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Magnetohydrodynamic Three-Dimensional Flow and Heat Transfer over a Stretching Surface in a Viscoelastic Fluid

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Abstract

In this paper, the problem of steady laminar three-dimensional magnetohydrodynamic (MHD) boundary layer flow and heat transfer over a stretching surface in a viscoelastic fluid is investigated. The equations which govern the flow are coupled nonlinear ordinary differential equations, which are solved numerically using a finite-difference scheme known as the Keller-box method. Various physical governing parameters such as the magnetic parameter M, the material or viscoelastic parameter K and the Prandtl number Pr are considered and the effects of these parameters are investigated. It is found that the material parameter K and the magnetic parameter M present opposite effects on the fluid flow and heat transfer characteristics. The numerical results obtained for the skin friction coefficient and the local Nusselt number are presented in tables. The features and profiles of the flow and heat transfer characteristics are illustrated in the forms of graphs.

Keyword : Magnetohydrodynamic, Three-Dimensional Flow, Heat Transfer, Viscoelastic Fluid

1. INTRODUCTION

Problems involving flows over stretching sheet are widely applied in various industries. Some of the applications can be found in the extrusion of a polymer in a melt-spinning, cooling of an infinite metallic plate in a cooling bath, hot rolling, wire drawing, glass–fiber production, manufacturing of plastic and rubber sheet and paper production. Pioneering work done by Crane (1970) to get the analytical solution for the steady two-dimensional boundary layer flow caused by the stretching surface with velocity varying linearly with distance from a fixed point has initiated many researchers to study different aspects of this problem either by integrating the problem with heat and mass transfer, MHD, chemical reaction, suction/injection, non-Newtonian fluids or other various situations. Some of the huge collections of research papers existing in literature can be found in Chakrabarti and Gupta (1979), Chen (1998), Fan et al. (1999), Sajid (2007), Xu and Liao (2009), Jat and Chaudary (2009), Ishak et al. (2009, 2010), Salleh et al. (2010) and Ali et al. (2011), to name just a few.

Due to the increasing interest and importance of non-Newtonian fluid flows, a great deal of work on non-Newtonian fluids has been done vastly. However, it is known that the governing equations of the non-Newtonian fluid are of higher order than the Navier-Stokes equations. Thus, the constitutive equations of the fluid are very complex as it involves a number of material parameters of the fluid. One of the simplest of the non-Newtonian fluids is the second grade fluid which has received considerable attention. Rajagopal et al. (1984) examined a special class of the second order fluids known as the viscoelastic fluids which exhibit both elastic and viscous properties. Further investigations on this second grade fluid were done by Dandapat and Gupta (1989) who discussed the flow of an incompressible second-order fluid due to stretching of a plane elastic surface in the approximation of boundary layer theory. Further, Andersson (1992) investigated the flow of viscolelastic fluid past a stretching sheet with the presence of transverse magnetic field, while Abel et al. (2001) carried out a study of the effect of magnetic field on the viscoelastic fluid flow and heat transfer over a non-isothermal stretching sheet with internal heat generation, and Cortell (2006) considered the flow and heat transfer of an incompressible homogeneous second grade fluid past a stretching sheet by considering two cases, i.e. sheet with constant surface temperature (CST case) and sheet with prescribed surface temperature (PST case). Again, Cortell (2007) investigated the problem of flow and heat transfer of an incompressible homogeneous second grade fluid over a non-isothermal stretching sheet in the presence of non-uniform internal heat generation/absorption. On the other hand, Abel et al. (2008) came out with the viscoelastic MHD flow and heat

transfer over a stretching sheet with viscous and Ohmic dissipations in which the governing equations were solved using the fifth order Runge-Kutta-Fehlberg method along with the shooting technique. Very recently, Arnold et al. (2010) studied the viscoelastic fluid flow and heat transfer characteristics over a stretching surface with frictional heating and internal heat genaration or absorption for the case of prescribed surface temperature (PST) and prescribed surface heat flux (PHF), and Prasad et al. (2010) has carried out a study on the momentum and heat transfer characteristics in an incompressible electrically conducting non-Newtonian boundary layer flow of a viscoelastic fluid over a stretching sheet which are then solved numerically using the shooting technique with fourth order Runge-Kutta integration scheme.

All the problems mentioned above dealt with the two-dimensional fluid flow. On the other hand, Hayat et al. (2008) solved the three-dimensional flow over a stretching surface in a viscoelastic fluid analytically, while Nazar and Latip (2009) solved the three-dimensional boundary layer flow due to a stretching surface in a viscoelastic fluid numerically but with different sign of k0 (viscoelastic parameter). Hence, motivated by their works, the present work is an extension of Nazar and Latip (2009) by considering the three-dimensional MHD flow and heat transfer of a viscoelastic fluid over a stretching surface. The present model which is a modification and extension of the existing model has been solved numerically by an implicit finite-difference scheme known as the Keller-box method.

2. ANALYSIS

Consider the steady, three-dimensional laminar flow of an incompressible electrically conducting fluid bounded by a stretching surface. Under the Boussinesq and boundary layer approximations, the system of equations which model the boundary layer flow is given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\nu\frac{\partial^2 u}{\partial z^2}+k_0\left[-\begin{pmatrix} u\dfrac{\partial^3 u}{\partial x\partial z^2}+w\dfrac{\partial^3 u}{\partial z^3} \\ \dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial z^2}+\dfrac{\partial u}{\partial z}\dfrac{\partial^2 w}{\partial z^2} \\ +2\dfrac{\partial u}{\partial z}\dfrac{\partial^2 u}{\partial x\partial z}+2\dfrac{\partial w}{\partial z}\dfrac{\partial^2 u}{\partial z^2} \end{pmatrix}\right]-\frac{\sigma B_0^{\,2}}{\rho}u, \tag{2}$$

$$u = u_w(x) = ax, \quad v = v_w(y) = by, \quad w = 0, \quad T = T_w \quad \text{at} \quad z = 0$$
$$u \to 0, \quad v \to 0, \quad T \to T_\infty, \quad \frac{\partial u}{\partial z} \to 0, \quad \frac{\partial v}{\partial z} \to 0 \quad \text{as} \quad z \to \infty \tag{5}$$

where a and b are positive constants, while $u_w(x)$ and $v_w(y)$ are the stretching velocities in the x and y directions, respectively. Equations (1) to (4) can be transformed into the ordinary differential equations by the following transformation:

$$u = axf'(\eta), \quad v = ayg'(\eta), \quad w = -\sqrt{av}\,[f(\eta) + g(\eta)], \quad \eta = \sqrt{\frac{a}{v}}\,z, \quad s = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}$$

where prime denotes the differentiation with respect to η, while T_∞ and T_w are the ambient temperature and wall temperature, respectively. The continuity equation (1) is identically satisfied and the transformed ordinary differential equations (2) to (4) are

$$f''' - \left[f'^2 - (f+g)f'' + Mf'\right] - K\left[(f+g)f^{iv} + (f''-g'')f' - 2(f'+g')f'''\right] = 0 \tag{7}$$
$$g''' - \left[g'^2 - (f+g)g'' + Mg'\right] - K\left[(f+g)g^{iv} + (f'-g')g'' - 2(f'+g')g'''\right] = 0 \tag{8}$$
$$\frac{1}{\Pr}s'' + (f+g)s' = 0 \tag{9}$$

subject to the boundary conditions (5) which become

$$f(0) = 0, \quad g(0) = 0, \quad f'(0) = 1, \quad g'(0) = c, \quad s(0) = 1,$$
$$f'(\infty) = 0, \quad g'(\infty) = 0, \quad f''(\infty) = 0, \quad g''(\infty) = 0, \quad s(\infty) = 0 \tag{10}$$

where $K = k_0 a/v$ is the dimensionless material or viscoelastic parameter, $M = \sigma B_0^2/\rho a$ is the dimensionless magnetic parameter, $c = b/a$ is the dimensionless stretching ratio, and Pr is the Prandtl number. It is worth mentioning that K=0 describes the classical Navier–Stokes equations for a viscous and incompressible fluid.

When $c = 0$ ($g = 0$) the problem reduced to the two-dimensional case, described by

$$f''' - f'^2 + ff'' - Mf' - K[ff^{iv} + f''^2 - 2f'f'''] = 0 \tag{11}$$

and when $c = 1$ ($f = g$), the problem reduced to the axisymmetric flow and the new Eqs.(7) and (8) are given by

$$f''' - f'^2 + 2ff'' - Mf' - 2K[ff^{iv} - 2f'f'''] = 0 \tag{12}$$

Equations (11) and (12) are subjected to the new transformed boundary conditions given by

$$f(0) = 0, f'(0) = 1, s(0) = 1, f'(\infty) = 0, f''(\infty) = 0, s(\infty) = 0 \tag{13}$$

The physical quantities of interest are the skin friction coefficient C_f on the surface along the x and y directions, which are denoted by C_{fx} and C_{fy}, respectively, and the local Nusselt number Nu_x which are defined as

$$C_{fx} = \frac{\tau_{wx}}{\rho u_w^2}, \quad C_{fy} = \frac{\tau_{wy}}{\rho u_w^2}, \quad Nu_x = \frac{x q_w}{\alpha (T_w - T_\infty)} \tag{14}$$

where τ_w and q_w are the wall shear stress and the wall heat flux, respectively. Thus, the wall shear stresses along the x and y directions are denoted by τ_{wx} and τ_{wy}, respectively. Using the variables (6), we obtain

$$\mathrm{Re}_x^{1/2} C_{fx} = f''(0), \quad \mathrm{Re}_x^{1/2} C_{fy} = \frac{1}{c}\frac{v_w}{u_w} g'(0), \quad \mathrm{Re}_x^{-1/2} Nu_x = -s'(0), \tag{15}$$

where Re_x is the local Reynolds number which is defined by $\mathrm{Re}_x = u_w x/\nu$.

3. RESULTS AND DISCUSSION

The nonlinear ordinary differential equations (7) to (9) subject to the boundary conditions (10) have been solved numerically using the implicit finite-difference scheme known as the Keller-box method as discussed and described in the book by Cebeci and Bradshaw (1988), for several values of parameters, namely the material parameter K, the magnetic parameter M, the stretching ratio c and the Prandtl number Pr. The values of the step size $\Delta\eta$ in η and the edge of boundary layer, η_∞, have to be adjusted for different values of the parameters to maintain accuracy. Throughout this study, we consider the value of $\Delta\eta = 0.02$ and it has been found to be satisfactory for a convergence criterion of 10^{-5} which gives four decimal places accuracy. On the other hand, the edge of the boundary layer is chosen to be between 5 to 10.

Tables 1 and 2 present the numerical results obtained for the wall skin friction coefficients $-f''(0)$, $-g''(0)$ and the local Nusselt number $-s'(0)$ for various values of the magnetic paramater M, viscoelastic parameter K and the stretching ratio c for Pr = 0.7 and 10, respectively. It is worth pointing out that the entire values of the skin friction coefficients $f''(0)$ and $g''(0)$ given in Table 1 are negative. Physically, the negative sign of the skin friction coefficient corresponds to the surface exerts a drag force on the fluid. For a specific value of c, it is seen that as K increases, the magnitude of the skin friction coefficient decreases and the local Nusselt number increases. The increase in the parameter M will lead to the increase of the magnitude of the skin friction coefficient. The opposite trend is observed for the local Nusselt number. It is also seen that as the sheet (surface) is stretched (given by the parameter c), the magnitude of the skin friction coefficients in both x and y directions and the local Nusselt number

will increase. The presence of the viscoelasticity in the fluid (given by the parameter *K*) also increases the magnitude of the skin friction coefficients and the local Nusselt number as the surface is stretched.

Table1. Values of $-f''(0)$ and $-g''(0)$ for different values of *K* and *M* when *c* = 0, 0.5 and 1.

M	K	c = 0	c = 0.5		c = 1
		$-f''(0)$	$-f''(0)$	$-g''(0)$	$-g''(0)$
0	0	1.0042	1.0932	0.4653	1.1748
	0.2	0.9225	0.9291	0.4066	0.9444
	1.0	0.7504	0.6513	0.2943	0.6461
10	0	3.3165	3.3420	1.6459	3.3667
	0.2	3.0276	2.8048	1.3840	2.6317
	1.0	2.3452	1.9175	0.9482	1.6667
100	0	10.0498	10.0582	5.0208	10.0663
	0.2	9.1742	8.4315	4.2096	7.8471
	1.0	7.1063	5.7552	2.8741	4.9551

Table 2. Values of the local Nusselt number $-s'(0)$ for several values of *K* and *M* when Pr = 0.7 and 10.

M	K	Pr = 0.7			Pr = 10		
		c = 0	c = 0.5	c = 1	c = 0	c = 0.5	c = 1
0	0	0.4909	0.5789	0.6800	2.3067	2.8609	3.3078
	0.2	0.5007	0.6099	0.7228	2.3243	2.8941	3.3587
	1.0	0.5221	0.6693	0.7867	2.3606	2.9538	3.4256
10	0	0.3497	0.2907	0.4584	1.8085	2.3572	2.8309
	0.2	0.3582	0.3278	0.5018	1.8682	2.4629	2.9807
	1.0	0.3902	0.4246	0.6027	2.0379	2.6663	3.1973
100	0	0.2353	0.2221	0.2739	0.8906	1.2415	1.5471
	0.2	0.2386	0.2317	0.2919	0.9842	1.4711	1.9430
	1.0	0.2499	0.2614	0.3433	1.1925	1.8513	2.4453

Figures 1 to 5 show the effects of the viscoelastic or material parameter *K* on the fluid flow and heat transfer characteristics, namely $f(\eta)$, $g(\eta)$, $f'(\eta)$, $g'(\eta)$ and $s(\eta)$, respectively, when c = 0.5, Pr = 0.7, M = 0 (without magnetic field) and *M* = 10. It is found from Figures 1 and 2 that the magnitudes of *f* and *g* increase by increasing the value of the fluid parameter *K* for a specific value of the magnetic parameter *M*, but this change is larger in *f* when compared to *g*. The presence of the magnetic field *M* in the fluid decreases the values of *f* and *g*. Furthermore, increasing the value of *K* will increase the boundary layer thickness. The same trend is also observed for the $f'(n)$ and $g'(\eta)$ profiles as

depicted by Figures 3 and 4. However, the opposite trend is observed from Figure 5 for the temperature profiles . It is seen that as K increases, the temperature profiles decrease, and thus, the thickness of thermal boundary layer also decreases. It is also found in this figure that the profiles increase with magnetic effect.

On the other hand, Figures 6 to 10 show the effects of the magnetic parameter M on the fluid flow and heat transfer characteristics, namely $f(\eta)$, $g(\eta)$, $f'(\eta)$, $g'(\eta)$ and $s(\eta)$, respectively, when c=0.5, Pr=0.7 and K=1. As illustrated in Figures 6 to 9 that, as M increases, the velocity profiles decrease and hence the effects of the magnetic parameter towards the flow will contribute to the decrease of the boundary layer thickness. The changes in f and f' almost double the changes of g and g' as shown in Figures 6 and 7 and 8 and 9, respectively. However, the opposite trend is observed again from Figure 10 for the temperature profiles $s(\eta)$. It is seen that as M increases, the temperature profiles also increase, and thus, the thickness of thermal boundary layer also increases. It is seen in all figures that the material or viscoelastic parameter K and the magnetic parameter M presents opposite effects on the fluid flow and heat transfer characteristics.

Finally, it is worth mentioning that all the profiles presented in Figures 1 to 10 satisfy the boundary conditions (10), and thus support the numerical results obtained.

4. CONCLUSION

In the present study, we have investigated theoretically the three-dimensional MHD flow and heat transfer of a viscoelastic fluid over a stretching surface. Numerical computation has been carried out to study the effects of the material or viscoelastic parameter K, the magnetic parameter M, the stretching ratio parameter c and the Prandtl number Pr on the skin friction coefficients, the local Nusselt number, as well as the velocity and temperature profiles. Results are presented in tables and figures for certain parameter conditions. It is found that the material parameter K and the magnetic parameter M present opposite effects on the fluid flow and heat transfer characteristics.

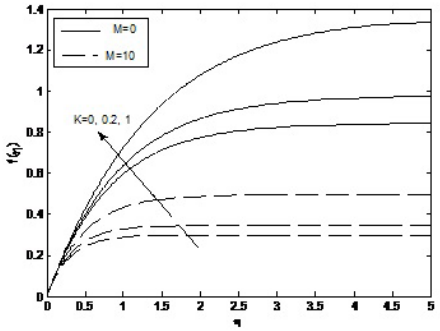

Figure 1. Variations of *f(η)* for various values of *K* when *c*=0.5, Pr=0.7, *M*=0 and 10.

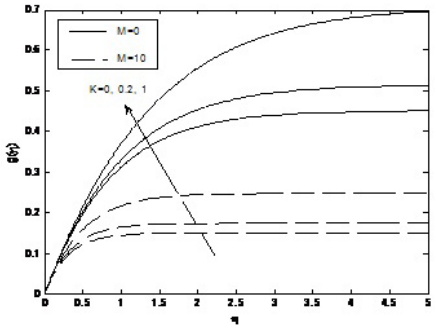

Figure 2. Variations of *g(η)* for various values of *K* when *c*=0.5, Pr=0.7, *M*=0 and 10.

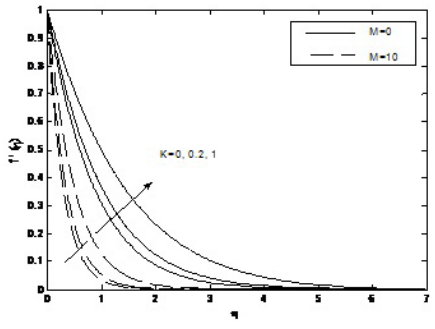

Figure 3. Variations of *f'(η)* for various values of K when c=0.5, Pr=0.7, M=0 and 10.

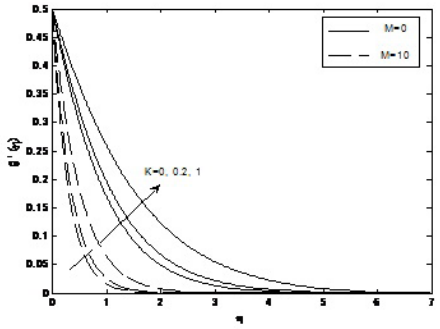

Figure 4. Variations of *g'(η)* for various values of *K* when *c*=0.5, Pr=0.7, *M*=0 and 10.

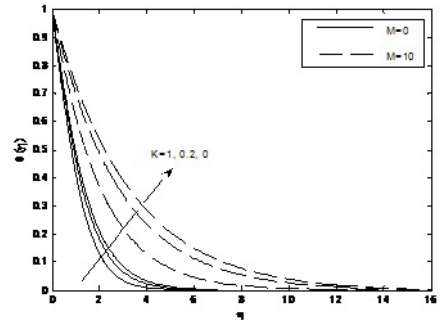

Figure 5. Variations of *s(η)* for various values of *K* when *c*=0.5, Pr=0.7, *M*=0 and 10.

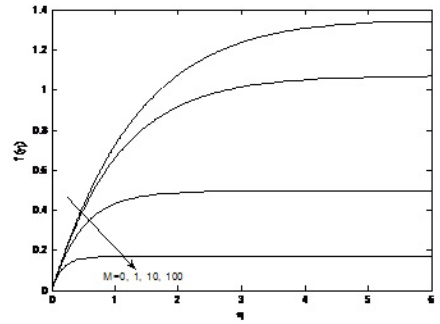

Figure 6. Variations of *f(η)* for various values of *M* when *c*=0.5, Pr=0.7, *K*=1

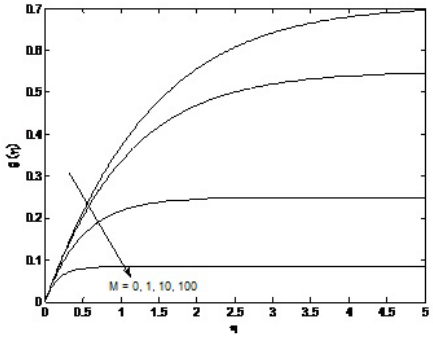

Figure 7. Variations of *g(η)* for various values of *M* when *c*=0.5, Pr=0.7, *K*=1

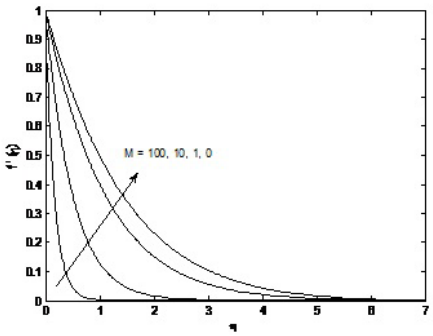

Figure 8. Variations of *f ′(η)* for various values of *M* when *c*=0.5, Pr=0.7, *K*=1

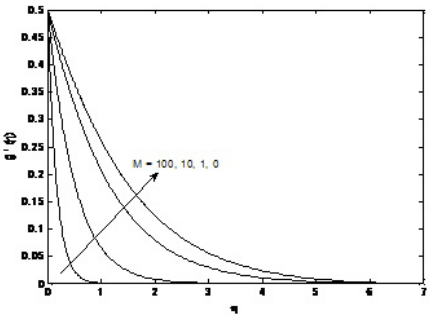

Figure 9. Variations of *g′(η)* for various values of *M* when *c*=0.5, Pr=0.7, *K*=1

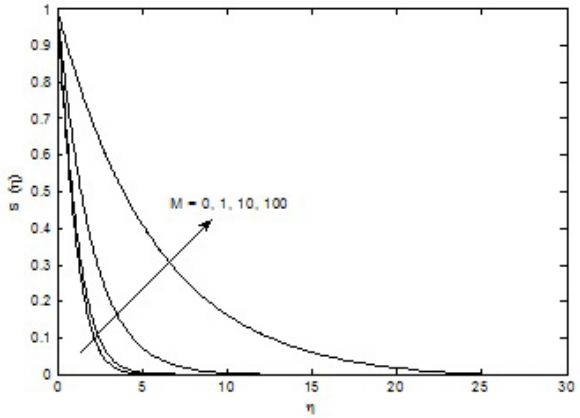

Figure 10. Variations of $s(\eta)$ for various values of M when c=0.5, Pr=0.7 and K=1.

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