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Stress-Strain State Analyses of the Composite Steel and Concrete Grid Structure

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Abstract: The study provides developing a method of the tress-strain state analyses of the composite steel and concrete grid structure, the main assumptions, and analysis of the geometry of the structure, the elementary states of equilibrium of the structures are considered, and the systems of determining equations are solved. The method of the stress-strain state analyses is proposed to be performed using the basic provisions of the technical theory of calculating thin shells. The method provides obtaining and solving the design models, and the equations needed for calculating the bearing capacity of the members of the composite steel and concrete grid structure. In particular, the method is for determining the essential area of the cross-section of the prestressed reinforcement, as well as to determine the forces that are acted in the composite steel and concrete structure. In general, the method provides designing the composite steel and concrete structure, determining the bearing capacity, and selecting the necessary dimensions of the structural elements of the composite steel and concrete structure.

Keywords: Stress-strain state, steel and concrete structures, grid structures

1. Introduction

Grid composite steel and concrete structures are relatively new types of structures that began to be developed and researched in the second half of the last century. The works of the authors and other researchers are dedicated to the study of structures, methods of calculation, analysis, improvement of the design, study of the influence of various types of load on the stress-strain state, etc. (Lapenko et al. 2020, Gasii et al. 2020, Johnson 2018). A detailed analysis of existing designs in comparison with the grid composite steel and concrete structures is given in the work (Storozhenko et al. 2020, Storozhenko & Gasii 2020).

In design, the grid composite steel and concrete structures are similar to traditional structural structures. The design experience indicates that the existing approaches to the calculation sometimes lead to significant overspends of materials and construction enlargement. Therefore, when designing long-span structures, it is advisable to take into account structural features.

The design of the structure, experimental research, and analysis are given in detail in previous works (Storozhenko et al. 2020), and therefore in the current work, the authors do not stop about it but solve the important issue of the calculation method, which is acceptable for use by engineers for grid composite steel and concrete structures designing.

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2. Materials and Methods

The composite steel and concrete grid structure (Fig. 1) can be erected and operated in such a way as to ensure full use of the strength properties of materials due to its features behavior under load. Therefore, to perceive the bending moments from the action of the external load, in the direction perpendicular to the cross-section (Z0X plane), the composite steel and concrete outside units that have openings (Fig. 2) through which the prestressed reinforcement is passed (following Ukrainian standards bar reinforcement of classes A600, A800, A1000, Vr1200, Vr1300, Vr1400, Vr1500, rope K1400 (K–7), K1500 (K–7), K1500 (K–19)). Prestressing of reinforcement is carried out by tensioning on concrete (Fig. 3) by force P.

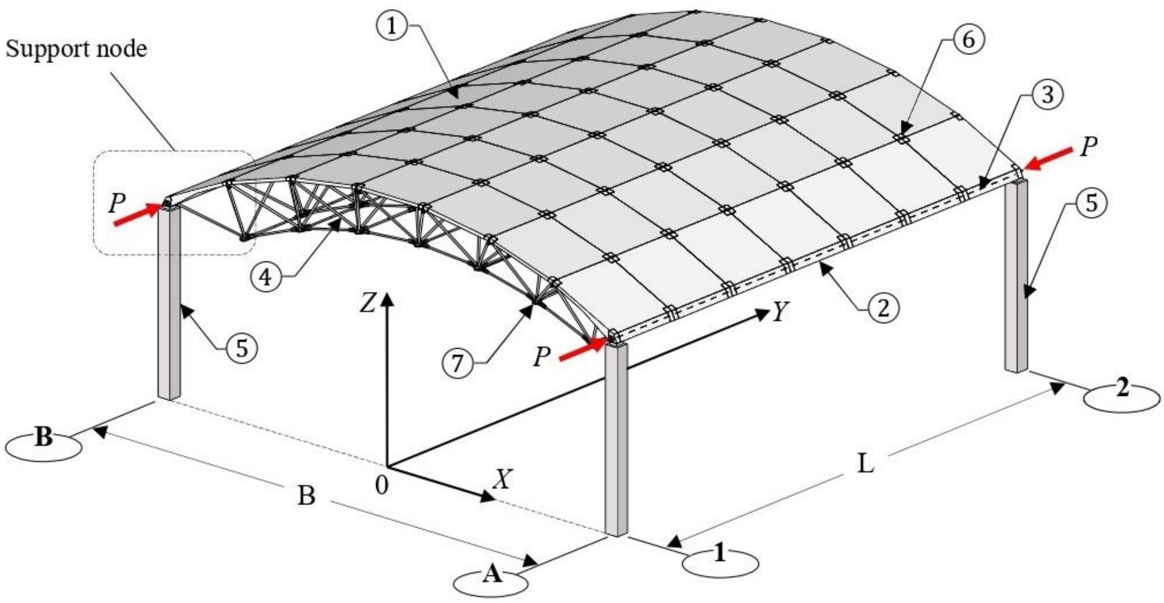

Fig. 1 - Design model of the composite steel and concrete grid structure

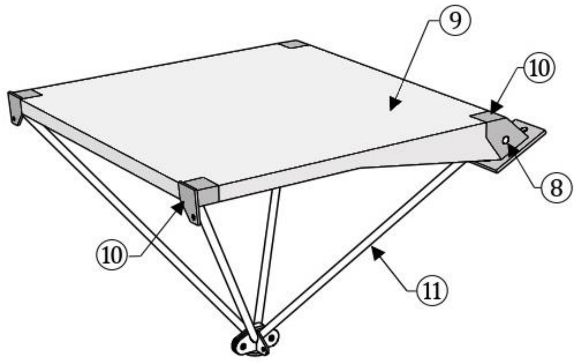

Fig. 2 - The composite steel and concrete outside unit with openings through which the prestressed reinforcement is passed

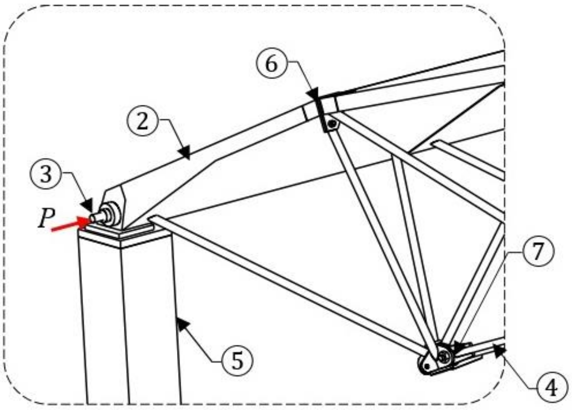

Fig. 3 - Prestressing of reinforcement by tensioning on concrete by force *P*

Numbers in Fig. 1, 2, and 3 show 1 – the composite steel and concrete unit; 2 – the composite steel and concrete outside unit (board element) with an opening for the passage of prestressed reinforcement; 3 – prestressed reinforcement; 4 – grid, which performs the function of a diaphragm; 5 – support; 6, 7 – connection nodes of the composite steel and concrete units along the top and bottom chords; 8 – openings for passage of prestressed reinforcement; 9 – reinforced concrete slab; 10 – steel embedded parts; 11 – tubular rods.

The calculation of the composite steel and concrete grid structure, taking into account its spatial work, is proposed to be performed using the basic provisions of the technical theory of calculating thin shells (Pietraszkiewicz & Konopińska 2015, Calladine 1988, Gol'Denveizer 2014, Palyvoda & Lapenko 2019). According to the theory, the composite steel and concrete grid structure is considered one for which the hypothesis of straight normals is valid, i.e., a rectilinear element perpendicular to the middle surface of the reinforced concrete slab before deformation remains straight and perpendicular to the deformed middle surface without changing its length. At the same time, normal stresses on sections parallel to the middle surface are considered so insignificant compared to other perpendicular ones that they can be neglected. Using the specified prerequisites, a set of internal forces acting on an infinitely small element is determined (Fig. 4).

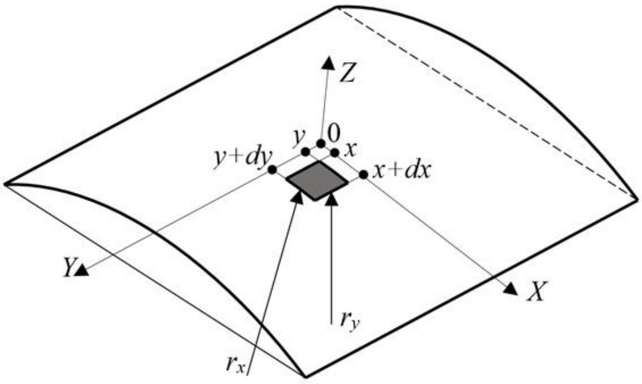

Fig. 4 - Selection of an infinitesimally small element of the composite steel and concrete grid structure

In the considered infinitesimal element with dimensions $dx \times dy$ (Fig. 5), the following internal forces will generally arise from the action of the external load q: bending moments M_x and M_y, normal compressive forces N_x and N_y, shear forces S_x and S_y, transverse forces V_x and V_y, torques H_x and H_y.

External load is according to the formula (1).

$$q = g + \upsilon \tag{1}$$

where g and υ – permanent and temporary loads, respectively.

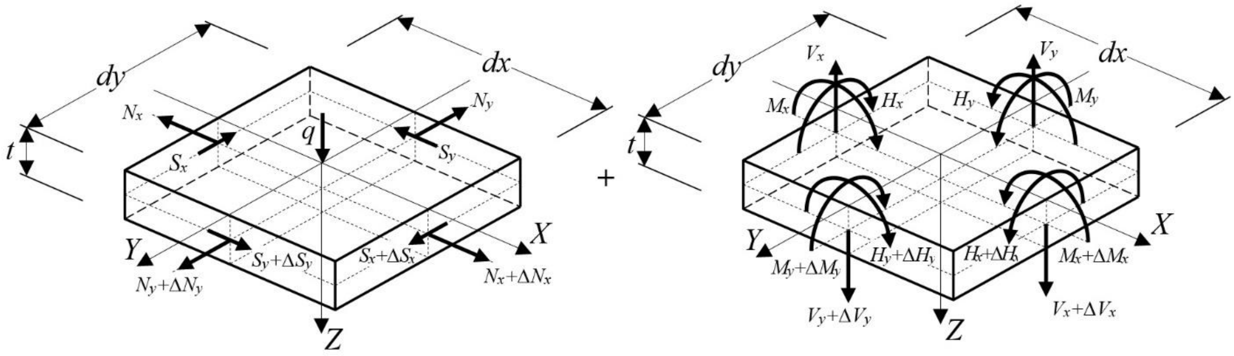

Fig. 5 - Normal and shear and transverse force, bending, and torque moments, which act on an infinitesimally small element (consider forces simultaneously)

Fig. 5 shows forces, which act on one element at the same time, but for convenience and clarity, they are conditionally shown on different.

The forces shown in Fig. 5 are found according to the formulas:

$$\Delta N_x = \left(\frac{\partial N_x}{\partial x}\right) dx \text{ and } \Delta N_y = \left(\frac{\partial N_y}{\partial y}\right) dy; \tag{2}$$

$$\Delta S_x = \left(\frac{\partial S_x}{\partial x}\right) dx \text{ and } \Delta S_y = \left(\frac{\partial S_y}{\partial y}\right) dy; \tag{3}$$

$$\Delta M_x = \left(\frac{\partial M_x}{\partial x}\right) dx \text{ and } \Delta M_y = \left(\frac{\partial M_y}{\partial y}\right) dy; \tag{4}$$

$$\Delta H_x = \left(\frac{\partial H_x}{\partial x}\right) dx \text{ and } \Delta H_y = \left(\frac{\partial H_y}{\partial y}\right) dy. \tag{5}$$

On the element $dx \times dy$, these forces, if denoted as $F(x;\ y)$, will have an increase of $\frac{\partial F_x}{\partial x} \cdot dx$ and $\frac{\partial F_y}{\partial y} \cdot dy$ (Fig. 5) In the composite steel and concrete grid structure, the generatrix is a gentle curve with a radius of curvature r_x. For such constructions, we can assume that $S_x = -S_y = S$ and $H_x = -H_y = H$. At the same time, it is assumed that the indicated forces acting on the infinitesimal element $dx \times dy$ are in equilibrium.

3. Results and Discussion

It is possible to determine the internal forces M_x, M_y, N_x, N_y, H and S, using physical, geometric relations and expressions of transverse forces V_x and V_y through bending M_x and M_y, and torque H moments, with the system of equations (6), which is obtained according to Fig. 5.

$$\begin{cases} \dfrac{\partial N_y}{\partial y} + \dfrac{\partial S}{\partial x} = 0; \\ \dfrac{\partial N_x}{\partial x} + \dfrac{\partial S}{\partial x} = 0; \\ \dfrac{\partial^2 M_y}{\partial y^2} + 2 \cdot \dfrac{\partial^2 H}{\partial y dx} + \dfrac{\partial^2 M_x}{\partial x^2} + \dfrac{N_x}{r_x} + \dfrac{N_y}{r_y} + q = 0; \\ \dfrac{M_y}{r_y} + \dfrac{M_x}{r_x} + \dfrac{D}{B} \cdot \left(\dfrac{\partial^2 N_x}{\partial y^2} - 2 \cdot \dfrac{\partial^2 S}{\partial y dx} + \dfrac{\partial^2 N_y}{\partial x^2} \right) = 0. \end{cases} \quad (6)$$

In the last system of equations (6), the cylindrical stiffness D is calculated according to the formula

$$D = E_c \cdot t^3 / 12, \quad (7)$$

and stiffness B according to the formula

$$B = E_c \cdot t. \quad (8)$$

The analysis of the system of differential equations of equilibrium of the infinitesimal element (6) shows that its solution concerning the specified forces even under the accepted simplifications is a time-consuming task. Therefore, it is impractical to use it in engineering practice. To simplify the equation of the system, the influence of various factors on the above-mentioned forces that occur in the reinforced concrete part (top chord) of the composite steel and concrete grid structure was experimentally obtained. Experimental (Storozhenko et al. 2018), as well as theoretical studies, showed that of the total evenly distributed load of intensity q applied to the top chord of the composite steel and concrete grid structure, only 4–9% affects the values of the component equations M_x, M_y, H, V_x and V_y, which corresponds to the stress-strain state during bending. The rest 91–96% of the load q falls on the components N_x, N_y, S_x and S_y, and this corresponds to the «momentless» stress-strain state. Based on such data, it was concluded that the load q, evenly distributed over the surface of the top chord of the composite steel and concrete grid structure, mainly leads to a «momentless» stress-strain state, that is, the stress-strain state of the structure is characterized mainly by normal N_x and N_y and shear S forces.

The results presented above indicate that the following prerequisites must be observed for the practice of using the composite steel and concrete grid structure:

• the line describing the composite steel and concrete grid structure along the bend (plane Z0X, Fig. 1) should be gentle and thin;

• the thickness of the reinforced concrete part (top chord) of the composite steel and concrete grid structure should change smoothly;

• the load perceived by the composite steel and concrete grid structure must be applied continuously without sudden changes;

• fastening of the composite steel and concrete grid structure must allow the movement of its supporting part.

The specified prerequisites allow accepting the assumption that differential equations include $D = 0$, $M_x = 0$, $M_y = 0$ and $H = 0$. Then the system of differential equations (6) will turn into the system (9).

$$\begin{cases} \dfrac{\partial N_y}{\partial y} + \dfrac{\partial S}{\partial x} = 0; \\ \dfrac{\partial N_x}{\partial x} + \dfrac{\partial S}{\partial x} = 0; \\ \dfrac{N_x}{r_x} + \dfrac{N_y}{r_y} + q = 0. \end{cases} \quad (9)$$

It should be noted that this simplification is a consequence of the condition when the steel structural grid of the composite steel and concrete grid structure prevents the occurrence of significantly larger deflections than the thickness t of the reinforced concrete part (top chord).

The accepted assumptions and operating conditions of the composite steel and concrete grid structure make it possible to calculate its bearing capacity using the limit equilibrium method. Then the forces N_x, N_y and S can be determined from equation (9) by introducing the stress function $F(x;\ y)$ in such a way as to obtain the following expressions:

$$N_x = \frac{\partial^2 F}{\partial y^2};\ N_y = \frac{\partial^2 F}{\partial x^2};\ S = \frac{\partial^2 F}{\partial x \partial y}. \tag{10}$$

If we substitute dependencies (10) into the system of differential equations (9), then the first two equations will turn into identities, and the third will have the form of Poisson's equation

$$\frac{1}{r_x} \cdot \frac{\partial^2 F}{\partial y^2} + \frac{1}{r_y} \cdot \frac{\partial^2 F}{\partial x^2} + q = 0. \tag{11}$$

From equation (11), can be found the function $F(x;\ y)$ with the help of which forces N_x, N_y and S are determined from formulas (10).

For the practice of designing the composite steel and concrete grid structure, taking into account the fact that for an infinitely small element $r_y \Rightarrow \infty$ it is advisable to take $1/r_y = 0$. This allows calculating the composite steel and concrete grid structure in the transverse and longitudinal directions separately. At the same time, the final calculation in the transverse direction (plane Z0X, Fig. 1) for shear forces S and bending moments M_x is performed for the condition of equilibrium of the elementary transverse strip of the composite steel and concrete grid structure, and the calculation of the bearing capacity in the longitudinal direction is based on the condition (12).

$$M_{Ed} \le M_{yu}, \tag{12}$$

where

$$M_{Ed} = q \cdot B \cdot L^2 / 8. \tag{13}$$

$$M_{yu} = 2 \cdot \int_0^{\Theta_c} f_{cd} \cdot dA_c \cdot r_x \cdot \cos\Theta - f_{yd} \cdot A_s \cdot z, \tag{14}$$

where $dA_c = t \cdot r_x \cdot sin\, d\Theta$ (Fig. 6); for small values of $\Theta \to sin\,\Theta = \Theta$.

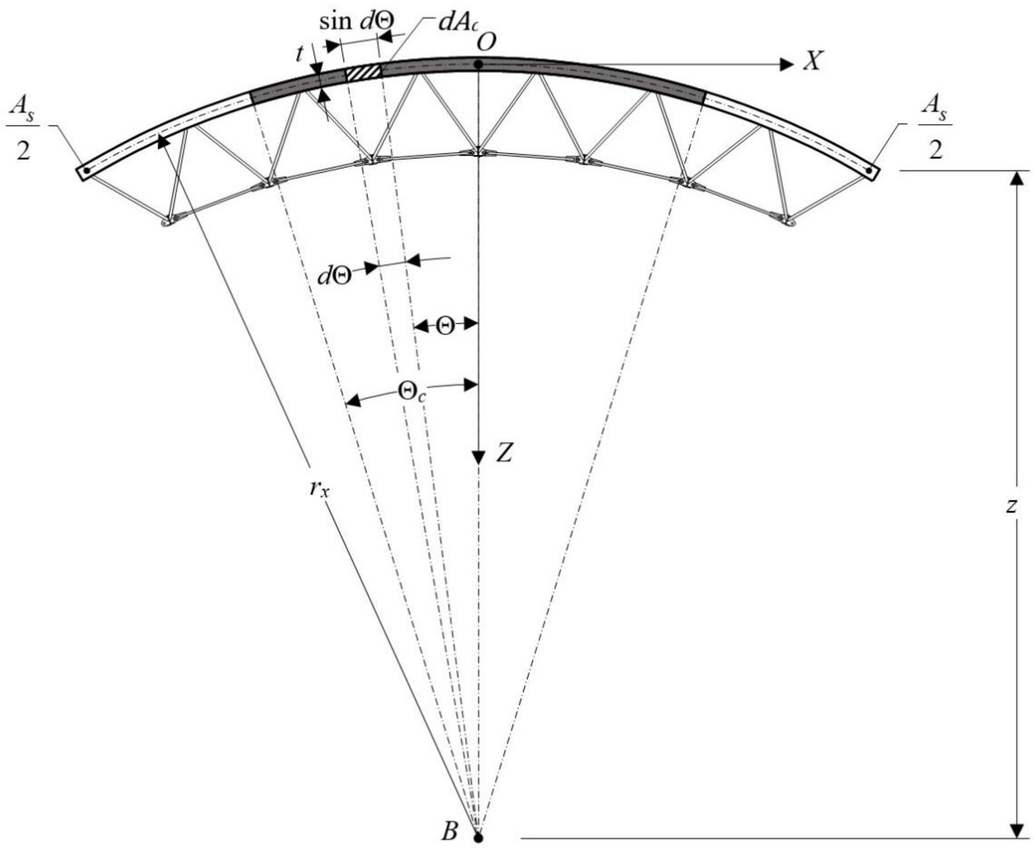

Fig. 6 - The design model of the elementary transverse strip of the composite steel and concrete grid structure

Then

$$dA_c = t \cdot r_x d\Theta \tag{15}$$

If expression (15) is substituted into formula (14), the first one is turned into (16).

$$M_{yu} = 2 \cdot f_{cd} \cdot r_x^2 \cdot t \cdot \int_0^{\Theta_c} \cos\Theta d\Theta - f_{yd} \cdot A_s \cdot z \tag{16}$$

and after transformation

$$M_{yu} = 2 \cdot f_{cd} \cdot r_x^2 \cdot t \cdot \sin\Theta_c - f_{yd} \cdot A_s \cdot z \tag{17}$$

The position of the neutral line is determined by the angle Θ_c, to calculate the value of which, the formula can be obtained from the equation ΣY=0.

$$N_y = N_{cu} = 2 \cdot \int_0^{\Theta_c} f_{cd} \cdot dA_c = 2 \cdot \int_0^{\Theta_c} f_{cd} \cdot t \cdot r_x \cdot d\Theta = 2 \cdot f_{cd} \cdot t \cdot r_x \cdot \Theta_c = f_{yd} \cdot A_s. \tag{18}$$

To calculate the bearing capacity of the composite steel and concrete grid structure, it is necessary to determine Θ_c from equation (18).

$$\Theta_c = \frac{f_{yd} \cdot A_s}{2 \cdot f_{cd} \cdot t \cdot r_x}. \tag{19}$$

After substituting formula (19) into (17), the last one turns into an expression

$$M_{yu} = 2 \cdot f_{cd} \cdot r_x^2 \cdot t \cdot \sin\left(\frac{f_{yd} \cdot A_s}{2 \cdot f_{cd} \cdot t \cdot r_x}\right) - f_{yd} \cdot A_s \cdot z \tag{20}$$

The necessary reinforcement area A_s, which ensures the bearing capacity of the composite steel and concrete grid structure under the action of the external bending moment M_{Ed} is determined by the combined solution of equations (17) and (18) with condition (12).

$$\sin \Theta_c - z \cdot \frac{\Theta_c}{r_x} - \frac{M_{Ed}}{2 \cdot f_{cd} \cdot t \cdot r_x} = 0 \tag{21}$$

The value of the angle Θ_c is determined from equation (21).

Then, if the angle Θ_c substitute into dependence (19), the formula for determining the required area of reinforcement A_s will have the view

$$A_s = \frac{2 \cdot f_{cd} \cdot t \cdot r_x \cdot \Theta_c}{f_{yd}}. \tag{22}$$

Bending moments acting in the plane of the cross-section (plane Z0X in Fig. 6) of the composite steel and concrete grid structure are proposed to be determined for a strip of unit width (Fig. 7).

Design dependences are obtained for the case of action on the composite steel and concrete grid structure of a vertically uniformly distributed load q (1). As can be seen from Fig. 7, except for the load q, tangential forces T and $T + \Delta T$ also act on a strip of unit width. The last one is located in the planes of conventional intersection. Using the relationship between forces T and V, the formula is obtained

$$\Delta T = -\frac{V \cdot S}{2 \cdot I \cdot B} + \frac{(V + \Delta V) \cdot S}{2 \cdot I \cdot B} = \frac{\Delta V \cdot S}{2 \cdot I \cdot B}, \tag{23}$$

where ΔV is the increase in the transverse force in the area under consideration;

S – the static moment of the cross-section of the composite steel and concrete grid structure.

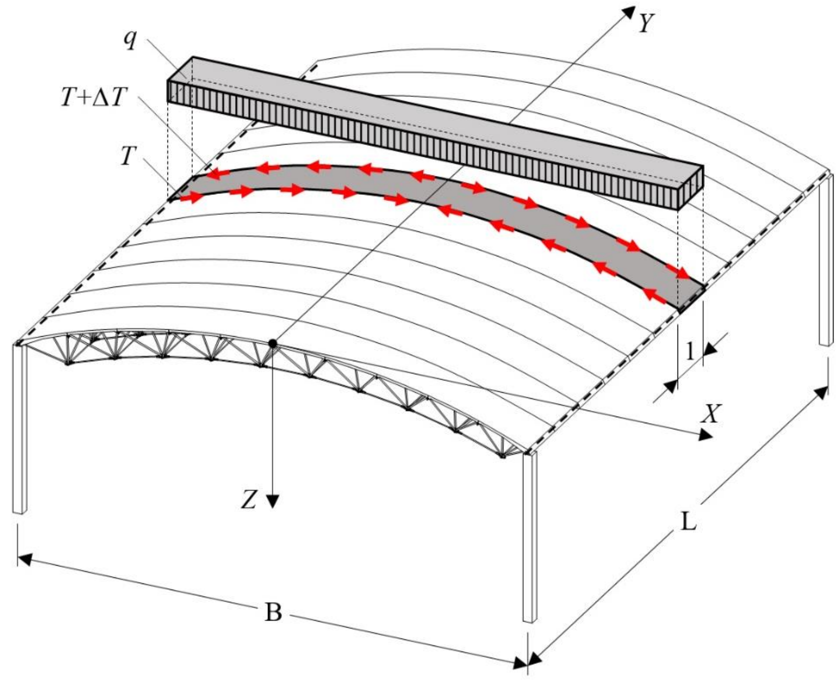

Fig. 7 - Design model for calculating the bearing capacity of the composite steel and concrete grid structure in the transverse direction

The bending moment M_x in the transverse direction can be determined if the condition of static equilibrium of the forces acting in it is applied (Fig. 8).

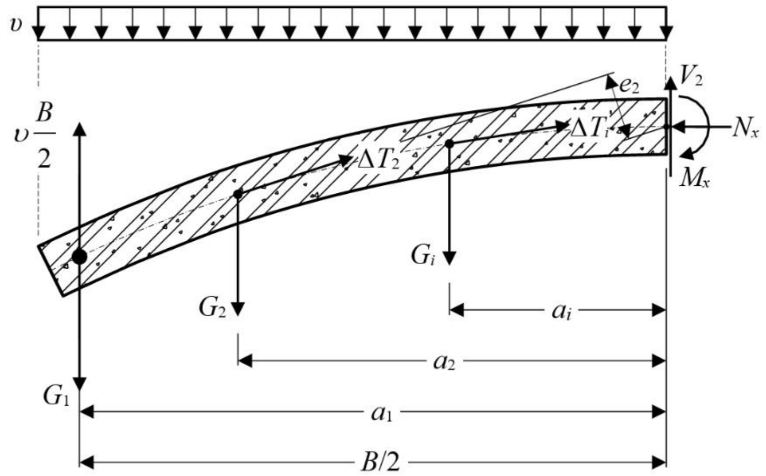

Fig. 8 - Design model for calculating the structure in the transverse direction

Fig. 8 shows that

$$M_x = M_0 + M_{\Delta T}, \tag{24}$$

where M_0 is the moment from the external load and the mass of the composite steel and concrete grid structure

$$M_0 = \sum_{i=1}^{n} G_i \cdot a_i + \upsilon \cdot \frac{B^2}{8}, \tag{25}$$

where G_i is the load from the mass of the i-th section of the composite steel and concrete grid structure;

$M_{\Delta T}$ – the bending moment due to shear forces relative to the section under consideration

$$M_{\Delta T} = \sum_{i=1}^{n} \Delta T_i \cdot t \cdot e_i, \quad (26)$$

where e_i is the eccentricity of applying the shear force ΔT_i.

4. Conclusion

Therefore, based on the results of the research, a method for analyzing the stress-strain state of the composite steel and concrete structure was developed. The method of stress-strain state analysis is developed based on the technical theory of thin shells, which allows making some assumptions to simplify mathematical dependencies and their further use for engineering practice, that is, for design. The presented method allows for determining the required cross-sectional area of the prestressed reinforcement in the outside elements, as well as to determine the basic forces acting on the composite steel and concrete structure, which in turn makes it possible to design the composite steel and concrete structure, determine the bearing capacity and select the necessary dimensions of the structural elements of the composite steel and concrete structure, etc.

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References

Lapenko, O., Baranetska, D., Makarov, V., & Baranetskyi, A. (2020). Designing of Structural Construction and Orthotropic Slabs from Steel Reinforced Concrete. In *Materials Science Forum* (Vol. 1006, pp. 173-178). Trans Tech Publications Ltd.

Gasii, G., Hasii, O., & Klimenko, V. (2020). Testing of the combined structural elements of support of a mine opening. In *E3S Web of Conferences* (Vol. 168, p. 00028). EDP Sciences.

Storozhenko, L., & Gasii, G. (2020). Experience and current issues of designing of steel and concrete composite structures of roof and floor systems. *Industrial Machine Building, Civil Engineering*, *2*(55), pp. 15-25.

Johnson, R. P. (2018). *Composite Structures of Steel and Concrete: beams, slabs, columns and frames for buildings*. John Wiley & Sons.

Pietraszkiewicz, W., & Konopińska, V. (2015). Junctions in shell structures: a review. *Thin-Walled Structures*, *95*, 310-334.

Calladine, C. R. (1988). The theory of thin shell structures 1888–1988. *Proceedings of the Institution of Mechanical Engineers, Part A: Power and Process Engineering*, *202*(3), 141-149.

Gol'Denveizer, A. L. (2014). *Theory of elastic thin shells: solid and structural mechanics* (Vol. 2). Elsevier.

Palyvoda, O., & Lapenko, O. (2019). Study of joint work of shell and core of tube confined concrete elements with strengthened core. In *Materials Science Forum* (Vol. 968, pp. 258-266). Trans Tech Publications Ltd.

Storozhenko, L., Yermolenko, D., & Gasii, G. (2018). Investigation of the deformation state of a composite cable space frame structures with a photogrammetric method. *International Journal of Engineering and Technology(UAE)*, *7*(3.2), 442-446.

Storozhenko, L., Gasii, G., Hohol, M., & Hasii, O. (2020, June). Preparation Technique of Experimental Specimens of Steel and Concrete Composite Slabs. *Lecture Notes in Civil Engineering*, *181*, pp. 147-154. Springer, Cham.