATLAB 'unwrap' command as shown in Fig. 4 (b) [10]. Fig. 4 (c) shows measurement result for $|S_{11}|$ of air and samples. The error in measurement of $|S_{11}|$ is able to be minimized for value of $|S_{11}|$ of air that is smaller than 0.2 [11]. However, the error could not be minimized for value of $|S_{11}|$ of air that is larger than 0.2, this was probably due to the imperfection of designed CCTF. The TEM wave becomes more

sensitive especially at higher frequency due to shorter wavelength and this could be overcome by high precision and very well machined CCTF.


(a)

(b)

(c)

(d)
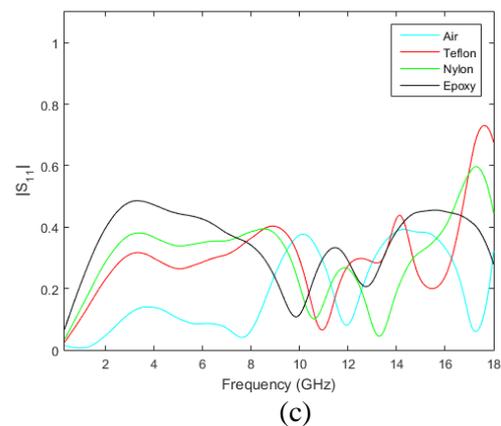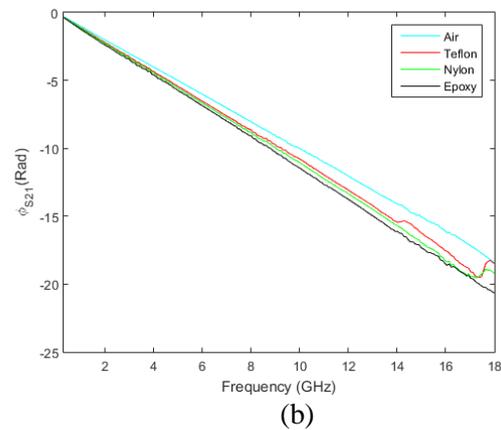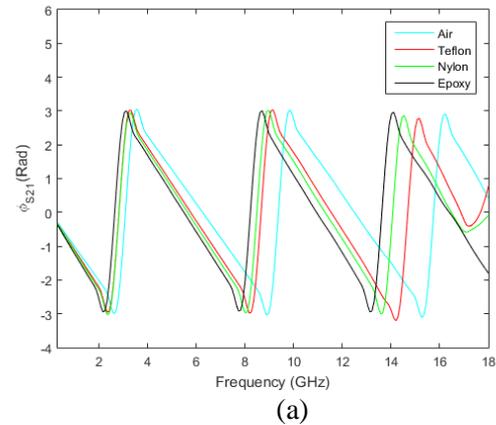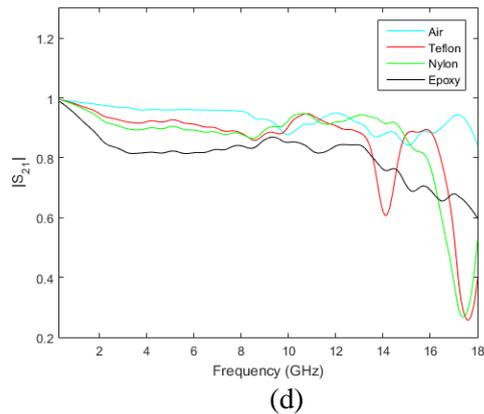
Fig. 4 (a) Phase shift ($j\phi_{21}$) of air and non-magnetic sample, (b) Ambiguity region of $j\phi_{21}$ eliminated by phase unwrapping method, linear magnitude of air and non-magnetic samples (c) $\left|S_{11}\right|$, (d) $\left|S_{21}\right|$.

Complex relative permittivity of non-magnetic $\left(\mu_r = 1\right)$ sample Teflon, Nylon and Epoxy was calculated by using Eq. 8. The calculated $\varepsilon_r^{'}$ and $\varepsilon_r^{''}$ of the three samples are shown in Fig. 5 (a) and (b).

The ideal expectation of the S-parameters measurement should be in satisfactory up to 18 GHz even though the CCTF has been designed with 32 GHz cut off frequency. However due to several reasons as mentioned above, the calculated dielectric constant and dielectric loss seemed to be only acceptable for frequency that was below than 14 GHz. This is probably due to signal distortion at higher frequency transmission wave in CCTF [12].

Table 1, 2 and 3 show comparison between calculated values of $\varepsilon_r^{'}$ and $\varepsilon_r^{''}$ for Teflon, Nylon and Epoxy samples and the one obtained from literatures [6,13-15]. It can be seen that the real part of permittivity is almost the same as the value calculated from other methods in literatures at several selected frequencies. These values have also been compared with manufacturer specification datasheet. However, the imaginary part of permittivity values are clearly deviated for all tested standard samples. This indicates that the measurement

accuracy of the imaginary part for low loss materials is usually high using S-parameter-based methods as reported in literature [16].

(a)

(b)
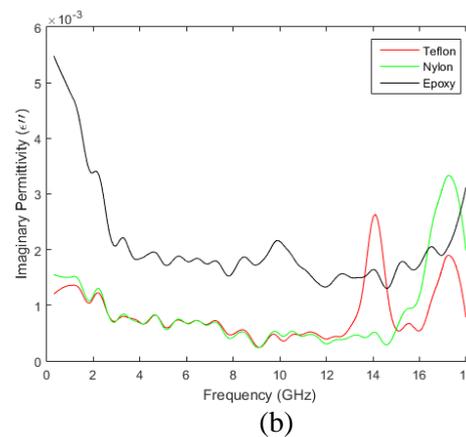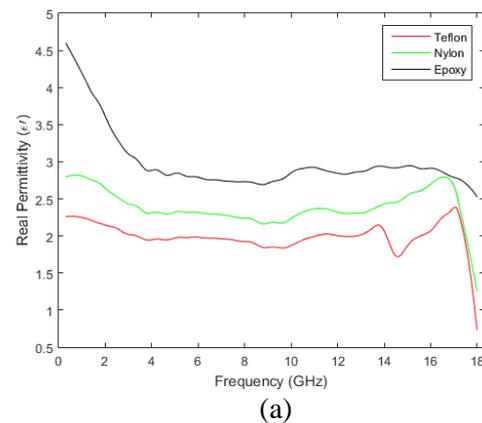
Fig. 5 (a) Dielectric constant $\varepsilon_r^{'}$ and (b) loss factor $\varepsilon_r^{''}$ of Teflon, Nylon and Epoxy.

Table 1 $\varepsilon_r^{'}$ and $\varepsilon_r^{''}$ of Teflon.

Frequency (GHz)	$\varepsilon_r^{'}$		$\varepsilon_r^{''}$	
	Theory [6,13]	This work	Theory [6,13]	This work
2		2.2		0.0012
4		2.0		0.0008
6		2.1		0.0008
8	2.1	2.0	0.0002	0.0006
10		1.9		0.0004
12		2.2		0.0005
14		2.4		0.0028

Table 2 $\varepsilon_r^{'}$ and $\varepsilon_r^{''}$ of Nylon.

Frequency (GHz)	$\varepsilon_r^{'}$		$\varepsilon_r^{''}$	
	Theory [13,14]	This work	Theory [13,14]	This work
2		2.7		0.12
4		2.4		0.07
6		2.4		0.06
8	2.5	2.3	0.03	0.04
10		2.4		0.05
12		2.3		0.03
14		2.5		0.05

Table 3 $\varepsilon_r^{'}$ and $\varepsilon_r^{''}$ of Epoxy.

Frequency (GHz)	$\varepsilon_r^{'}$		$\varepsilon_r^{''}$	
	Theory [13,15]	This work	Theory [13,15]	This work
2		3.5		0.34
4		3		0.19
6		2.9		0.19
8	3.1	2.9	0.01	0.18
10		3		0.22
12		3.1		0.15
14		3		0.17

Absolute error for $\varepsilon_r^{'}$ and $\varepsilon_r^{''}$ of Teflon, Nylon and Epoxy can be defined as:

$$\text{Absolute error} = |\text{Average calculated value} - \text{Theoretical value}| \quad (10)$$

Table 4 Absolute error for $\varepsilon_r^{'}$ of Teflon, Nylon and Epoxy.

Sample	Average value of $\varepsilon_r^{'}$	Absolute error
Teflon	2.11	0.01
Nylon	2.42	0.08
Epoxy	3.05	0.05

Table 5 Absolute error for $\varepsilon_r^{"}$ of Teflon, Nylon and Epoxy.

Sample	Average value of $\varepsilon_r^{"}$	Absolute error
Teflon	0.001	0.0008
Nylon	0.0006	0.0294
Epoxy	0.002	0.008

6. Conclusion

In this work, complex relative permittivity has been determined by using transmission phase shift method for non-magnetic samples. The proposed method has invariant position and calibration independent features where samples can be placed in any arbitrary within the CCTF and no sample calibration needed within the CCTF. The customized CCTF shows a good agreement with the dielectric constant value obtained from literatures. This finding will be very useful in designing a well functioned test fixture in determination of complex relative permittivity of samples by utilizing S-parameters measured from VNA.

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